Optimized Resource Management in Heterogeneous Wireless Networks

S. M. Ahsan Kazmi, Nguyen H. Tran, *Member, IEEE*, Walid Saad, *Senior Member, IEEE*, Long Bao Le, *Senior Member, IEEE*, Tai Manh Ho, *Student Member, IEEE*, Choong Seon Hong, *Senior Member, IEEE*

***Abstract*—The dense and pervasive deployment of wireless small cells can boost the performance of existing macrocellular networks; however, it poses significant challenges pertaining to the cross-tier interference management. In this paper, the downlink resource allocation problem for an underlay small cell network is studied. In this network, the protection of the macrocell tier is achieved by imposing cross-tier interference constraints in the resource allocation problem. To solve the underlying mixed-integer resource allocation problem, we propose two different algorithms. The first algorithm is developed by applying the duality-based optimization approach for the relaxed problem, which enables distributed implementation. The second distributed algorithm which enables coordination is devised based on matching theory. Simulation results show that the proposed duality-based algorithm outperforms the greedy approach by 4% in terms of sum-rate whereas the matching-based algorithm with tier-coordination yields performance gains up to 17% compared with the duality-based approach.**

***Index Terms*—resource allocation, heterogeneous networks, matching games, wireless small cell networks.**

I. Introduction

One promising solution for improving the capacity of wireless networks is via the dense deployment of small cell base stations (SBSs). However, effective operation of such SBSs requires meeting several technical challenges including resource allocation (RA), interference management (IM), and emerging network engineering issues.

A centralized approach using a greedy algorithm for RA has been proposed in [1]; however, this requires heavy message passing and suffers from scalability issues for densely deployed SBSs. In [2], another distributed scheme has been proposed which achieves a sub-optimal RA solution. However, this scheme may not be suitable for dense heterogeneous networks (HetNets) because of high signaling overhead required to establish reference users for individual SBSs. A distributed RA scheme with macro-tier protection is proposed in [3] which is shown to converge to a centrally calculated solution. However, this approach has slow convergence, which may not be desirable in dense small cell network.

We address these challenges by introducing two novel algorithms for RA and IM. Both proposed distributed algorithms are designed to simultaneously enable protection for the macrocell tier and to operate in large-scale dense networks. Furthermore, unlike existing algorithms such as in [1], [2], and [3], the proposed matching-based approach achieves efficient coordination between network tiers with *minimal message passing* compared to previous works [1], [2]. Our main contributions can thus be summarized as follows:

- We formulate the RA problem with the macro-tier protection as a mixed-integer optimization problem.
- To solve this problem, we develop a duality-based RA algorithm that can achieve a near optimal solution by uniformly assigning cross-tier interference budget for the SBSs. This approach enables distributed implementation and affordable computational complexity.
- By exploiting non-uniform cross-tier interference using a tier-coordination approach, the network performance can be improved. However, a duality-based approach with tier-coordination induces heavy information exchange [3]. Therefore, based on matching theory, we propose a second distributed approach with limited information exchange.
- Numerical results show that, the proposed duality-based algorithm outperforms the greedy approach by 4% in terms of sum-rate and the use of matching theory can significantly improve the overall network sum-rate. This performance advantage can reach up to 17% compared to the duality-based optimization approach.

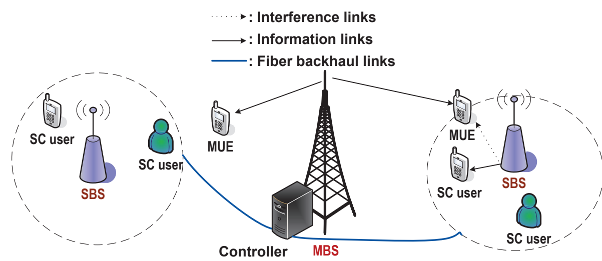

Fig. 1: Proposed system model. Solid line showing the downlink information links while dotted line showing the cross tier interference.

