Multi-Dimensional Incentive Mechanism in Mobile Crowdsourcing with Moral Hazard

Yanru Zhang, *Member, IEEE,* Yunan Gu, Miao Pan, *Member, IEEE,* Nguyen H. Tran, *Member, IEEE,* Zaher Dawy, *Senior Member, IEEE,* and Zhu Han, *Fellow, IEEE*

Abstract—In current wireless communication systems, there is a rapid development of location based services, which will play an essential role in the future 5G networks. One key feature in providing the service is the mobile crowdsourcing in which a central cloud node denoted as the principal collects location based data from a large group of users. In this paper, we investigate the problem of how to provide continuous incentives based on user's performances to encourage users' participation in the crowdsourcing, which can be referred to the moral hazard problem in the contract theory. We not only propose the one-dimensional performance-reward related contract, but also extend this basic model into the multi-dimensional contract. First, an incentive contract which rewards users by evaluating their performances from multiple dimensions is proposed. Then, the utility maximization problem of the principal in both one-dimension and multi-dimension are formulated. Furthermore, we detailed the analysis of the multi-dimensional contract to allocate incentives. Finally, we use the numerical results to analyze the optimal reward package, and compare the principal's utility under the different incentive mechanisms. Results demonstrate that by using the proposed incentive mechanism, the principal successfully maximizes the utilities, and the users obtain continuous incentives to participate in the crowdsourcing activity.

Index Terms—Crowdsourcing, incentive mechanism, multi-dimension, moral hazard, contract theory.

1 Introduction

Nowadays, people are used to accessing various sophisticated location based services (e.g., Yelp and Google Map) by their smartphones via/through wireless access networks [1]. Most location based services are essentially based on crowdsourcing which is a technology that requires user to regularly transmit data to the for the service provider which is denoted as principal here after. The data is obtained by the embedded sensor such as GPS, accelerometer, digital compass, gyroscope, and camera, or users themselves [2]. Once the data is aggregated and processed by the principal, the location-based service is provided to the users for free or with purchase. The brief illustration of crowdsourcing is shown in Fig. 1. One well-known application is the live auto traffic map offered by Google. Smartphone users transmit the traffic information which includes the time, location, and velocity to Google. Google collects and processes the data to provide free live traffic map to mobile users [3].

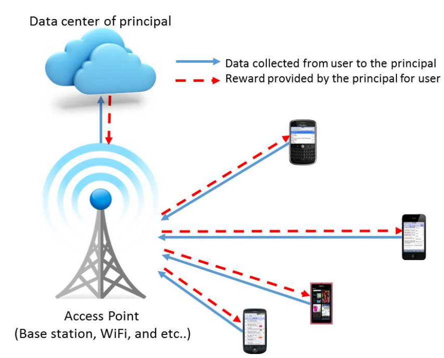


Fig. 1: An illustration of crowdsourcing.

With the drastic growth in the global location based service market, and the rapid development of big data technology, more data as well as user participation are required to support more sophisticated services [4]. Although the users receive the satisfaction from enjoying the location based service, there are many concerns that stop users from providing location based data for the principal. When participating in a crowdsourcing activity, users contribute their effort, time, knowledge and/or experience, and consume the battery power and computing capacity of their smartphones. In addition, the users expose their locations with potential privacy threats [5]. Hence, many users hesitate to participate in with those concerns, which becomes one of the serious impediments to the development of location based services

- *Y. Zhang (yrzhang@ie.cuhk.edu.hk) is with the Information Engineering Department, Chinese University of Hong Kong, New Territory, Hong Kong.*
- *Y. Gu (guyunan@huawei.com) is with the IP Technology Research Division of Huawei, Beijing, China.*
- *M. Pan (mpan2@uh.edu) is with the Department of Electrical and Computer Engineering, University of Houston, Houston, Texas 77004.*
- *N. H. Tran (nguyenth@khu.ac.kr) is with the Department of Computer Science and Engineering, Kyung Hee University, South Korea.*
- *Z. Dawy (zd03@aub.edu.lb) is with the Electrical and Computer Engineering Department, American University of Beirut, Beirut, Lebanon.*
- *Z. Han (zhan2@uh.edu) is with the University of Houston, Houston, TX 77004 USA, and also with the Department of Computer Science and Engineering, Kyung Hee University, Seoul, South Korea.*

Manuscript received October 03, 2015; revised September 05, 2016; accepted July 20 2017.

[6]. Thus, necessary incentive mechanisms that motivate the users to participate in crowdsourcing are needed to address those critical demands.

Many researches have already noticed that there is an urgent need to alleviate these challenges by providing incentive mechanisms to the users. For some kinds of location-based data, users are incentivized to upload data with simple rewards, such as allowing them to use the location-based app for free like Google Map. But this kind of incentive is not a solution for all problems. But for some kinds of crowdsourcing activities, the data collection process may require extra effort from users, instead of simply turn on the smartphone or app. For example, the recent popular app MoBike for public bike sharing encourages users to take a photo of the place where the bike has been parked after being used, also description of the location is preferred, which together serve as supplementary information despite the GPS. In this case, extra credit is needed to motivate users to help. For similar location based services, more complicated incentive mechanisms are needed to better drive users' action.

The design proposed in [7] is to give users a one-time reward after users have accomplished a certain task. A problem with this mechanism is its inability to provide continuous incentives to users to stay active after receiving the opening reward [8]. Inspired by the effort-based reward from the labor market, several works studied this problem by providing users with the amount of reward that is consistent with their performances. The work in [5] and [9] have derived the performance and reward dependent function for users that induces the maximum profit for the principal. In one of our previous works [10], we have also proposed a contract that includes user's current performance related reward and user's satisfaction from enjoying the free service in the reward package.

The works above capture the fundamental aspect of providing necessary incentive for user to participate in the crowdsourcing activity. However, beyond these insights, the simplified one-dimensional models are too abstract to capture the main features of the user's contributions, since users are supposed to work on several different tasks [11]. For example, a user's contribution to Yelp involves many dimensions and cannot adequately be reduced to a simple problem of effort choices. Users do not only make location based check-ins, upload photos, and write reviews for the restaurants and bars. But, they are also encouraged to invite new friends to sign up, and to give feedbacks and suggestions to Yelp for the website to determine the future overall strategy [12]. Generally speaking, in the real world crowdsourcing, the user's action set is considerably richer than in the previous literatures have described, and the variables in the contract can be conditioned on are much more difficult to specify or to observe precisely.

The complexity of real world scenarios makes one dimensional incentive mechanisms hard to adapt; in addition, other considerations also arise if we only reward users based on one aspect of the performance [13]. Still taking Yelp for example, suppose we introduce a mechanism that links user's reward to the number of his/her reviews, the advantage of this mechanism is that it provides an independent measure of the user's performance. But there is also an disadvantage that it measures only a part of what users are encouraged to contribute to the website. To put it in another way, if the crowdsourcing is a single-task problem, in which the only thing user needs to do is writing reviews, the quality of a review such as length, correctness, and objectiveness is not considered. If the crowdsourcing is a multi-task problem such as Yelp, the other tasks such as checking-ins, uploading photos, and inviting friends will be ignored. In a nutshell, there is a definite risk that this policy will induce users to overwhelmingly focus on the part that will be rewarded and to neglect the other components that can enrich the content of the crowdsourcing activity [14].

Thus, a qualified mechanism can both reward user's effort in a comprehensive way, and drive user's incentive to undertake actions that affect the principal's utility, in return. To capture the incentive problem in crowdsourcing, the one-dimension incentive mechanism needs to be modified into a number of dimensions. At the very least the user's action set must include the range of different tasks it is responsible for. Furthermore, performance measures must be multi-dimension rather than one-dimension for all, so that the principal can drive user's incentives by assigning different reward weights on different tasks [15].

Based on this motivation, we aim at offering a contract that considers different aspects of user's contributions, and assigns different reward weights on their performance in order to incentivize them to provide high quality information to the principal. Fortunately, the moral hazard problem from contract theory provides us a useful tool to design such a mechanism that can solve the employees' multi-dimension action problems when performing multiple tasks [16]. Indeed, the moral hazard model can be adopted to solve the crowdsourcing incentive problem. From the principal's perspective, it "employs" the users to upload location based data and reward them by multi-dimension measures. The principal makes profit by extracting useful information from the collected data, which also incurs a cost such as the reward given back to users. Thus, to maximize its own payoff, the principal needs to find an optimal mechanism that can properly reward user's efforts and drive user's incentives [17].

The main contributions of this paper are summarized as follows. First, we are first to propose a performance and reward consistent contract to maximize the principal's utility as well as to provide users with a continuous incentive to participate in crowdsourcing activities. Second, we extend the incentive mechanism from one-dimension to multi-dimension, which characterizes the general situation in real world and provides comprehensive reward package to the users. Last, through simulations, we reveal different parameter's impacts on the optimal reward package, and compare the principal utility under six different incentive mechanisms. Our results show that by using the proposed incentive mechanism, the principal successfully maximizes the utilities and the users obtain the continuous incentives to participate in the crowdsourcing activity.

The remainder of this paper is organized as follows. First, we will introduce the network model in Section 2. Then, the problem formulation is described in Section 3, and we give the extended analysis of the multi-dimensional case. The performance evaluation is conducted in Section 4.

Finally, Section 5 draws the conclusion.

