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Dynamics of service selection and provider pricing game in heterogeneous cloud market

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ABSTRACT

This paper studies price competition in a heterogeneous cloud market formed by public providers and a cloud broker, all of which are also known as cloud service providers (CSPs). We formulate the price competition between CSPs as a two-stage noncooperative game. In stage I, in which CSPs set their service prices to maximize their revenues, we model the pricing game using the noncooperative static game. We provide sufficient conditions for the existence and uniqueness of Nash equilibrium prices, which can be obtained using an iterative algorithm. The convergence properties of the iterative algorithm are characterized using the contract mapping theorem. In stage II, given the prices set by CSPs, cloud users can select the services that provide them the best payoff in terms of performance (i.e., delay) and price. We apply an evolutionary game to study the evolution and dynamic behavior of cloud users. Furthermore, we use the Wardrop equilibrium and replicator dynamics to determine the equilibrium and its convergence properties of the service selection game. To attract users to the equilibrium, we implement the service selection algorithms using population evolution and reinforcement learning approaches. Numerical results illustrate that our game models can provide comprehensive understanding of the heterogeneous CSPs market and service selection in cloud computing.

1. Introduction

Recently, cloud computing has become more and more popular in large-scale computing due to its ability to share data and computations over a network of scalable nodes. The number of infrastructure as a service (IaaS) providers is increasing quickly as the cloud computing market grows. Hence, cloud users have to deal with many different service types, pricing schemes and cloud interfaces. In the beginning of cloud deployment, an IaaS public cloud provider, e.g., Amazon, Google, Microsoft, which we call a public provider (PP), dominates the market. However, the cloud computing market trend shows an increasing market share of the multi-cloud or federated clouds as IaaS providers by a cloud broker (CB) (Yaw et al., 2015; Panda et al., 2014; Panda and Jana, 2015; Manvi and Shyam, 2014). The emergence of this paradigm is forming a heterogeneous market leading to the complicated economic marketplace. The performance delivered by IaaS providers to cloud user depends on both the resource allocation (as traditional issues) and the strategic incentives that come from the multi-tiered economic interactions that consists of two components as follows. The first interaction is competition among IaaS providers for cloud users and among IaaS providers. The second interaction is between cloud users who are both price-sensitive and performance-sensitive when choosing an IaaS provider. Thus, the precise pricing model considered in a heterogeneous market is significant. However, to the best of our knowledge, till now no research has considered both of pricing method and user service selection.

In this paper, by incorporating the heterogeneity of CSPs (i.e., CB and PP) and the dynamical behavior of users, we study price competition in a heterogeneous CSP market in cloud computing. We focus on the pricing problem of CSPs, who compete with each other by setting service prices to maximize their revenues. At the same time, we consider a fundamental question of cloud users, i.e., from which CSP it is better for a cloud user to select the cloud service. By tackling this problem, our contributions can be presented as follows.

- We propose a game theoretical model in a heterogeneous CSP market in which there are two stages of competition. In stage I,

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on the CSP side, we formulate the competition among CSPs for selling service opportunities as a noncooperative game, where each CSP can set a service price such that its revenue is maximized. We use $M/M/1$ and $M/M/\infty$ queue models to show correlations among the expected task finishing times, resource capacity, and the request rates (from cloud users to the CB and PP, respectively). To the best of our knowledge, this is the first study that discusses heterogeneous CSP competition queueing models ($M/M/1$ and $M/M/\infty$). Since the pricing strategy of a CSP depends on its competitors, we use the game theoretic approach to study the strategic situation in the noncooperative static game (NSG). We provide the sufficient conditions for existence and uniqueness of closed form Nash equilibrium prices. Furthermore, we use an iterative algorithm to determine the Nash equilibrium in a distributed manner.
- In stage II, on the cloud user side, given prices set by CSPs, rational cloud users can select a service from a CSP that provides them the best payoff in terms of performance (i.e. delay) and price. We use the Wardrop equilibrium to derive a steady-state equilibrium achievable by cloud users in the service selection game. Wardrops principles are used extensively in modeling the traffic distribution of communication networks and transportation networks and has received significant attention in the algorithmic game theory community (Roughgarden and Tardos, 2002). We further focus on modeling the dynamic behavior of cloud users using the evolutionary game approach (Sandholm, 2010), which characterizes the strategic interactions among a large number of users, whose behaviors are modeled as a dynamic adjustment process. Evolutionary game theory was first used in biology to study the change of animal populations, and then later applied in economics to model human behaviors. It is most useful to understand how a large population of users converges to Nash equilibria in a dynamic system. We use replicator dynamics (Sandholm, 2010), which are expressed as a set of differential equations, to model the evolution of the cloud users since such cloud users adapt their service selection based on the observed system state. We then analyze equilibrium and convergence properties of the proposed game. In the case of perturbation in the evolutionary game, we use a Markov chain to investigate behavior in the cloud user society over very long time spans. To attract cloud users to the equilibrium, we implement service selection algorithms through population evolution and reinforcement learning approaches.