II. System Model and Problem Definition

Consider a HetNet consisting of a set of SBSs, $\mathcal{B} = \{1, 2, ..., J\}$, located within the coverage of one macrocell base station (MBS) as shown in Fig. 1. The set of macrocell users (MUEs) and small cell users (SUEs) are denoted by $\mathcal{M} = \{1, 2, ..., M\}$ and $\mathcal{S} = \{1, 2, ..., S\}$, respectively. The MBS and SBSs use the same set of orthogonal resources $\mathcal{R} = \{1, 2, ..., R\}$.[1] However, for any given resource $r \in \mathcal{R}$, a predefined interference threshold $I^r_{\max}$ must be maintained for protecting the MUEs.

A. Resource Allocation and Link Models

We assume that all SBSs transmit using a fixed power (e.g., any feasible power for each SBS transmitter) [2]. However, each SBS can have its own, and different power budgets. In addition, we assume that the transmit power of each SBS is equally divided among its resources and, thus, the interference

This research was supported by Basic Science Research Program through National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2014R1A2A2A01005900). Dr. CS Hong is the corresponding author.

S. M. A. Kazmi, N. H. Tran, T. M. Ho, and CS Hong are with the Department of Computer Science and Engineering, Kyung Hee University, Korea (e-mails: ahsankazmi, nguyenth, hmtai, cshong@khu.ac.kr).

W. Saad is with Wireless@VT, Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA, USA (e-mail: walids@vt.edu).

L. B. Le is with the Institute National de la Recherche Scientifique (INRS), Universite du Quebec, Montreal, QC, Canada (e-mail: long.le@emt.inrs.ca).

[1] One resource corresponds to one subcarrier or subchannel of the LTE network [3].

power on each resource is constant. For RA optimization, we introduce binary variables $x_{j,k}^r$, as follows:

$$x_{j,k}^r = \begin{cases} 1, & \text{if SUE } k \text{ in SBS } j \text{ is assigned resource } r, \\ 0, & \text{otherwise.} \end{cases}$$

We always set $x_{j,k}^r = 0$ for any SUE k, which is not associated with SBS j. The received SINR pertaining to the transmission of SBS j to SUE k over resource r with transmit power P_j^r is:

$$\gamma_{j,k}^r = \frac{P_j^r g_{j,k}^r}{P_M^r g_{M,k}^r + \sum\limits_{i \in \Omega_r,} P_i^r g_{i,k}^r + \sigma^2}, \tag{1}$$

where P_M^r and $P_i^r, \forall i \in \Omega_r$, represent the transmit powers of the MBS and SBS, respectively, in the set Ω_r which are using resource r. The channel gain between SBS j and SUE k is $g_{j,k}^r$ whereas $g_{M,k}^r$ and $g_{i,k}^r$ are, respectively, the channel gains from the MBS and other underlay SBSs i to SUE k. The noise power is assumed to be σ^2. Then, the data rate of user k associated with SBS j on resource r is given by $R_{j,k}^r = W^r \log(1+\gamma_{j,k}^r)$ where W^r is the bandwidth of resource r.

B. Problem Statement

Our objective is to maximize the sum rate of all SBSs by reusing the macrocell resources. The data rate achieved by an SBS j over all allocated resources is:

$$R_j = \sum\nolimits_{r \in \mathcal{R}} \sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r W^r \log(1+\gamma_{j,k}^r). \tag{2}$$