2 System Model

In this section, we will first introduce the principal-user model by constructing the reward package offered by the principal. Then, we will give the utility functions of both the user and principal before proceeding to the solution of the optimal contract. We assume that the crowdsourcing is a multi-task problem, in which there are n tasks that the user can work on and will be rewarded based on its performances on the different tasks.

2.1 Operation Cost

When crowdsourcing for the principal, the user encounters an operation cost which includes the consumption of power due to signal processing, execution, and data uploading activities, in addition to power consumption due to data transmission. But the operation cost does not only restrict to the power consumption, but also the user's effort, time, knowledge and/or experience. Consider a user who participates in a crowdsourcing activity who makes a one-time choice of a vector of efforts $a = (a_1, \ldots, a_n)$, $n \geq 1$, for those tasks. When exerting efforts, the operation cost incurred is defined in a quadratic form [18],

$$\psi(a) = \frac{1}{2} a^T C a, \tag{1}$$

where C is a symmetric $n \times n$ matrix with the form of

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix}. \tag{2}$$

The diagonal element c_{ii} of C reflects the user's task-specific operation cost coefficient, and the off-diagonal elements c_{ij} represent the relationship between two tasks i and j.

The sign of c_{ij} indicates technologically substitute, complementary, independent between two tasks i and j, if $c_{ij} > 0, < 0, = 0$, respectively. If two tasks are technologically substitute, raising the effort on one task raises the marginal operation cost of the effort on the other task. The example of technologically substitute is dynamic route planning and traffic jam detection. When the roads are detected as highly congested, the navigation app will start to recalculate the route so that the driver can avoid them. Thus, more power is consumed. In contrast, raising the effort on one task decreases the marginal operation cost of the effort on the other task if they are technologically complementary. There are two examples for technologically complementary: 1) mapping GPS traces to road segments and route/travel time estimation, 2) traffic jam detection and visualization. In both examples, good achievements in one task ease the work in the other task, and thus save the power. For technologically independent tasks, their operation cost is not dependent on how much efforts are exerted on other tasks. There are many technologically independent examples in crowdsourcing, such as reporting of location, time, and speed in the dynamic traffic map.

Therefore, under different scenarios, the exact form of the operation cost function $\psi(a)$ varies. In return, the optimal reward varies with the shape of the operation cost functions. In particular, the user decision on the effort level for one task affects the marginal operation cost of undertaking other tasks, will be discussed in the next section. In this paper, we do not consider the technologically complementary case, since it does not provide further insights of this model, but increases the mathematical complexity. Thus, the operation cost coefficient matrix is a positive semi-definite matrix with every element in C is non-negative.

2.2 Performance Measurement

The location based data received by the principal may differs from the user's actual situation. The error may come from the measurement system. For example, there are usually GPS position errors due to the device and signal diversity, especially in "urban canyons" near tall buildings or tunnels [19]. Another example is the urban noise mapping system, in which the sound level meter (SLM) has a precision of ± 2.7 dB [20]. The phone position and context can induce errors and enlarge the variance of errors.

We assume that the effort a the user exerts is hidden from the principal, but the user's contribution can be observed as a vector of information $q = (q_1, \ldots, q_n)$, $n \geq 1$, which can be regarded as the user's performance. Due to the previous mentioned reasons such as the different measurability on tasks, the received information q varies [21]. Therefore, the performance of the user is a noisy signal of its effort:

$$q = a + \varepsilon, \tag{3}$$

where the random component $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$, $n \geq 1$, is assumed to be normally distributed with mean zero and covariance matrix Σ. Thus, the user's performance follows the distribution of $q \sim N(a, \Sigma)$.

The variance Σ is a symmetric $n \times n$ covariance matrix with the form of

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix}, \tag{4}$$

where σ_i^2 denotes the variance of ε_i, and σ_{ij} is the covariance of ε_i and ε_j [22]. The variance denotes the difficulty to guarantee the correctness of measuring effort [23], and also reflects the relationship between the effort exerted by the user and the performance observed by the principal. If the variance is large, the measurability of the performance is difficult, and there is a high probability that the performance is poorly measured and far away from the true effort user exerted. An example is the use of a smartphone microphone as a SLM, which incurs large errors when the phone is put in a pocket or when making a phone call [24]. In contrast, if the performance is easy to measure, the variance will be small or even zero. For example, the report of time is an independent measure with variance 0. The covariance of two measurements exists because the measurement on one task may affect the measurement of the others; for example the detection of a pothole and a bump have a strong connection. Due to this measurement error, both the principal and user will face the measurement cost when integrating multiple tasks.

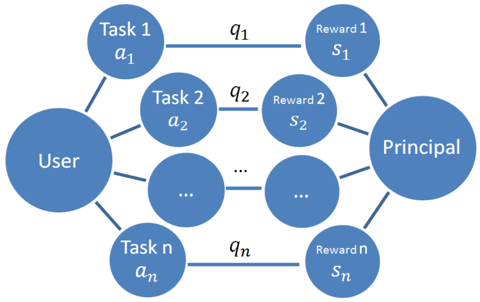


Fig. 2: The multi-task reward contract.

2.3 Reward Package

Inspired by the manager's reward package in industry, which comprises a fixed salary, a bonus related to the firm's profits, and stock options related reward based on the firm's share price [25], we define the user's reward package w in crowdsourcing as a linear combination of a fixed salary and several performance related rewards [26]. By restricting the reward package offered by the principal in the linear form, the reward package w user receives by participating in the crowdsourcing activity can be written as

$$w = t + s^T q, \tag{5}$$

where t denotes the fixed reward salary, which is a constant and is independent of performance, and $s = (s_1, \ldots, s_n)$, $n \geq 1$, is the reward related to the user's performance q. As q is a random variable which follows $q \sim N(a, \Sigma)$, the reward package w is also a random variable with a mean of $t + s^T a$. From the scaling property of covariance, we know that $Var(s^T q) = s^T \Sigma s$. Thus, the reward package follows the distribution $w \sim N(t + s^T a, s^T \Sigma s)$.

At this point, we can propose the contract that is offered by the principal as (a, t, s), where a and s are $n \times 1$ vectors, and t is a constant value. Under this contract, the principal offers the user a reward package which includes a fixed salary t, and n performance related rewards $(s_1, \ldots, s_n)$. Fig. 2 illustrates how this contract works. The user exerts effort a_i for task i, which is observed as a performance q_i by the principal. The principal offers a reward related to q_i, with the reward assigned to the task as s_i.

2.4 Utility of User

In this model, we assume that the user has constant absolute risk averse (CARA) risk preferences, which means the user has a constant attitude towards risk as its income increases. Due to the conservative property of user, we want to define the user's utility function as a concave form. Furthermore, due to the symmetric matrix form of the measurement error and cost coefficient, we need to have the utility function in an exponential form, so that we can transfer the utility function to another form and simplify the problem solving process. Thus, we adopt the negative exponential utility form [27],

$$u(a, t, s) = -e^{-\eta[w - \psi(a)]}, \tag{6}$$

where $\eta > 0$ is the agent's degree of absolute risk aversion

$$\eta = -\frac{u''}{u'}, \tag{7}$$

where u is the user's utility function. A larger value of η means more incentive for the user to implement an effort. The utility and operation cost of the user are measured in such monetary units that they are consistent with the reward from the principal. Thus we have the user's utility function as a concave function, and can easily transform the utility function to certainty equivalent which will be explained later, to simplify the problem solving process.

From (6), we see that the user's utility is a strictly increasing and concave function. For lower computation complexity, we can make use of the exponential form of the utility function, and use *certainty equivalent* as a monotonic transformation of the user's expected exponential utility function [28].

Proposition 1. *The user's utility can be equally represented by certainty equivalent:*

$$CE_u = t + s^T a - \frac{1}{2} a^T C a - \frac{1}{2} \eta s^T \Sigma s. \tag{8}$$

The certainty equivalent consists of the expected reward minus the operation cost and measurement cost. The detail proof of this transformation can be found in the Appendix.

2.5 Utility of Principal

In this model, we regard the principal as a "buy and hold" investor, who cares only about the direct performance of the user [29]. That is, the principal is not concerned about its profit from the location based service in the secondary market (e.g., advertisement selling). Therefore, the effort a leads to an expected gross benefit of $V(a)$, which accrues directly to the principal. Thus, we define the utility of the principal as the expected gross benefits of $V(a)$ minus the reward package w to the user. Thus, the principal's expected utility is written as

$$U(a, t, s) = V(a) - w, \tag{9}$$

where $V(\cdot)$ is the evaluation function which follows $V(0) = 0$ and $V'(\cdot) > 0$. Different from the user who has CARA risk preferences, the principal here is assumed to be risk neutral, i.e., $V''(\cdot) = 0$. Thus, the expected profit of the principal can be simplified to

$$U(a, t, s) = \beta^T a - w, \tag{10}$$

where $\beta = (\beta_1, \ldots, \beta_n)$, $n \geq 1$, characterizes the marginal effect of the user's effort a on the principal's utility $V(a)$. Similar to the definition of user's certainty equivalent, we can derive the principal's certainty equivalent as

$$\begin{aligned} CE_p &= E[\beta^T a - w], \\ &= \beta^T a - s^T a - t. \end{aligned} \tag{11}$$

2.6 Social Welfare

With the definitions of both user's and principal's utility functions and certainty equivalent payoffs, we can have the social welfare defined as their joint surplus, i.e., the summation of user's and principal's equivalent certainty:

$$
\begin{aligned}
R &= CE_u + CE_p, && (12)\\
&= \beta^T a - \frac{1}{2} a^T C a - \frac{1}{2}\eta s^T \Sigma s.
\end{aligned}
$$

The social welfare is the effort exerted by the user, minus the operation cost and the cost incurred by inaccurate measurement. Notice that this expression is independent of the fixed salary t, which serves as an intercept term in the contract. Thus, the fixed salary t can only be used to allocate the total certainty equivalent between the two parties [30]. Later we will see that, under the optimal contract, the social welfare has the same value as the utility of the principal, as the user receives zero utility in crowdsourcing by receiving the optimal reward package.