The remainder of this paper is organized as follows. Section 2 discusses the related work. Section 3 introduces the system models. In Section 4, we present pricing competition in a duopoly heterogeneous CSP market. Section 5 presents the dynamic service selection game of cloud users. Section 6 shows the numerical results. Section 7 presents the multiple CSP scenario. Section 8 draws the conclusions.

2. Related work

We first review the most notable works on resource allocation and pricing in cloud systems. Then, we discuss relevant works that use game theory to study price competition and user behaviors.

Considering prices charged by cloud providers, the authors in Hong et al. (2011) and Tsakalozos et al. (2011) used dynamic programming and microeconomics, respectively, to solve the resource allocation problems for cloud users. Xu and Li (1952) studied dynamic cloud pricing using a proposed revenue management framework. In Kantere et al. (2011), the authors proposed a price-demand model for the cloud cache and found the optimal price that maximizes the cloud provider's profit. Auction is one of the pricing schemes widely applied to solve the resource allocation problems (Zhang et al., 2013). Teng and Magoules (2010) and Mihailescu and Teo (2010b) applied auction mechanisms to find optimal prices in the cloud, in which cloud users had budgetary and deadline constraints, respectively. In Lee et al. (2013), based on a combinatorial double auction, the authors proposed a real-time group auction system that improves resource efficiency and monetary benefits for both users and providers in the cloud instance market. Qiu et al. (2014) applied a two-stage Stackelberg game where one broker is the leader and the private clouds are the followers. However, in the heterogeneous cloud computing market, we assume that the cloud brokers and the public cloud providers play the same role and compete each other. Thus, we resort to a limiting regime (a non-cooperative static game) to be able to provide analytical results.

Nevertheless, most of these works focused on provider pricing and the responses of cloud users via their demand functions. In this paper, we focus on the pricing mechanisms and their impacts on the equilibrium behaviors of users in a strategic queueing system, where arriving users can take the delay and service price into account to make their service selection strategically, which can be traced back from the work of Naor (1969), Edelson and Hilderbrand (1975), Stidham (2009), and Hassin and Haviv (2003). Several works have addressed this paradigm of wireless network, including Do et al. (2012a, 2012b, 2014), Tran et al. (2013), and Nan et al. (2012). In the paradigm of cloud computing, Feng et al. (2014) and Anselmi et al. (2011) examine the optimal prices that can be determined in a competitive environment with more than one cloud provider. However, they only consider the market formed by homogeneous IaaS providers, all of which use the same $M/M/1$ queue to derive the expected delay of cloud users. Feng et al. (2014) and Anselmi et al. (2011) do not consider the dynamic behavior of users in such a multiple CSP scenario. Recent works have considered evolutionary games to study user behavior in cognitive radio and heterogeneous wireless networks (Niyato and Hossain, 2009; Niyato et al., 2009; Elias et al., 2013). However, no works use them for cloud computing.

There are similar pricing in the literature for cloud market. Ardagna et al. (2013) consider a two-tier model capturing the interaction between SaaSs and a single IaaS and study the existence and efficiency of equilibrium allocations. Similarly, Anselmi et al. (2011), Feng and Li (2013) and Nan et al. (2014) considered two-tier models capturing the interaction between users and SaaSs or between SaaSs and PaaSs/IaaSs, and studied the existence and efficiency of equilibrium allocations. A typical approach to solve a two-tier pricing game is the Stackelberg game (Nan et al., 2014; Wahab et al., 2016). Here, we also apply a two-stage Stackelberg game, which is widely used in wireless and cloud computing services (Nan et al., 2014; Niyato et al., 2009). Feng and Li (2013) studied price competition between multiple IaaS cloud providers in a homogeneous market but with different resource capacities. The authors applied an $M/M/1$ queue to capture correlations among the expected task finishing times, the IaaS provider resource capacity and the request rates from users. Yet, the context of this paper is price competition and user service selection in a heterogeneous cloud computing environment, which has a different system model. Firstly, in a heterogeneous market, we use an $M/M/1$ queue for a public IaaS provider and an $M/M/\infty$ queue for multi-cloud, and such heterogeneity in IaaS provider resource capacities (i.e., service-level agreements) makes our analyses much more challenging. Secondly, the model considered in this paper is to capture the two-tier competing dynamics between users and IaaS providers simultaneously.