Moreover, the interference experienced by MUE m on resource r is given by $I^r = \sum_{j \in \mathcal{B}} \sum_{k \in \mathcal{S}} x_{j,k}^r P_j^r g_{j,m}^r$, where $g_{j,m}^r$ is the channel gain between SBS j and MUE m, on resource r. Note that the binary RA variables $x_{j,k}^r$ ensure that we only account for the interference created by SUEs that are assigned the same resource. The considered RA problem can be stated as follows:

$$\textbf{P1:} \underset{x_{j,k}^r \in \mathcal{X}, \forall k,j,r}{\text{maximize}} \quad \sum\nolimits_{j \in \mathcal{B}} R_j \tag{3}$$

$$\text{subject to} \quad \sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r \leq 1, \ \forall r \in \mathcal{R}, \forall j \in \mathcal{B}, \tag{4}$$

$$I^r \leq I_{\max}^r, \ \forall r \in \mathcal{R}. \tag{5}$$

In **P1**, constraint (4) ensures that each resource can be allocated to at most one user in each SBS to avoid strong intra-cell interference ; additionally, constraint (5) ensures the MUE protection by keeping its aggregate interference below a predefined threshold. Problem **P1** is a non-convex, integer problem, which is difficult to solve for a practical setting with large sets of users and resources [4]. Typically, solutions presented for problems similar to **P1** requires significant message exchanges [2], [3]. Therefore, by using optimization and matching theory, we present two distributed *novel and practical* algorithms with minimal message passing (i.e., no message exchange in Alg. 1 due to relaxation and only SBS proposals to MBS in Alg. 2) which are suitable for a large-scale dense networks of SBSs.

III. Optimization-based Resource Allocation

A. Problem Relaxation and Dual Decomposition

To develop a practical distributed algorithm for the RA problem, we decompose the original problem into multiple problems which can be solved at individual SBSs. Toward this end, we relax the coupled interference constraint (5) of **P1** by dividing the interference threshold into J parts corresponding to J SBSs [5]. This guarantees that each SBS is allocated the same cross-tier interference budget on each resource. Note that more complex designs can allocate different cross-tier interference budgets for different SBSs; however, such design would require heavy message passing among SBSs, which is impractical in dense HetNets. Thus, the decomposed problem can be stated as follows:

$$\textbf{P2:} \underset{x_{j,k}^r \in \mathcal{X}, \forall k,r}{\text{maximize}} \quad R_j \tag{6}$$

$$\text{subject to} \quad \sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r \leq 1, \ \forall r \in \mathcal{R},$$

$$\sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r P_j^r g_{j,m}^r \leq I_{\max}^r / J, \ \forall r \in \mathcal{R}.$$

The partial Lagrangian with respect to the interference constraint of **P2** can be presented as $L(\{x_{j,k}^r\}, \alpha_k^r) = \sum_{r \in \mathcal{R}} L_r(x_{j,k}^r, \alpha_k^r)$, where α_k^r represents the interference price, and $L_r(x_{j,k}^r, \alpha_k^r)$ is equal to:

$$\sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r W^r \log(1+\gamma_{j,k}^r) - \alpha_k^r \Big(\sum\nolimits_{k \in \mathcal{S}} x_{j,k}^r P_j^r g_{j,m}^r - \frac{I_{\max}^r}{J}\Big).$$

The dual function $g(.)$ is then given as

$$g(\alpha_k^r) = \begin{cases} \underset{x_{j,k}^r \in \{0,1\} \forall k, \forall r}{\text{maximize}} & L(x_{j,k}^r, \alpha_k^r) \\ \text{subject to} & \sum_{k \in \mathcal{S}} x_{j,k}^r \leq 1. \end{cases} \tag{7}$$

Algorithm 1 Optimization-based distributed RA

initialize: $t = 0$, $\alpha_k^r(0) \geq 0$, step-size $\kappa_r(0) > 0$;
repeat
$t \leftarrow t+1$
Each SBS j updates $x_{j,k}^r$ for its SUEs k and α_k^r as follows:

- $x_{j,k}^r(t+1) = \begin{cases} 1, & \text{if } r = r^* \text{and } k = k^*, \\ 0, & \text{otherwise} \end{cases}$

where, $r^* = \arg\max\limits_{r \in R} (W^r \log(1+\gamma_{j,k}^r) - \alpha_k^r(t) P_j^r g_{j,m}^r)$,
$k^* = \arg\max\limits_{k \in \mathcal{S}} (W^{r^*} \log(1+\gamma_{j,k}^{r^*}) - \alpha_k^{r^*}(t) P_j^{r^*} g_{j,m}^{r^*})$;