3 Problem Formulation

With the system model, we can formulate the principal's utility maximization problem while providing the user necessary incentives to participate. The principal's problem can be written as

$$
\begin{aligned}
&\max_{a,t,s} && U(a^*, t, s), && (13)\\
&s.t. && (a)\;\; a^* \in \arg\max_a u(a,t,s),\\
& && (b)\;\; u(a^*, t, s) \geq u(\overline{w}),
\end{aligned}
$$

where $u(\overline{w})$ is the reservation utility of the user when not taking any effort ($a = \mathbf{0}$) in the crowdsourcing. The principal maximize its own utility under the incentive compatible (IC) constraint (a) that the user selects the optimal effort a^* maximizing its own utility, and the individual rationality (IR) constraint (b) that the utility user received is no less than its reservation utility.

In the following subsections, we will first solve this problem in the one-dimension case. Then, we will extend this problem to multiple dimensions, which is the general case in reality. Then, we will exam three specific scenarios to have deeper understanding of the multi-dimension incentive problem.

3.1 One-Dimension Moral Hazard

When this incentive problem is one-dimension, i.e., $n = 1$, the user makes a single effort choice a, and the distribution of the effort measurement error ε reduced to $N(0, \sigma_1^2)$. Therefore, the user's performance distribution is $q \sim N(a, \sigma_1^2)$. As a result, the reward package now is written as

$$w = t + sq, \tag{14}$$

where s is also a constant value. The user's operation cost is reduced to

$$\psi(a) = \frac{1}{2} c_{11} a^2. \tag{15}$$

Typically, the user and principal's utility and certainty equivalent can be written, respectively, as

$$u(a,t,s) = -e^{-\eta(t+sq-\frac{1}{2}c_{11}a^2)}, \tag{16}$$

$$CE_u = t + sa - \frac{1}{2}c_{11}a^2 - \frac{1}{2}\eta s^2 \sigma_1^2. \tag{17}$$

$$U(a,t,s) = \beta a - w, \tag{18}$$

$$CE_p = \beta a - sa - t. \tag{19}$$

As the certainty equivalent is a monotonic transformation of the expected utility function, maximizing the principal's and user's expected utilities is equivalent to maximizing their equivalent certainty payoffs. Thus, we can rewrite the optimization problem in terms of their certainty equivalent wealth, and thus obtain the following simple reformulation of the principal's problem:

$$
\begin{aligned}
&\max_{a,t,s} && (\beta - s)a - t, && (20)\\
&s.t. && (a)\;\; a^* \in \arg\max_a [t + sa - \frac{1}{2}c_{11}a^2 - \frac{1}{2}\eta s^2\sigma_1^2],\\
& && (b)\;\; t + sa - \frac{1}{2}c_{11}a^2 - \frac{1}{2}\eta s^2\sigma_1^2 \geq \overline{w},
\end{aligned}
$$

where $\overline{w}$ denotes the reservation reward of the user when not participating in the crowdsourcing activity.

This one dimensional problem is easy to solve by using the first-order approach. In the first step, we reduce the IC constraint in (a) by taking the first derivative of the user's certainty equivalent regarding a, and setting $u'(a,t,s) = 0$. Then, we obtain the effort $a = s/c_{11}$. Accordingly, we substitute the IR constraint in (b) with the optimal effort a^* and simplify the principal's problem to

$$
\begin{aligned}
&\max\nolimits_{a,t,s} && (\beta - s)\tfrac{s}{c_{11}} - t, && (21)\\
&s.t. && (a)\;\; s\tfrac{s}{c_{11}} + t - \tfrac{1}{2}c_{11}\left(\tfrac{s}{c_{11}}\right)^2 - \tfrac{1}{2}\eta s^2\sigma_1^2 = \overline{w}.
\end{aligned}
$$

Substituting for the value of t in the IR constraint and maximizing with respect to s, we then have the fraction of reward s^* related to performance in the optimal linear reward package as:

$$s^* = \frac{\beta}{1 + \eta c_{11}\sigma_1^2}. \tag{22}$$

With s^*, we have the optimal effort

$$a^* = \frac{\beta}{c_{11} + \eta c_{11}^2 \sigma_1^2}. \tag{23}$$

Representing t by $\overline{w}$, s^* and a^*, we obtain the fixed salary t in the optimal linear reward package as:

$$
\begin{aligned}
t^* &= \overline{w} + \frac{1}{2}\left(\eta\sigma_1^2 - \frac{1}{c_{11}}\right)s^2, && (24)\\
&= \overline{w} + \frac{1}{2}\left(\eta\sigma_1^2 - \frac{1}{c_{11}}\right)\left[\frac{\beta}{1 + \eta c_{11}\sigma_1^2}\right]^2.
\end{aligned}
$$

Under the single task problem, we see that the user's reward package and optimal effort are all decreasing with the operation cost coefficient and the variance of measurement. In other words, the higher the operation cost, or the more difficulty to measure a performance, the user will be less likely to exert effort in the crowdsourcing.

3.2 Multi-Dimension Moral Hazard

When this problem has multiple dimensions, i.e., $n \geq 2$, the problem becomes more complicated to solve. In this subsection, we will first solve the general case where we assume that the measurement error is stochastic dependent and the user's effort is technologically dependent. After this general solution, we will move on to the bench mark case with both stochastic and technological independence.

Under the assumption of stochastic dependent, the error terms are stochastically interacted, i.e., $\sigma_{ij} \neq 0$. For technologically dependent, we mean that the activities are technologically correlated with each other, i.e., $c_{ij} > 0$ and C is a positive definite matrix.

Similar to the previous section, we still solve this multi-dimensional problem by using certainty equivalent model with the following simple reformulation of the principal's problem:

$$\max_{a,t,s} \quad \beta^T a - s^T a - t, \tag{25}$$

$$s.t. \qquad (a) \;\; a^* \in \arg\max_a [t + s^T a - \frac{1}{2} a^T C a - \frac{1}{2} \eta s^T \Sigma s],$$
$$(b) \;\; t + s^T a - \frac{1}{2} a^T C a - \frac{1}{2} \eta s^T \Sigma s \geq \overline{w},$$

where $\overline{w}$ also denotes the reservation reward of the user when not participating in the crowdsourcing activity. The IC constraint represents the rationality of the user's effort choice. The IR constraint in (b) ensures that the principal cannot force the user into accepting the contract.

Similar to the one-dimension case, we first solve the optimal effort by reducing the IC constraint first. The user's certainty equivalent is concave, since its second-order derivative with respect to a is a negative definite matrix $-C$. Thus, the optimal effort can be determined by taking the first-order derivative of the user's certainty equivalent regarding a, and set $u'(a, t, s) = 0$. In the matrix differentiation, if we define $\alpha = a^T C a$, as C is a symmetric matrix, we have $\partial \alpha / \partial a = 2a^T C$ [21]. Since C is symmetric positive definite, its inverse is existent. Thus, through numerical derivations, we can finally have $a = C^{-1} s$ in this multi-dimension case. Accordingly, we substitute the IR constraint in (b) with the optimal effort a^* and simplify the principal's problem to

$$\max\nolimits_{a,t,s} \quad \beta^T C^{-1} s - s^T C^{-1} s - t, \tag{26}$$
$$s.t. \quad (a) t + s^T C^{-1} s - \tfrac{1}{2}(C^{-1}s)^T C (C^{-1}s) - \tfrac{1}{2}\eta s^T \Sigma s = \overline{w}.$$

Substituting the value of t in the IR constraint to the objective and differentiating the objective function with respect to s, we have the performance related reward s^* in the optimal multi-dimension reward package as:

$$s^* = (C^{-1} + \eta \Sigma)^{-1} C^{-1} \beta = (I + \eta C \Sigma)^{-1} \beta. \tag{27}$$

With s^*, we have the optimal effort in the multi task case as

$$a^* = C^{-1}(I + \eta C \Sigma)^{-1} \beta. \tag{28}$$

Representing t by $\overline{w}$, s^* and a^*, we obtains the fixed salary t in the optimal linear reward package as:

$$t^* = \overline{w} + \frac{1}{2} s^T (\eta \Sigma - C^{-1}) s, \tag{29}$$
$$= \overline{w} + \frac{1}{2} \left[(I + \eta C \Sigma)^{-1} \beta\right]^T \left(\eta \Sigma - C^{-1}\right) \left[(I + \eta C \Sigma)^{-1} \beta\right].$$

Comparing this equation with the first order results, we see that the first order reward package is one special case of this general case and can be derived from this general case directly by setting the matrixes as one dimension ($n = 1$).