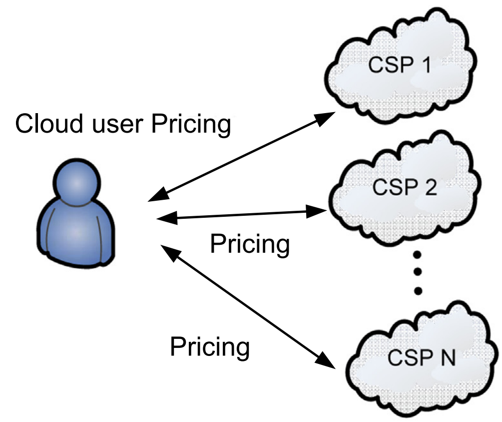
3. The system model

This paper aims to introduce and analyze a stylized model capturing the multi-tiered interaction between users and cloud providers in a manner that exposes the interplay of congestion, pricing, and performance issues. To accomplish this, we introduce a novel two-tier model for the heterogeneous cloud computing market. This model considers the strategic interaction between users and IaaS provider (the first and second tiers), within each tier there is also competition among users and IaaS providers, respectively. The details of the model are provided by three key features: (i) IaaS providers compete by strategically determining their price in order to maximize profit, which depends on the number of users they attract; (ii) users strategically determine which IaaS provider to use depending on a combination of performance and price; (iii) the performance experienced by the users is affected by the congestion of the resources procured at the IaaS chosen by the users, and that this congestion depends only on total traffic of users to the IaaS. We first introduce preliminaries of the system model. We then present the CB and PP models.

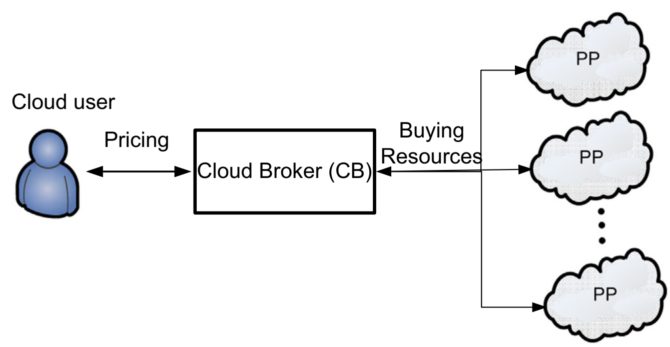
3.1. Preliminaries

The multi-cloud model can integrate resources from different IaaS providers, which increases scalability/reliability and reduces cost, while access to the resources is transparent to users (Toosi et al., 2012; Mihailescu and Teo, 2010a). In the multi-cloud architecture, illustrated in Fig. 1, a cloud broker (CB) is essential to transform the cloud computing market into a commodity-like service (Tordsson et al., 2012). A CB offers a provisioner that analyzes the workload, schedules Virtual Machine (VM) placement among multiple clouds providers and optimizes deployments (Calheiros et al., 2011). Moreover, the VM manager in the CB can be used to provide a uniform interface with VM management, independent of the particular cloud provider technology (Tordsson et al., 2012). Currently, an IaaS PP (e.g., Amazon) operates as a standalone IaaS cloud service provider (CSP) (Fig. 2(a)). However, in a multi-cloud model (Fig. 2(b)), a CB offers cloud services to users as a CSP (Hossain et al., 2013). The CB (a third party) acts as a mediator between the cloud user and the IaaS provider. Cloud users buy resources from the CB instead of the cloud provider in order to obtain additional benefit (e.g., compensation) (Hossain et al., 2013). The cloud brokerage model can be used to offer a commendable pricing mechanism that considers the least expensive service for a cloud user, as well as more profit for CSPs (Hossain et al., 2013).