- $\alpha_k^r(t+1) = \alpha_k^r(t) - \kappa_r(t)(\sum\limits_{k \in \mathcal{S}} x_{j,k}^r(t) P_j^r g_{j,m}^r - I_{\max}^r / J)$,

where, $\kappa_r(t) > 0$ is a step-size.
if $x_{j,k}^r = 1$ and $P_j^r g_{j,m}^r > I_{\max}^r / J$ **then**
  $x_{j,k}^r = 0$
until $\alpha_k^r(t+1) - \alpha_k^r(t) \leq \epsilon$

B. Algorithm Design

Based on the above analysis, we propose, in Alg. 1, an optimization-based distributed resource allocation algorithm. In this algorithm, the updates of the interference price $\alpha_k^r(t)$ can be conducted in a distributed way since these updates only require information on the channel gain $g_{j,m}^r$, which can be obtained from the underlying SBS via the MBS. This requires a relatively small exchange of information compared to the previous works in [1], [2], and [3]. The convergence can be proved using the gradient-based standard technique [4], and it converges to a near optimal solution due to the simplified uniform allocation of the cross-tier interference budget [5].

IV. Matching-based Resource Allocation

Although Algorithm 1 provides a low-complexity solution, it requires relaxing the interference constraints to eliminate coordination between tiers and maintain the IM process. This can degrade the network performance. To improve it, a certain coordination between network tiers for non-uniform cross-tier interference allocation is required. Hence, a central controller can be implemented at the MBS to enable coordination among macro-SBS tiers [1] and to maintain the interference limit $I_{\max}^r$. In the presence of coordination, we propose a second solution approach based on matching theory [6].

A. Matching theory preliminaries

The RA problem can be formulated as a *two-sided matching game*. We assume each SUE can use a single resource. However, different SBSs can use the same resource to improve the spectrum efficiency. Our design corresponds to a *many-to-one matching*[7] given by the tuple $(\mathcal{B}, \mathcal{R}, q_r, \succ_{\mathcal{B}}, \succ_{\mathcal{R}})$. Here, $\succ_{\mathcal{B}} \triangleq \{\succ_j\}_{j\in\mathcal{B}}$ and $\succ_{\mathcal{R}} \triangleq \{\succ_r\}_{r\in\mathcal{R}}$ represent the set of the preference relations of the SBSs and resources, respectively.

Definition 1: A *matching* μ is defined by a function from the set $\mathcal{B} \cup \mathcal{R}$ into the set of elements of $\mathcal{B} \cup \mathcal{R}$ such that: (i) $|\mu(j)| \leq 1$ and $\mu(j) \in \mathcal{R}$, (ii) $|\mu(r)| \leq q_r$ and $\mu(r) \in \mathcal{B} \cup \phi$, where q_r is the quota of r, and (iii) $\mu(j) = r$ if and only if j is in $\mu(r)$.

1) Preferences of the players: Matching is performed on the basis of preference profiles that can be built by the SBSs $\mathcal{P}_j$ and the controller $\mathcal{P}_r$ to rank potential matchings based on the local information. Note that, on each r, each SBS j will choose its user k with highest data rate $R_j^r = \max_k R_{j,k}^r$. Then, an SBS j ranks a resource r based on the following preference function:

$$\mathcal{U}_j(r) = R_j^r. \tag{8}$$

Similarly, for the controller side, each resource r also ranks the SBSs according to the following preference function:

$$\mathcal{U}_r(j) = R_j^r - \beta I_j^r, \tag{9}$$

where $I_j^r = P_j^r g_{j,m}^r$ represent interference produced by SBS j to the MUE assigned that resource. The first term in (9), represents the achievable data rate on resource r, the second term accounts for a penalty due to the interference produced by SBS j, and β represents a weight parameter. The second term implies that the controller gives less utility to the SBSs which cause higher interference to the MUE on resource r.