Using the formulas (27) for s^* we can indeed determine how the optimal linear incentive reward varies with the accuracy of output measures for each task and the operation cost coefficient of each task. Assume, for example, when two tasks are technologically substitution $c_{ij} > 0$, if the measurability of task i worsens, that is, σ_i^2 increases, then, as is intuitive, s_j^* goes up, but s_i^* goes down. Thus, there is a measurement complementarity between the s_i^* and s_j^* in the presence of technologically substitutes problems [16].

A higher incentive reward can induce the user to implement a higher effort, but it will also expose the user to a higher risk. It, therefore, requires a premium to compensate the risk averse user for the risk he/she bears. The optimal power of incentive is therefore determined by the tradeoff between incentive and insurance.

3.2.1 *Stochastic Independent and Technologically Independent*

In this benchmark case, the error terms are stochastically independent (i.e., $\sigma_{ij} = 0$, Σ is a diagonal matrix), and the tasks are technologically independent (i.e., $c_{ij} = 0$, C is a diagonal matrix). Thus, the optimal incentive contract for each task is similar to the single-task problem, and the solution in (27) simplifies to

$$s_i^* = \frac{\beta_i}{1 + \eta c_{ii} \sigma_i^2}, \quad \forall i \in \{1, \ldots, n\}. \tag{30}$$

The user's optimal choice of effort becomes

$$a_i^* = \frac{s_{ii}}{c_{ii}} = \frac{\beta_i}{(1 + \eta c_{ii} \sigma_i^2) c_{ii}}, \quad \forall i \in \{1, \ldots, n\}. \tag{31}$$

Representing t by $\overline{w}$, s^* and a^*, we obtain the fixed salary t in the optimal linear reward package as:

$$t_i^* = \overline{w} + \frac{1}{2} \left(\eta \sigma_i^2 - \frac{1}{c_{ii}}\right) \left[\frac{\beta_i}{1 + \eta c_{ii} \sigma_i^2}\right]^2. \tag{32}$$

In this case, efforts are set independently of each other since the operation cost of inducing the user to perform any given task is independent of the other tasks. As expected, s is decreasing in risk aversion degree η, operation cost coefficient c_{ii} and measurement error variance σ_i^2. We can also prove the relationship between reward s_i and effort a_i from $a = C^{-1}s$. As in this technologically independent case, C is a diagonal matrix with elements c_{ii} on the diagonal. Thus, we can take the partial derivatives as

$$\frac{\partial s_i}{\partial a_i} = c_{ii}, \quad \text{and} \quad \frac{\partial a_i}{\partial s_i} = c_{ii}^{-1}. \tag{33}$$

Thus, we see that the reward s_i for effort a_i is decreasing in c_{ii}, and the higher of s_i, the more effort the user is like to exert.

The algorithm for solving the formulated problems is summarized in Algorithm 1.

Algorithm 1: Optimal Contract

Input: β, n, C, η, Σ, $\bar{w}$
Output: **a**, **s**, t
1. Optimal Effort;
Represent optimal effort $\mathbf{a}^*$ by **s** and t from first derivative of (20/25a);
2. Optimal Reward Package;
Take the optimal effort $\mathbf{a}^*$ into (20/25);
Obtain the reward **s** and fixed salary t, as well as **a**;

3.3 Extending Analysis

3.3.1 Zero Incentive

In this part, we analyze one special case, in which the principal does not provide any incentive for some tasks. In other words, the reward s_i for task i is less than or equal to zero. In the general multi-dimension case, the optimal effort a is affected by those cross-partial of C due to technological substitutes. To illustrate how the operation cost coefficients affect the principal to assign a zero reward, we consider the two-dimension case with stochastic independent, i.e., $\sigma_{12} = \sigma_{21} = 0$. We assume that task 2 is easy to measure, i.e., σ_2^2 is finite and small, while task 1 is impossible to measure, i.e., $\sigma_1^2 \to \infty$. In this case, effort a_1 cannot be measured, nor can we assign specific reward s_1 to task 1. Thus, the only way to provide incentives for task 1 is to reduce the reward s_2 for task 2. If task 1 is a critical work that the principal cares extremely about, it may be optimal to punish effort on task 2 ($s_2 < 0$) or give no reward at all for task 2 ($s_2 = 0$). In this case, *zero incentive* happens for task 2.

Proposition 2. *When efforts are technologically substitutes, providing incentives for a given task can be implemented either by increasing the reward for that task or by reducing the rewards for the other tasks.*

The second case when *zero incentive* may happen is, when $c_{12} = \sqrt{c_{11}c_{22}}$, the effort for the two tasks are "perfect substitutes," i.e., $a = a_1 + a_2$. Thus, we have $s_1 = s_2$ as the user must equate the marginal return to effort in various tasks. In the case of $\sigma_1^2 \to \infty$, we thus have $s_1 = s_2 = 0$.

The third case when *zero incentive* happens is that the user has a deep love for task 1. Then it will be willing to exert all its effort even in the absence of any financial reward. This *zero incentive* case can be found in many online applications, in which the user receives incentives through the other user's praise and self-esteem, instead of the principal's reward. In this case, the effort choice of the user will also equate the marginal nonfinancial benefit with the marginal cost [16].

3.3.2 Missing Incentive

In some cases, the incentive mechanism cannot provide specific incentives for some aspects of user's contribution. *Missing incentive* differs from *zero incentive* in the sense that, in *zero incentive*, the principal measures the user's performance on the task, but rewards zero. However, the principal neither takes into consideration of user's contribution on the task, nor give any reward in the *Missing incentive*. One example in crowdsourcing is the NoiseTube which is designed to measure and map urban noise pollution using smartphones sensors such as microphone and GPS. Those data can be used directly to construct the dynamic noise map. Furthermore, they can be used to support decision and policy making in different domains such as public health, urban planning, environmental protection and mobility, which will bring far more great benefit in the future [24]. Even though those contributions are important, the principal is unable to account for such explicit incentive provisions in actual contracts.

3.3.3 Groupings of Tasks

In the single-user multi-task problem, the performance related rewards $(s_1, \ldots, s_n)$ serve three purposes: allocating risk, motivating work, and directing the user's efforts among the various tasks [30]. However, a trade-off arises when these objectives are in conflict with each other. For example, risk-sharing may be inconsistent with motivating work, and motivating hard work may distort the user's effort allocation across tasks. If we have multiple users, the principal can group the tasks, which enables lowering the cost of incentive by using more sensitive measure of actual performance.

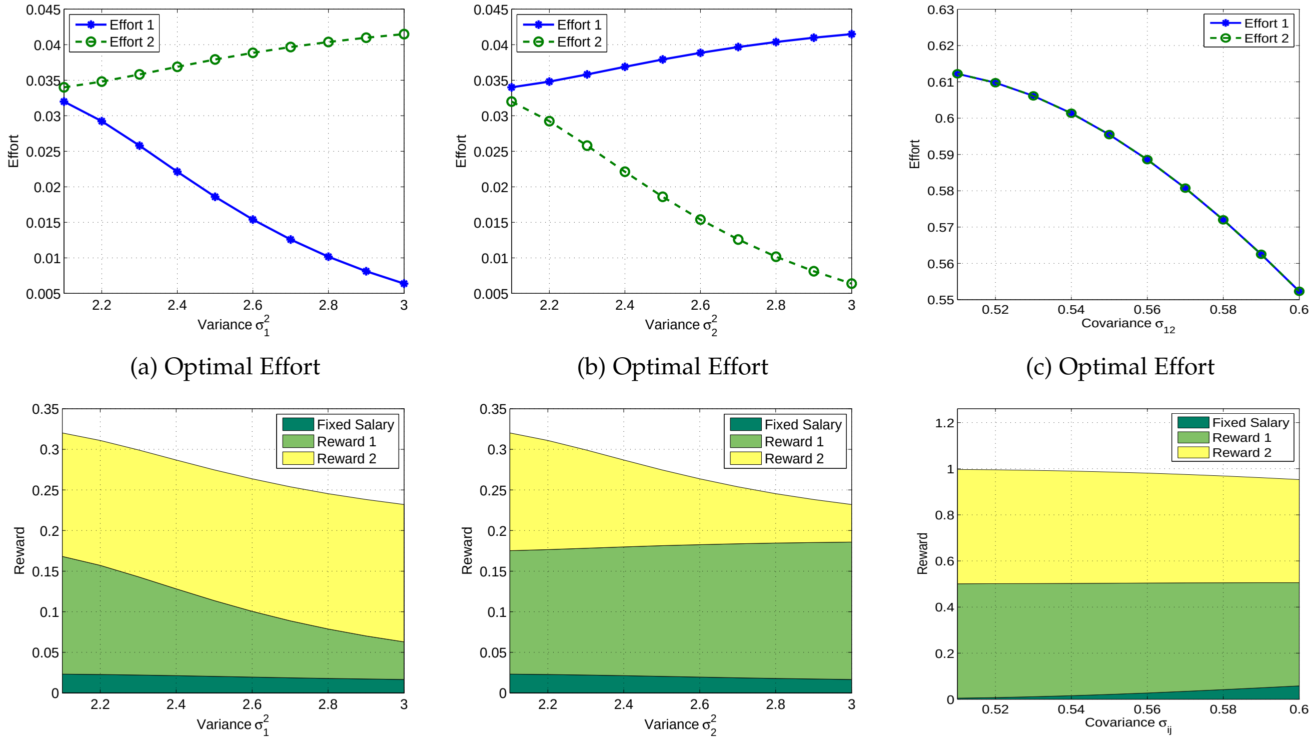
To alleviate those conflicts, we consider grouping tasks into different jobs that can assign to different users. One application can be used in Nericell [31], in which varied road and traffic condition need to be detected. The part of common traffic detection tasks such as traces, traffic flow speed, and driving patterns can be grouped and assigned to users with basic sensing functions, such as GPS and accelerometer. The other parts of the newly introduced tasks such as the detection of crashes, potholes and bumps, can be grouped and assigned to users equipped with a special-purposed device with 3-axis accelerometers.