A heterogeneous CSP market consists of the PP and CB offering services to the same pool of cloud users. The cloud users arrive according to a Poisson process with rate λ. Upon arrival, each cloud user has to make a decision: (1) acquiring a multi-cloud service from a CB for guaranteed service; or (2) using the legacy public cloud service from a PP (i.e., Amazon). We assume that the CB has an $M/M/\infty$ queue of virtualized instances because the CB can overcome the resource limitation by buying resources from cloud providers. Then, the CB can offer services with guaranteed Quality of Service (QoS). However, the PP only has $M/M/1$ queue of virtualized instances due to the limitation of resources. There are two stages of competition, illustrated in Fig. 3, in this heterogeneous market. In stage I, CSPs (CB and PP) are competitive to sell services to cloud users. If the price and delay offered by one CSP are high,



(a) Standalone Cloud Service Providers (CSPs).



(b) Multi-cloud with a Cloud Broker (CB).

Fig. 2. Pricing in standalone and multi-cloud.

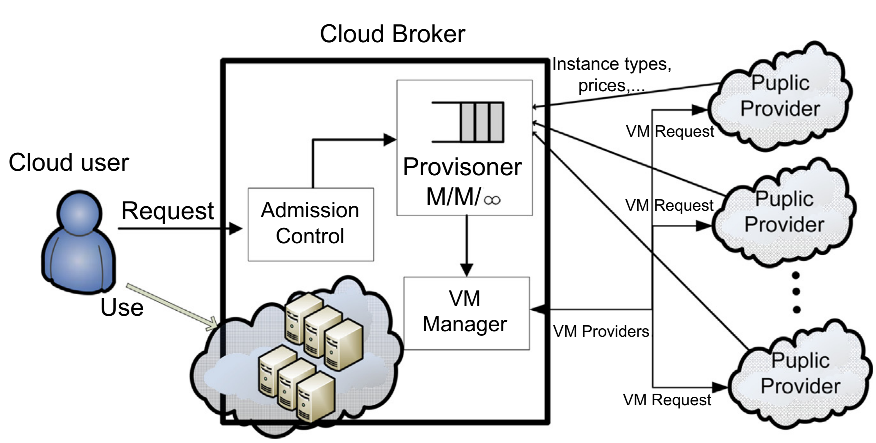


Fig. 1. Multi-cloud architecture with a cloud broker.

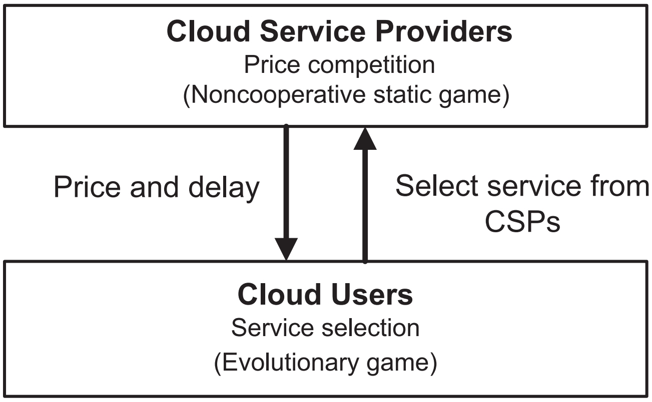


Fig. 3. Two-stage CSP price competition and user service selection games in a heterogeneous cloud market.

cloud users will deviate to choose the service from another CSP. Therefore, each CSP must individually set the price carefully so that its revenue is maximized. Therefore, the game-theoretic formulation is a natural solution for the competition between CSPs due to the mutually opposing interest of CSPs. In this paper, we use a noncooperative static game (NSG) (Osborne, 1994) to model the pricing competition between CSPs in stage I. In stage II, given the prices set by CSPs, cloud users select the CSP to join. If many cloud users choose a cloud service provided by the same CSP, the corresponding service becomes congested, which may result in performance degradation (i.e., increased the delay). As a result, cloud users will evolve to choose a CSP offering lower price and better performance. The evolution of a cloud user will stop when the cost (the service price plus the delay cost) becomes identical to the average cost of all cloud users. Thus, we formulate the service selection process of cloud users as an evolutionary game (Sandholm, 2010). Here, we mainly consider a heterogeneous duopoly market that has one PP and one CB. We then consider the multiple CSP scenario in Section 6. Let us denote by λ_1 the overall cloud user arrival rate at the CB and by λ_2 the rate of cloud users at the PP, so that $\lambda = \lambda_1 + \lambda_2$.