For the formulated two-sided matching game, our goal is to seek a *stable matching*, which is a key solution concept [8]. To find a stable matching, the deferred-acceptance algorithm can be employed [8]. Traditionally, in one-to-many matching, a fixed, per player quota on one side is assumed according to which a fixed number of players of the opposite side can be matched. However, our formulated matching game involves a *dynamic quota* as the controller allows a number of SBSs (with heterogeneous interference) to use each resource as long as the interference constraint on that resource is not violated. This heterogeneous interference of SBSs and dynamic quota of resources introduces new challenges that prevent the use of standard deferred-acceptance algorithm. Therefore, we formally define the blocking pair for the formulated game as follows:

Definition 2: A pair (j, r) is a *blocking pair* for μ if:

a) $I_{res}^r \geq I_j^r,\ j \succ_r \emptyset$ and $r \succ_j \mu(j)$,

b) $I_{res}^r < I_j^r,\ I_{res}^r + \sum_{j'\in\mu(r)} {I_{j'}}^r \geq I_j^r$,
$j \succ_r j'$ and $r \succ_j \mu(j)$,

where $I_{res}^r = I_{max}^r - I^r$ represent the residual of the interference tolerance (remaining quota) on the resource r. *The quota of a resource $r \in \mathcal{R}$ is filled when $I_{res}^r < I_j^r$ for a requesting $j \in \mathcal{B}$.* Definition 2 is based on the following intuition [9]. Whenever an SBS j prefers a resource r to its assigned resource $\mu(j)$, if either: i) r has sufficient interference tolerance I_{res}^r and is willing to admit j (i.e., $j \succ_r \emptyset$), or ii) its quota is filled but it is able to admit j by rejecting some accepted SBSs which are ranked lower than j, then j and r can deviate from their assigned $\mu(j)$ and $\mu(r)$, respectively. A matching is stable if no blocking pair exists.

Algorithm 2 Matching-based distributed RA

1: **input**: $\mathcal{P}_j$, $\mathcal{P}_r$, $\forall r, j$
2: **initialize**: $t = 0$, $\mu^{(t)} \triangleq \{\mu(j)^{(t)}, \mu(r)^{(t)}\}_{j\in\mathcal{B}, r\in\mathcal{R}} = \emptyset$, ${I_{res}^r}^{(t)} = I_{\max}^r$, ${\mathcal{K}_r}^{(t)} = \emptyset$, ${\mathcal{P}_j}^{(0)} = \mathcal{P}_j$, ${\mathcal{P}_r}^{(0)} = \mathcal{P}_r$, $\forall r, j$
3: **repeat**
4: $t \leftarrow t + 1$
5: **for** $r \in \mathcal{R}$ **do**
6: **for** $j \in \mathcal{B}$ with r as its most preferred in ${\mathcal{P}_j}^{(t)}$ **do**
7: **while** $j \notin \mu(r)^{(t)}$ and $\mathcal{P}_j^{(t)} \neq \emptyset$ **do**
8: **if** ${I_{res}^r}^{(t)} \geq I_j^r$, **then**
9: $\mu(r)^{(t)} \leftarrow \mu(r)^{(t)} \cup \{j\}$; ${I_{res}^r}^{(t)} \leftarrow {I_{res}^r}^{(t)} - I_j^r$;
10: **else**
11: ${\mathcal{P}'}_r^{(t)} = \{j' \in \mu(r)^{(t)} | j \succ_r j'\}$
12: $j_{lp} \leftarrow$ the least preferred $j' \in {\mathcal{P}'}_r^{(t)}$;
13: **while** $({\mathcal{P}'}_r^{(t)} \neq \emptyset) \cup ({I_{res}^r}^{(t)} < I_j^r)$ **do**
14: $\mu(r)^{(t)} \leftarrow \mu(r)^{(t)} \setminus \{j'\}$; ${\mathcal{P}'}_r^{(t)} \leftarrow {\mathcal{P}'}_r^{(t)} \setminus \{j_{lp}\}$;
15: ${I_{res}^r}^{(t)} \leftarrow {I_{res}^r}^{(t)} + I_{j'}^r$;
16: $j_{lp} \leftarrow$ the least preferred $j' \in {\mathcal{P}_r'}^{(t)}$;
17: **if** ${I_{res}^r}^{(t)} \geq I_j^r$, **then**
18: $\mu(r)^{(t)} \leftarrow \mu(r)^{(t)} \cup \{j\}$; ${I_{res}^r}^{(t)} \leftarrow {I_{res}^r}^{(t)} - I_j^r$;
19: **else**
20: $j_{lp} \leftarrow j$;
21: ${\mathcal{K}_r}^{(t)} = \{k \in {\mathcal{P}_r}^{(t)} | j_{lp} \succ_r k\} \cup \{j_{lp}\}$
22: **for** $k \in {\mathcal{K}_r}^{(t)}$ **do**
23: ${\mathcal{P}_k}^{(t)} \leftarrow {\mathcal{P}_k}^{(t)} \setminus \{r\}$; ${\mathcal{P}_r}^{(t)} \leftarrow {\mathcal{P}_r}^{(t)} \setminus \{k\}$;
24: **until** $\mu^{(t)} = \mu^{(t-1)}$
25: **output**: $\mu^{(t)}$