Providing incentives for an user in any task incurs a fixed cost such as the measurement error. Thus, in the two-dimension case, assigning joint responsibility for any task would incur two fixed costs, which is unnecessary. If some tasks are jointly responsible, it is optimal to split them among the users without affecting either the total effort required from each user or the total effort allocated to any task. This grouping of tasks is possible to eliminate some of the user's risk, so increasing the utilities of both the principal and users [15].

The issue of how the tasks should be grouped can be found in [30]. For the two-dimension case, tasks should be grouped such that all the hardest-to-monitor tasks are assigned to user 1 and all the easiest-to-monitor tasks are assigned to user 2. Separating tasks according to their measurability characteristics allows the principal to give strong incentives for tasks that are easy to measure without fearing that the user will substitute efforts away from other harder-to-measure tasks.

4 Simulation Results and Analysis

In this section, we will first give a detailed analysis of reward package in the multi-dimensional case. We will look at how different reward items in the reward package change by varying the parameters such as the operation cost coefficients and measurement error covariance. Then, we



(a) Optimal Effort (b) Optimal Effort (c) Optimal Effort

(d) Reward package (e) Reward package (f) Reward package

Fig. 3: The optimal effort and reward package as the measurement error covariance Σ matrix varies.

will conduct a comparison of the principal's utility among different incentive mechanisms.

In the simulation set up, we assume that, the reservation reward of the user $\overline{w} = 0$ when not participating in the crowdsourcing ($a = 0$). The reason we do not consider the user's utility is that, from the optimal reward package we have derived, no matter how those parameters change, the user's utility will remain the same. The optimal reward package will bring user the utility the same as the reservation utility $-e^{-\eta\overline{w}}$, which in our case is -1 as we set $\overline{w} = 0$.

4.1 Optimal Reward Package Analysis

4.1.1 Measurement Error

To look into the detail of how the variance and covariance of measurement error affect the optimal effort and reward package, we set up the multi-dimensional space as $n = 2$. Since the measurement error covariance matrix is symmetric, there are three variables that we can vary: the variances of measurement error for tasks 1 and 2: σ_1^2 and σ_2^2, and the covariance σ_{12}/σ_{21}. We fix the operation cost matrix C, and risk averse degree η, and show the results in Fig. 3, where the first row gives the optimal efforts, and the second row gives the reward packages.

In Figs. 3(a, b, d, e), we are going to see how the variances of the measurement error on the performances affect the user's selection of efforts for the two tasks and the rewards offered by the principal. When we vary one variance, the other one keeps fixed.

Fig. 3a show the measurement error variance σ_1^2 for task 1 increases, the optimal effort a_1 for task 1 decreases, while the effort a_2 for task 2 shows opposite properties. From Fig. 3d, we see that as measurement error variance σ_1^2 for task 1 increases, reward package w, the fixed salary t and reward s_1 are decreasing, while the reward for task 2 is increasing. This result is because the measurement error becomes more volatile (σ_1^2 increases), the user's benefit from task 1 decreases (s_1 becomes smaller), but the share from task 2 increases so that the use's utility can be maintained at the reservation utility.

Figs. 3(b, e) show similar properties as Figs. 3(a, d). At this time we fixed σ_1^2 but increase σ_2^2, thus Figs. 3(b, e) show the opposite behavior compare to the previous case. As σ_2^2 increases, i.e., the measurement error for task 2 becomes more volatile, user prefers to exert more effort for task 1 instead of task 2. As we can see from Fig. 3b that, the effort for task 1 is increasing while effort for task 2 is decreasing. Similarly, from Fig. 3e we see that the user's reward from the task 2 and the fixed salary t are decreasing at the same time, but the reward from the task 1 goes up.

From Figs. 3(d, e), we have learned that, as the user's utility remains the same (i.e., -1) in all situations, the reward package offered to the user will mostly rely on the part that is more stable, such as the reward with fixed measurement error variance: reward 2 when σ_1^2 increases and reward 1 when σ_2^2 increases. In summary, the reward design lowers the proportion of bonus from the less predictable part. By this mechanism, the risk of losing user's incentive in all kinds of situations can be canceled.

In Figs. 3(c, f), we investigate the impacts of covariance σ_{12}/σ_{21} on the optimal effort and reward package, while fixing σ_1^2 and σ_2^2 the same. The simulation results show that, as the covariance σ_{12}/σ_{21} increases, the optimal effort a and reward package w are all decreasing. Since we assign the same operation cost for tasks 1 and 2, the optimal effort of them overlaps in Fig. 3c. Meanwhile, from Fig. 3f we see that, within the reward package, reward 1 and reward 2 are decreasing except the fixed salary t. When the relationship between the performance observed by the principal and the effort exerted by the user becomes more volatile, it is harder to predict them to identify an effort. Thus, the user becomes more reluctant to exert effort, and the principal receives less utility and rewards the user less.

4.1.2 Operation Cost

To see how the operation cost coefficients affect the optimal effort and reward package, we also set up the multi-dimensional space as $n = 2$. The operation cost coefficient is also a symmetric matrix, and we can vary three of the elements: task-specific operation cost coefficient for task 1 and 2: c_{11} and c_{22}, and the technologically substitution coefficient c_{12}/c_{21}. We fix the measurement error covariance matrix Σ, and risk averse degree η, and show the results in Fig. 4, where the first row gives the optimal efforts, and the second row gives the reward packages as what we have done in Fig. 3.

Figs. 4(a, b, d, e) show how the task-specific operation cost affects the user's effort choice for the two tasks and the reward items in reward package. We keep one operation cost coefficient fixed when vary the other operation cost coefficient.

In Fig. 4a, we see that as the operation cost coefficient c_{11} for task 1 increases, the optimal effort a_1 for task 1 decreases,but effort a_2 for task 2 increases. In Fig. 4d, reward package w and reward s_1 are decreasing, while the reward for task 2 and fixed salary t are increasing. This result is intuitive, since if exerting effort for task 1 encounters more operation cost, (c_{11} increases), the user will be more likely to switch effort to task 2, which consumes less operation.

Figs. 4(b, e) show similar properties as Figs. 4(a, d). At this time we fixed c_{11} but increase c_{22}, thus Figs. 4(b, e) shows the opposite behavior compared to the previous case. As c_{22} increases, i.e., the operation cost for task 2 increases, user prefers to exert more effort for task 1 instead of task 2. We can see from Fig. 4b that the effort for task 1 is increasing while effort for task 2 is decreasing. Similarly, from Fig. 4e we see that the user's reward from the task 2 is decreasing. While the reward from the task 1 and the fixed salary t go up at the same time.

From both Fig. 4d and Fig. 4e, we observe that, the user is more likely to exert effort on the task that incurs less operation cost, and thus the reward package will reward more on the task with a smaller operation cost coefficient. Thus, we see that the principal reward 2 when c_{11} increases and reward 1 when c_{22} increases.

In Figs. 4(c, f), we investigate the impacts of technologically substitution c_{12}/c_{21} on the optimal effort and reward package, while fixing the task specific operation cost coefficients c_{11} and c_{22} the same and unchanged. As the technologically substitution c_{12}/c_{21} increases, the optimal effort a and reward package w are all decreasing. Since we assign the same task-specific cost coefficients for both tasks, the optimal effort of them two overlap in Fig. 4c. Meanwhile, from Fig. 4f we see that, reward s_1 and reward s_2 are both decreasing except the fixed salary t. This is due to less efforts are exerted from the user, less performance related rewards will be offered. However, in order to keep user incentivized, the principal has to increase the fixed salary t, so that the user's utility is guaranteed.

4.2 Incentive Mechanism Comparison

In the previous section, we have solved the optimal reward package when the measurement error is stochastic dependent and effort is technologically dependent. As this multi-dimensional case is the most general case in reality, we name this mechanism by *General*. In addition, we also obtained the optimal reward package when the measurement error and effort are independent, and thus we name it by *Independent*. We also have a third one called *Single Bonus* that is the reward package obtained in the one dimensional case. In this one-dimensional case, we can regard the principal rewards user on only one task. In this subsection, we will propose another three incentive mechanisms as the comparisons with the previous two. Those three mechanisms are generally based on our current model, while they are different from each other in the construction of their reward packages.

The first two are special cases of the *General*: one is stochastic independent but technologically dependent, the other one is technologically independent but stochastic dependent, and are named by *Stochastic Independent* and *Technologically Independent*, respectively. The last one is called *Opening Reward*, that is the reward package only contains a fixed salary t. We can regard this mechanism as a company which will offer each user an opening reward as the Karma which is mentioned in Section I. But this *Opening Reward* mechanism does not care about user's future performance.

4.2.1 Stochastic Independent

When tasks are stochastic independent, the co-variances of the error measurement are zero, and we have $\sigma_{ij} = 0$ and Σ becomes a diagonal matrix. The optimal performance related rewards for each task in (26) is simplified to

$$s^* = (I + \eta C \mathrm{Diag}(\Sigma))^{-1}\beta, \tag{34}$$

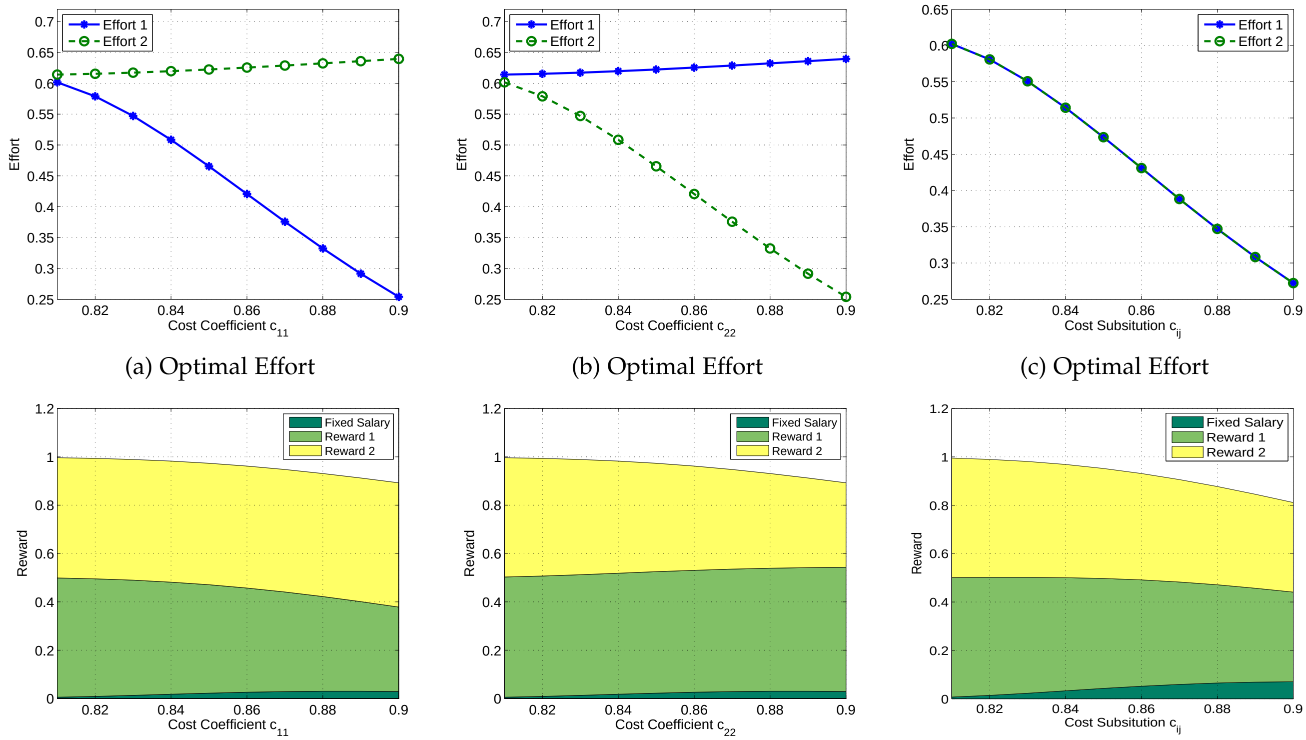
where $\mathrm{Diag}(\Sigma)$ is the a $n \times n$ diagonal matrix with element σ_i^2, $\forall i \in \{1, \dots, n\}$ on the diagonal. Based on $a = C^{-1}s$ and (29), we can easily obtain the user's optimal choice of effort and the fixed salary t in this stochastic independent but technologically dependent package.

4.2.2 Technologically Independent

When tasks are technologically independent, the cross-partials of the cost function are zero, i.e., $c_{ij} = 0$ and C becomes a diagonal matrix. The optimal incentive contract for each task in (26) simplifies to

$$s^* = (I + \eta \mathrm{Diag}(C)\Sigma)^{-1}\beta, \tag{35}$$

where $\mathrm{Diag}(C)$ is the a $n \times n$ diagonal matrix with element c_{ii}, $\forall i \in \{1, \dots, n\}$ on the diagonal. Based on $a = C^{-1}s$ and (29), we can easily obtain the user's optimal choice of effort and the fixed salary t in this technologically independent but stochastic dependent package.



(a) Optimal Effort (b) Optimal Effort (c) Optimal Effort

(d) Reward package (e) Reward package (f) Reward package

Fig. 4: The optimal effort and reward package as the operation cost coefficient C matrix varies.

4.2.3 Opening Reward

When no performance related reward is offered, the problem is formulated as

$$\max_{a,t} \quad \beta^T a - t, \tag{36}$$

$$s.t. \quad (a) \quad a = \arg\max_a [t - \frac{1}{2}a^T Ca - -\frac{1}{2}\eta s^T \Sigma s],$$

$$(b) \quad t - \frac{1}{2}a^T Ca - -\frac{1}{2}\eta s^T \Sigma s = \overline{w}.$$

The optimal effort a^* and opening reward t^*, respectively, have the form of

$$a^* = C^{-1}\beta, \tag{37}$$

$$t^* = \overline{w} + \frac{1}{2}a^T Ca = \overline{w} + \frac{1}{2}(C^{-1})^T \beta^T \beta. \tag{38}$$

4.2.4 Comparisons

In Fig. 5, we compare the principal's utility from the six incentive mechanisms as we vary the task-specific operation cost coefficient c_{ii}. From the simulation results we see that, as the cost coefficient c_{ii} increases, the principal's utility is decreasing as well. The reason for this phenomenon is that larger cost coefficient c_{ii} means more operation cost when implying an effort. Therefore, the user is less likely to exert effort in the crowdsourcing activity. With less data are collected from the users, the principal's utility will certainly decrease. In addition, from Fig. 5, we see that the principal obtains the largest utility in the *Independent* case. Followed by the *Opening Reward*, *Stochastic Independent*, and *Technologically Independent*, the *General* case proposed by us brings the fifth highest utility to the principal, while the *Single Bonus* gives the least utility.

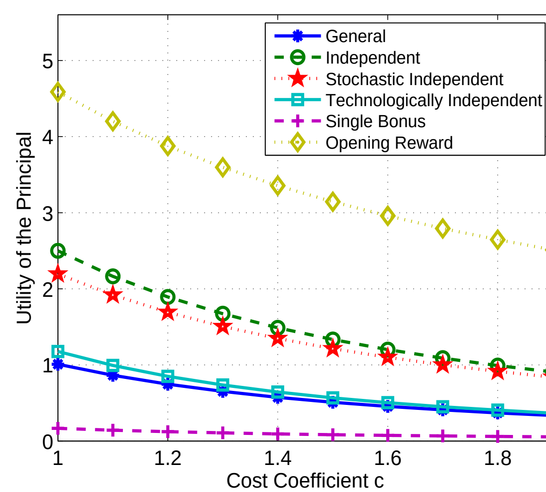


Fig. 5: The principal's utility as the operation cost coefficient c_{ii} varies.

In Fig. 6, we analyze the impact of user's risk averse degree η on the principal's utility. As the principal's utility $V = a - t$ in the *Opening Reward* is independent of the risk averse degree η, we cannot see any change in the principal's utility. For the other five mechanisms, we see that the principal's utility is decreasing as the user's risk averse degree η increases. This result is intuitive as a larger η means the user becomes more conservative and sensitive to risk, thus less likely to participate in. With less effort obtained from the user, the principal's utility will certainly decrease. From Fig. 6 we also obtains the similar ranking of the principal's

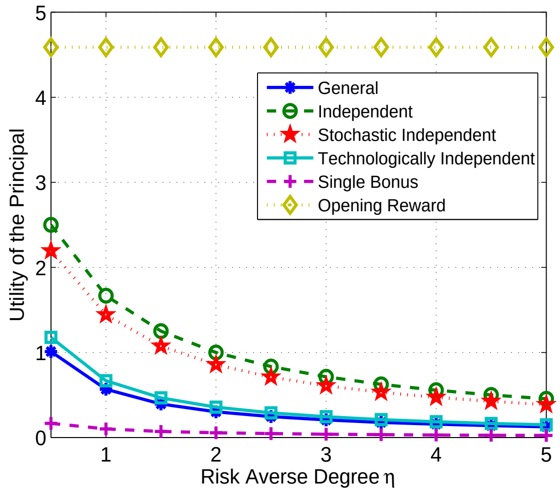


Fig. 6: The principal's utility as risk averse degree η varies.

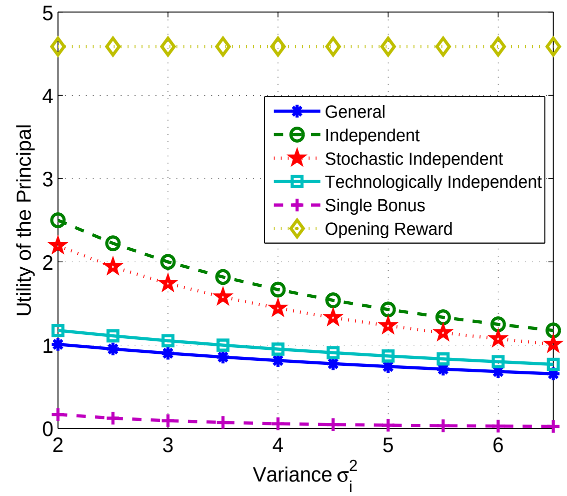


Fig. 7: The principal's utility as measurement error variance σ_i^2 varies.

utility as in the previous figure: the *Independent* case brings higher utility than the *Stochastic Independent*, *Technologically Independent*, and *General* one, and the *Single Bonus* one brings the smallest utility for the principal.