2) Proposed algorithm: As a solution to this game, we propose a novel RA scheme to produce a stable matching in Alg. 2 which guarantees macro-tier protection captured in constraint (5). At each iteration t, each r receives proposals from unassigned SBSs j that rank r as the highest in ${\mathcal{P}_j}^{(t)}$(lines 5-7). i) If r has sufficient quota ${I_{res}^r}^{(t)}$ to admit j, it accepts the proposal and updates ${I_{res}^r}^{(t)}$ and $\mu(r)^{(t)}$(lines 8-9). ii) Otherwise, if the quota of r is filled, then r finds all of its current matched j' which have a lower ranking than j according to ${\mathcal{P}_r}^{(t)}$(lines 10-11). Each least preferred SBS $j_{lp} \in {\mathcal{P}'}_r^{(t)}$ is then sequentially rejected, and ${I_{res}^r}^{(t)}$, ${\mathcal{P}'}_r^{(t)}$, and j_{lp} are updated until j can be admitted or there is no additional j' to reject (lines 12-16). After rejecting all $j' \in {\mathcal{P}'}_r^{(t)}$, if r still has an insufficient quota to admit j, then j is rejected and j is set to the j_{lp} (lines 17-20). Finally, the controller removes j_{lp} and its less preferred SBSs from the ${\mathcal{P}_r}^{(t)}$, and similarly these SBSs also remove r from their respective ${\mathcal{P}_j}^{(t)}$(lines 21-23). *With this process, we guarantee that any less preferred SBS will not be accepted by that resource even if it has sufficient quota to do so, which is crucial for the matching stability of our design.* This process is repeated until the matching converges (line 24).

Theorem 1: Alg. 2 converges to a stable allocation.

Proof: We prove this theorem by contradiction. Assume that Alg. 2 produces a matching μ with a blocking pair (j, r) by Definition 2. Since $r \succ_j \mu(j)$, j must have proposed to r and has been rejected due to interference violation on r (lines 19-20). When j was rejected, then j' was rejected either before j (lines 13-16), or was made unable to propose because r is removed from j' preference list (lines 22-23). Thus, $j' \notin \mu(r)$, a contradiction. ∎

The output $\mu^{(t)}$ of Alg. 2 can be transformed to a feasible allocation vector $\mathcal{X}$ of problem **P1** (line 25). Note that, the worst case running time complexity of Alg. 2 is *linear* in the size of input preference profiles (i.e., $\mathcal{O}(JR)$ where J and R represent SBSs and resources, respectively) similar to Alg. 1 which also has a *linear* complexity (i.e., $\mathcal{O}(KR)$, where K represents the number of SUEs).