In Fig. 7, we increase the variance σ_i^2 to see how the principal's utility varies. Similar to the previous case, the principal's utility $V = a - t$ in the *Opening Reward* is independent of the covariance matrix. Thus, we cannot see any change of the principal's utility. For the other mechanisms, the principal's utility is decreasing with the variance, which is in accordance with our conclusion in the previous section. The variance σ_i^2 of measurement error denotes the relationship between effort levels exerted by the user and the performance observed by the principal. As σ_i^2 increases, it indicates a weaker relationship between effort levels and the expected reward achieved. As a result, the users are likely to exert lower levels of effort with increases in uncertainty, and thus a lower cost of participation. With the decrease of optimal effort, less data is obtained from the user, the principal's utility will certainly decrease. From Fig. 7 we also obtain the similar ranking of the principal's utility as in the previous figure: the *Independent* case brings higher utility than *Stochastic Independent*, followed by *Technologically Independent* and *General* one, the *Single Bonus* one brings the lowest utility for the principal.

The reason for the performance ranking of the six mechanisms in Fig. 5, Fig. 6, and Fig. 7 is as follows. The *Independent* mechanism is the ideal case of the *General* multi-dimension case. As less measurement cost is occurred when predicting the outcome and less operation cost is encountered due to effort substitution, a higher utility is obtained than the other mechanisms. The *Stochastic Independent* and *Technologically Independent* are partial independent cases of the *General* multi-dimension one, thus, the principal's utility lies between the *Independent* and *General* mechanisms. But as we have assigned larger values for the covariance matrix of the the measurement error than the operation cost coefficient matrix, more effort will be exerted in the *Stochastic Independent* than in the *Technologically Independent* mechanism. Therefore, the principal's utility is higher in the *Stochastic Independent* than in the *Technologically Independent* case, while the *Single Bonus* only reward user with only one dimension evaluation. As a result, the users have less incentive to exert more effort in other tasks. In return, less utility is obtained by the principal. For the result of the *Opening Reward* case, it seems unreasonable at the first sight, as it brings the principal the highest utility than the other three mechanisms. While we notice that *Opening Reward* is a "once-for-all" deal which does not provide continuous incentives for the users, i.e., after the users have fulfilled their duty and receive the reward, they are more likely to stop participating in crowdsourcing.

5 Conclusions

In this paper, we have investigated the problem of providing incentives for users to participate in the crowdsourcing by rewarding user from multi-dimension evaluations. We solve the principal's utility maximization problem in both one-dimension and multi-dimension cases. Furthermore, we give analysis of special scenario of the multi-dimension model. Finally, we use the numerical results to analyze the optimal reward package by varying different parameters. In addition, we compare the principals' utility under the six different incentive mechanisms, and show that the principal's utility deteriorates with large operation cost coefficient, higher risk aversion of users, and large measurement error variance.

Appendix A
Proof of Proposition 1

We have the user's utility function in (6) as $u = -\exp\{-\eta[w - \psi(a)]\}$. From Section II, we know that $w \sim N(t + s^T a, s^T \Sigma s)$. As the user incurs an operation cost ψ, the actual income w' has the distribution

$$w' = w - \psi(a) \sim N(t + s^T a - \frac{1}{2}a^T C a, s^T \Sigma s). \qquad (39)$$

Let μ denotes $t + s^T a - \frac{1}{2} a^T C a$, and σ^2 denotes $s^T \Sigma s$, we have $w' \sim N(\mu, \sigma^2)$ for simplification. The corresponding density function for w' is

$$f(w') = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(w'-\mu)^2}{2\sigma^2}\right]. \tag{40}$$

The corresponding expected exponential utility function is

$$\begin{aligned} E[u(w')] &= -E[\exp(-\eta w')] \qquad (41)\\ &= -\int_{-\infty}^{+\infty} \exp(-\eta w') f(w') dw' \\ &= -\int_{-\infty}^{+\infty} \exp(-\eta w') \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(w'-\mu)^2}{2\sigma^2}\right] dw' \\ &= -\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\eta w' - \frac{(w'-\mu)^2}{2\sigma^2}\right] dw'. \end{aligned}$$

For the exponential part, we see that $-\eta w' - \frac{(w'-\mu)^2}{2\sigma^2}$

$$\begin{aligned} &= -\eta w' - \frac{(w'-\mu)^2}{2\sigma^2} + \eta\mu - \eta\mu + \frac{\eta^2\sigma^2}{2} - \frac{\eta^2\sigma^2}{2} \qquad (42)\\ &= -\left[\eta w' + \frac{(w'-\mu)^2}{2\sigma^2} - \eta\mu + \frac{\eta^2\sigma^2}{2}\right] - \eta\mu + \frac{\eta^2\sigma^2}{2} \\ &= -\frac{1}{2}\left[\frac{(w'-\mu)^2}{\sigma^2} + 2\eta(w-\mu) + \eta^2\sigma^2\right] - \eta\mu + \frac{\eta^2\sigma^2}{2} \\ &= -\frac{1}{2\sigma^2}[(w'-\mu) + \eta\sigma^2]^2 - \eta\mu + \frac{\eta^2\sigma^2}{2}. \end{aligned}$$

Thus, the expected exponential utility function $E[u(w')]$ becomes

$$\begin{aligned} &= -\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\eta w' - \frac{(w'-\mu)^2}{2\sigma^2}\right] dw' \qquad (43)\\ &= -\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2\sigma^2}[(w'-\mu) + \eta\sigma^2]^2 - \eta\mu + \frac{\eta^2\sigma^2}{2}] dw' \\ &= -\exp\left(-\eta\mu + \frac{\eta^2\sigma^2}{2}\right) \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2\sigma^2}[(w'-\mu) + \eta\sigma^2]^2] dw' \end{aligned}$$

As the integration part is the density function of a random variable following a normal distribution with a mean of $\mu - \eta s^2$ and variance σ^2, we have

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}[(w'-\mu) + \eta\sigma^2]^2\right] dw' = 1 \tag{44}$$

Therefore, we have

$$E[u(w')] = -\exp\left(-\eta\mu + \frac{\eta^2\sigma^2}{2}\right) = -\exp\left[-\eta\left(\mu - \frac{\eta\sigma^2}{2}\right)\right]. \tag{45}$$

We rewrite the equation as $E[u(w')] = \exp(-\eta CE)$, where CE denotes the certainty equivalent of the user. From the original definition of the user's utility function, we have the user's certainty equivalent as

$$\begin{aligned} CE &= \mu - \frac{\eta\sigma^2}{2} \qquad (46)\\ &= t + s^T a - \frac{1}{2} a^T C a - \frac{1}{2}\eta s^T \Sigma s. \end{aligned}$$

From the derivation, we see that the certainty equivalent is a monotonic transformation of the user's expected exponential utility function u. Therefore, CE represents the same preference as $E[u]$.

ACKNOWLEDGMENTS

This work was made possible by NSF Grant CNS-1702850, Grant CNS-1646607, Grant ECCS-1547201, Grant CCF-1456921, Grant CMMI-1434789, Grant CNS-1443917, Grant ECCS-1405121, Grant CNS-1343361, and Grant CNS-1350230 from the US National Science Foundation.

This research was also supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (NRF-2016R1D1A1B01015320).

REFERENCES

[1] G. Chatzimilioudis, A. Konstantinidis, C. Laoudias, and D. Zeinalipour-Yazti, "Crowdsourcing with smartphones," *Internet Computing, IEEE*, vol. 16, no. 5, pp. 36–44, Sept. 2012.

[2] D. Zhao, X. Li, and H. Ma, "How to crowdsource tasks truthfully without sacrificing utility: Online incentive mechanisms with budget constraint," in *INFOCOM, 2014 Proceedings IEEE*, Toronto, Canada, Apr. 2014.

[3] J. Angmin and J. Valentino-Devries, "Apple, google collect user data," technic report, 2011. [Online]. Available: http://online.wsj.com/news/articles/SB10001424052748703983704576277101723453610

[4] S. Chernova, N. DePalma, E. Morant, and C. Breazeal, "Crowdsourcing human-robot interaction: Application from virtual to physical worlds," in *RO-MAN, 2011 IEEE*, Atlanta, GA, Jul. 2011.

[5] T. Luo, H. Tan, and L. Xia, "Profit-maximizing incentive for participatory sensing," in *INFOCOM, 2014 Proceedings IEEE*, Toronto, Canada, Apr. 2014.

[6] L. R. Varshney, "Privacy and reliability in crowdsourcing service delivery," in *SRII Global Conference (SRII), 2012 Annual*, San Jose, CA, Jul. 2012.

[7] KARMA, "Meet karma," technic report, 2012. [Online]. Available: https://yourkarma.com/

[8] L. Gao, G. Iosifidis, J. Huang, and L. Tassiulas, "Hybrid data pricing for network-assisted user-provided connectivity," in *INFOCOM, 2014 Proceedings IEEE*, Toronto, Canada, Apr. 2014.