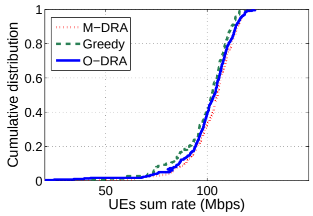

Fig. 2: Rate at $I^r_{\max} = -60$dBm.

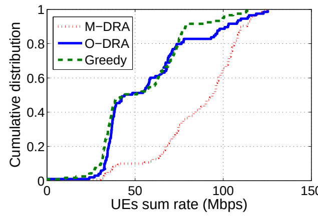

Fig. 3: Rate at $I^r_{\max} = -100$dBm.

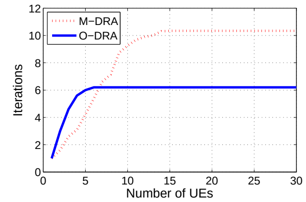

Fig. 4: Average number iterations of M-DRA vs O-DRA, for different number of users.

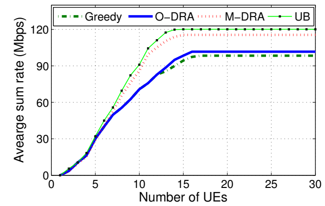

Fig. 5: Comparison of average sum rate of O-DRA, M-DRA, and Greedy with UB.

V. Numerical Results

For our simulations, we consider a network with 5 SBSs each of which supports 3 UEs, and 5 MUEs using 5 resources. All users are randomly located inside the coverage of an MBS which has a radius of $r_1 = 1000$ m, whereas the coverage distance of each small cell is $r_2 = 100$ m. The bandwidth of each resource W^r is set equal to 1 and the weighting parameter β is set to a normalized value of 1, whereas the background noise power is assumed to be -90 dBm. The channel power gain is modeled as $g^r_{j,k} = 10^{(-L(d_{j,k}))/10}$, where $L(d_{j,k})$ represents the path loss and $d_{j,k}$ is the distance between BS j and user k. We assume that $L(d_{M,k}) = 16.62 + 37.6\log_{10}(d_{M,k})$ for the channel gain from the MBS to UE k and $L(d_{j,k}) = 37 + 32\log_{10}(d_{j,k})$ for the channel gain from SBS j to UE k. The SBSs transmit with varying power over simulation runs ranging from 15 dBm to 23 dBm. For comparison purposes, we compare the proposed algorithms with a centralized greedy scheme that sequentially allocates resources to users in each SBS until the interference constraint is violated. All results are obtained by averaging over a large number of independent simulation runs, each of which realizes random locations of base stations, users, and channel power gains. Results corresponding to the optimization-based, matching-based, and greedy algorithms are denoted as “O-DRA”, “M-DRA”, and “Greedy”, respectively.

In Figs. 2 and 3, we compare the sum rate of SBSs achieved by different schemes for two different interference thresholds $I^r_{\max} = -60$ and -100 dBm. It can be observed that both proposed algorithms and greedy algorithm result in indistinguishable performance when $I^r_{\max} = -60$ dBm. However, for decreased macro-tier interference threshold ($I^r_{\max} = -100$ dBm), lower sum rate of the SBSs can be achieved since the interference protection constraint becomes stricter. Moreover, the matching approach outperforms the optimization-based approach in terms of the sum rate at this lower interference protection threshold. This is because the optimization approach splits the interference budget $I^r_{\max}$ uniformly among all SBSs and does not coordinate with the macro-tier. Moreover, each SBS may prevent its UEs from using resource r if the uniformly assigned interference limit is violated. On the other hand, in the matching-based approach, the controller performs coordination between the network tiers and, thus, it only rejects the least preferred SBS-UE pair from the set of potential SBS-UE pairs.

Fig. 4 compares the average number of iterations required by both M-DRA and O-DRA versus the number of users (i.e., network size) as $I^r_{\max} = -80$ dBm. We can see that, as the number of users increases, the average number of iterations also increases. Moreover, M-DRA has a reasonable convergence time that does not exceed an average of 11 iterations for all network sizes with 5 resources. Moreover, for O-DRA, the maximum number of iterations is smaller than 7 for all network sizes. This fast convergence time can be achieved due to a completely distributed design of O-DRA with no message passing.

In Fig. 5, the average sum rate of all UEs versus the number of UEs is shown for the proposed and greedy algorithms as $I^r_{\max} = -80$ dBm. Moreover, we use the upper bound (UB) of problem **P1** which is obtained by relaxing the binary indicator variable so that it can take any value in the range [0, 1] as a benchmark here. It can be inferred that the matching-based, optimization-based and Greedy approaches achieve up to 96.8%, 82.6%, and 80.2% of the average sum rate obtained by the UB, respectively for a network with 20 UEs. Thus, it is clear that the matching-based approach is close to optimal. Furthermore, it can be observed that the sum rate increases with more UEs, which, however, saturates as the number of UEs becomes sufficiently large. This is because of the limited number of resources at each SBS ($r = 5$). Additionally, the optimization-based approach achieves a performance benefit up to 4% compared to the greedy approach while the matching-based approach achieves 17% and 21% higher sum rate compared to the optimization-based and greedy approaches, respectively for a network with 20 UEs.

VI. Conclusion

In this paper, we have proposed two resource allocation algorithms for the two-tier heterogeneous network, namely the optimization-based and matching-based algorithms. Distributed implementation of these algorithms have also been discussed in details. Numerical studies have shown that the matching-based algorithm outperforms the optimization-based algorithm especially in the low macro-tier interference limit. However, the performance of both algorithms is almost indistinguishable as the macro-tier interference limit becomes sufficiently large.

References

[1] Y.-S. Liang, W.-H. Chung, G.-K. Ni, I.-Y. Chen, H. Zhang, and S.-Y. Kuo, “Resource allocation with interference avoidance in ofdma femtocell networks,” *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2243–2255, Jun 2012.

[2] K. Son, S. Lee, Y. Yi, and S. Chong, “Refim: A practical interference management in heterogeneous wireless access networks,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 6, pp. 1260–1272, 2011.

[3] A. Abdelnasser, E. Hossain, and D. I. Kim, “Tier-aware resource allocation in ofdma macrocell-small cell networks,” *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 695–710, 2015.

[4] S. Boyd and L. Vandenberhe, “Convex optimization,” *Cambridge University Press, 2004*.

[5] T. Ho, N. Tran, C. Do, S. Kazmi, E.-N. Huh, and C. Hong, “Power control for interference management and qos guarantee in heterogeneous networks,” *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1402–1405, Aug. 2015.

[6] Y. Gu, W. Saad, M. Bennis, M. Debbah, and Z. Han, “Matching theory for future wireless networks: fundamentals and applications,” *IEEE Commun. Mag.*, vol. 53, no. 5, pp. 52–59, May 2015.

[7] J. Sethuraman, C.-P. Teo, and L. Qian, “Many-to-one stable matching: Geometry and fairness,” *Mathematics of Operations Research*, vol. 31, no. 3, pp. 581–596, 2006.

[8] A. E. Roth, “Deferred acceptance algorithms: History, theory, practice, and open questions,” *Int. J. Game Theory*, vol. 36, no. 3-4, pp. 537–569, 2008.

[9] H. Xu and B. Li, “Anchor: A versatile and efficient framework for resource management in the cloud,” *IEEE Trans. Parallel Distrib. Syst.*, vol. 24, no. 6, pp. 1066–1076, Jun. 2013.