[9] L. Duan, T. Kubo, K. Sugiyama, J. Huang, T. Hasegawa, and J. Walrand, "Incentive mechanisms for smartphone collaboration in data acquisition and distributed computing," in *INFOCOM, 2012 Proceedings IEEE*, Orlando, FL, Mar. 2012.

[10] Y. Zhang, Y. Gu, L. Liu, M. Pan, Z. Dawy, and Z. Han, "Incentive mechanism in crowdsourcing with moral hazard," in *IEEE Wireless Communications and Networking Conference (WCNC)*, New Orleans, LA, Mar. 2015.

[11] A. Karmann, "Multiple-task and multiple-agent models: Incentive contracts and an application to point pollution control," *Annals of Operations Research*, vol. 54, no. 1, pp. 57–78, Dec. 1994.

[12] Yelp. [Online]. Available: http://www.yelp.com/

[13] D. Meng and G. Tian, "Multi-task incentive contract and performance measurement with multidimensional types," *Games and Economic Behavior*, vol. 77, no. 1, pp. 377 – 404, Jan. 2013.

[14] E. Fehr and K. M. Schmidt, "Fairness and incentives in a multi-task principal-agent model," *Scandinavian Journal of Economics*, vol. 106, no. 3, pp. 453–474, Sep. 2004.

[15] B. Holmstrom and P. Milgrom, "Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design," *Journal of Law, Economics, & Organization*, vol. 7, no. sp, pp. 24–52, Jan. 1991.

[16] P. Bolton and M. Dewatripont, *Contract Theory*. Cambridge, MA: The MIT Press, 2004.

[17] L. Werin and H. Wijkander, *Contract Economics*. Oxford, UK: Blackwell Publishers, 1992.

[18] D. Bergemann, J. Shen, Y. Xu, and E. Yeh, "Multi-dimensional mechanism design with limited information," in *Proceedings of the 13th ACM Conference on Electronic Commerce*, Valencia, Spain, Jun. 2012, pp. 162–178.

[19] A. Thiagarajan, L. Ravindranath, K. LaCurts, S. Madden, H. Balakrishnan, S. Toledo, and J. Eriksson, "Vtrack: Accurate, energy-aware road traffic delay estimation using mobile phones," in *Proceedings of the 7th ACM Conference on Embedded Networked Sensor Systems*, Berkeley, California, 2009, pp. 85–98.

[20] R. K. Rana, C. T. Chou, S. S. Kanhere, N. Bulusu, and W. Hu, “Ear-phone: An end-to-end participatory urban noise mapping system,” in *Proceedings of the 9th ACM/IEEE International Conference on Information Processing in Sensor Networks*, Stockholm, Sweden, 2010, pp. 105–116.
[21] Y. Barzel, “Measurement cost and the organization of markets,” *Journal of Law and Economics*, vol. 25, no. 1, pp. 27–48, Apr. 1982.
[22] B. Danforth. (2009) Variance-covariance matrix. [Online]. Available: http://www.unc.edu/\~jjharden/methods/vcv_week3.pdf
[23] M. N. Leger, L. Vega-Montoto, and P. D. Wentzell, “Methods for systematic investigation of measurement error covariance matrices,” *Chemometrics and Intelligent Laboratory Systems*, vol. 77, no. 1–2, pp. 181 – 205, May 2005.
[24] M. Stevens and E. D’Hond, “Crowdsourcing of pollution data using smartphones,” in *UbiComp ’10*, Copenhagen, Denmark, Sep. 2010.
[25] L. A. Bebchuk, J. M. Fried, and D. Walker, “Managerial power and rent extraction in the design of executive compensation,” *University of Chicago Law Review*, vol. 69, no. 2, pp. 751–846, Jul. 2002.
[26] P. Bolton, J. Scheinkman, and W. Xiong, “Executive compensation and short-termist behavior in speculative markets,” National Bureau of Economic Research, Working Paper, May 2003.
[27] J. Norstad, “An introduction to utility theory,” technic report, 1999. [Online]. Available: http://www.norstad.org/finance/util.pdf
[28] Investopedia. (2003) Certainty equivalent. [Online]. Available: http://www.investopedia.com/terms/c/certaintyequivalent.asp
[29] B. H. J. Tirole, “Market liquidity and performance monitoring,” *The Journal of Political Economy*, vol. 101, no. 4, pp. 678–709, Aug. 1993.
[30] B. Holmstrom and P. Milgrom, “Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design,” *Journal of Law, Economics, and Organization*, vol. Sp, no. 3, pp. 24–52, Jan. 1991.
[31] P. Mohan, V. N. Padmanabhan, and R. Ramjee, “Nericell: Rich monitoring of road and traffic conditions using mobile smartphones,” in *Proceedings of the 6th ACM Conference on Embedded Network Sensor Systems*, Nov. 2008, pp. 323–336.

Yanru Zhang (S’13-M’16) received the B.S. degree in electronic engineering from University of Electronic Science and Technology of China (UESTC) in 2012, and the Ph.D. degree from the Department of Electrical and Computer Engineering, University of Houston (UH) in 2016. She is now working as the Postdoctoral Fellow at the Network Communications and Economics Lab, the Chinese University of Hong Kong (CUHK). Her current research involves the contract theory and matching theory in network economics, Internet and applications, wireless communications and networking. She received the best paper award at IEEE ICCS 2016.

Yunan Gu (S’13) received her B.S. degree in electronic engineering from Nanjing University of Science and Technology in June 2011, her M.S. in computer science from Texas Southern University in 2013, and her Ph.D. from the University of Houston. Now, she is in an R&D engineer in the IP Technology Research Division of Huawei. Her research interests include matching theory, LTE-Unlicensed, D2D, V2X, and so on.

Miao Pan (S’07-M’12) received his BSc degree in Electrical Engineering from Dalian University of Technology, China, in 2004, MASc degree in electrical and computer engineering from Beijing University of Posts and Telecommunications, China, in 2007 and Ph.D. degree in Electrical and Computer Engineering from the University of Florida in 2012, respectively. He is now an Assistant Professor in the Department of Electrical and Computer Engineering at University of Houston. He was an Assistant Professor in the Computer Science at Texas Southern University from 2012 to 2015. His research interests include cognitive radio networks, cyber-physical systems, and cybersecurity. His work on cognitive radio network won Best Paper Award in Globecom 2015. Dr. Pan is currently Associate Editor for IEEE Internet of Things (IoT) Journal. He is a member of IEEE.

N guyen H. Tran (S’10-M’11) received the BS degree from Hochiminh City University of Technology and Ph.D degree from Kyung Hee University, in electrical and computer engineering, in 2005 and 2011, respectively. Since 2012, he has been an Assistant Professor with Department of Computer Science and Engineering, Kyung Hee University. His research interest is applying analytic techniques of optimization, game theory, and stochastic modelling to cutting-edge applications such as cloud and mobile-edge computing, datacenters, 5G, and deep learning for networks. He received the best KHU thesis award in engineering in 2011 and sereval best paper awards, including IEEE ICC 2016, APNOMS 2016, and IEEE ICCS 2016. He is the Editor of IEEE Transactions on Green Communications and Networking.

Zaher Dawy (SM’09) received the B.E. degree in computer and communications engineering from the American University of Beirut (AUB), Beirut, Lebanon, in 1998 and the M.E. and Dr.-Ing. degrees in communications engineering from Munich University of Technology (TUM), Munich, Germany, in 2000 and 2004, respectively. Since 2004, he has been with the Department of Electrical and Computer Engineering, AUB, where he is currently a Professor. His research and teaching interests include wireless communications, cellular technologies, context-aware mobile computing, mobile solutions for smart cities, computational biology, and biomedical engineering. Dr. Dawy is Editor for IEEE Communications Surveys and Tutorials, IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, and Elsevier Physical Communications. He has also served as Executive Editor for Wiley Transactions on Emerging Telecommunications Technologies (ETT) from 2011 to 2014. He received Abdul Hameed Shoman Award for Young Arab Researchers in 2012, IEEE Communications Society 2011 Outstanding Young Researcher Award in Europe, Middle East, and Africa Region, AUB Teaching Excellence Award in 2008, Best Graduate Award from TUM in 2000, Youth and Knowledge Siemens Scholarship for Distinguished Students in 1999, and Distinguished Graduate Medal of Excellence from Hariri Foundation in 1998.

Zhu Han (S’01–M’04-SM’09-F’14) received the B.S. degree in electronic engineering from Tsinghua University, in 1997, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, in 1999 and 2003, respectively.

From 2000 to 2002, he was an R&D Engineer of JDSU, Germantown, Maryland. From 2003 to 2006, he was a Research Associate at the University of Maryland. From 2006 to 2008, he was an assistant professor at Boise State University, Idaho. Currently, he is a Professor in the Electrical and Computer Engineering Department as well as in the Computer Science Department at the University of Houston, Texas. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. Dr. Han received an NSF Career Award in 2010, the Fred W. Ellersick Prize of the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the Journal on Advances in Signal Processing in 2015, IEEE Leonard G. Abraham Prize in the field of Communications Systems (best paper award in IEEE JSAC) in 2016, and several best paper awards in IEEE conferences. Currently, Dr. Han is an IEEE Communications Society Distinguished Lecturer.