3.2. Cloud broker model

The average cost incurred by a cloud user consists of two components: (i) the service price p_1 of the CB and (ii) the average delay cost. We assume that the CB always has a sufficient number of servers to serve the demand of cloud users. As illustrated in Fig. 1, whenever an arriving cloud user decides to request the service from the CB, the admission control unit sends the accepted request to the provisoner. The provisoner finds an allocation of VM among different cloud providers that optimizes the user criteria and meets the placement constraints. The VM manager can provide a unified management interface for operations, e.g., to deploy, monitor and terminate VMs, with multiple IaaS providers. The CB is modeled as an $M/M/\infty$ queue, serving a common pool of potential cloud users with infinite servers, which combines the resource capacity of multiple cloud providers operated by the VM manager. The $M/M/\infty$ queuing model has been adopted by a number of existing papers that analyzed data center or cloud provider operations. In Duffield and Whitt (1997), the authors represent the large multi-server system as an $M/G/\infty$ system. In Stolyar (2012), the authors solve the "packing" of VMs in the physical host machines problem in a network cloud having an infinite server system. We assume that the virtual resource capacity of the CB is represented by its service rate μ (Feng et al., 2014; Khazaei et al., 2012; Rao et al., 2010). Let α be the delay cost per unit time (i.e., α represents user urgency). The expected cost when acquiring the multi-cloud service from the CB is thus given by

$$C_1 = \frac{\alpha}{\mu} + p_1. \tag{1}$$

The revenue of the CB corresponds to the total revenue obtained by pricing users. As a consequence, the CB utility function is expressed as follows:

$$U_1 = \lambda_1 p_1. \tag{2}$$

3.3. Public provider model

If a cloud user selects a cloud service of the PP, it joins a queue of cloud users who have chosen the same PP. This queue is used in order to model the delay incurred when a few cloud users wish to use the same cloud infrastructure of the PP. Here, the PP system is modeled by an $M/M/1$ queue, serving a common pool of potential cloud users with one "virtual" server. In queueing theory, the multi-server system can be modeled as $G/G/m$ (i.e., general arrival distribution and general service rate distribution) (Khazaei, 1975) or $M/G/m/m + r$ (i.e., exponential arrival distribution and general service rate distribution with a finite buffer) (Khazaei et al., 2012). However, since the analysis will be very complex in the general model, researchers have to approximate the general model to solvable models in order to proceed with analysis and prediction. Thus, the $M/M/1$ queuing model has been used in cloud computing literature to offer closed form results (Feng et al., 2014; Gupta et al., 2010; Yu et al., 2014).

The PP system is modeled as an $M/M/1$ queue with a service rate μ. We assume that $\lambda < \mu$ for the queue stable condition. Here, we mainly consider homogenous service rates where the service rates of the PP and the CB are the same (i.e., μ). The main reason is that we focus on the competition of CSPs at the same class of service or similar configurations (e.g., consider two CSPs, Amazon and Google: both Amazon m1.medium and Google n1-standard1 have one virtual CPU and 3.75 GB RAM; Amazon c3.large and Google n1-standard-8 have eight virtual CPUs and 30 GB RAM). Thus, the service rates of CSPs are approximately equal at the same class of service. Based on queuing theory (Khazaei, 1975), the expected cost when acquiring the service from the PP is thus given by

$$C_2 = \frac{\alpha}{\mu - \lambda_2} + p_2. \tag{3}$$

The PP utility function is expressed as follows:

$$U_2 = \lambda_2 p_2. \tag{4}$$

4. Pricing competition in heterogeneous duopoly market

Due to the complexity of the model, we need to consider a limiting regime in order to be able to provide analytical results. There are monopoly market (one service provider dominates all the market), duopoly market (two service providers share the market), and oligopoly market (more than two service providers share the market). Motivated by the huge and growing number of IaaS providers, the limiting regime we consider is the duopoly market as a starting point. In this setting, we can attain an analytical characterization of the interacting markets which yield interesting qualitative insights.

In this section, we derive the equilibrium points, namely: (i) the equilibrium rates $(\lambda_1^e, \lambda_2^e)$ at which cloud users join the CB and PP, respectively; (ii) the equilibrium prices set by the CB and PP in a heterogeneous duopoly market (two CSPs share a market). This paper studies the duopoly scenario for the ease of analysis. However, the duopoly scenario can still provide insights into pricing and user dynamics of the heterogeneous CSPs market. The analysis introduced in Sections 4 and 5 can be similarly extended to the multiple PPs and CBs scenario in Section 6 at the expense of the increased complexity.

4.1. Wardrop equilibrium of service selection game

Given prices (p_1, p_2), the equilibrium rates $(\lambda_1^e, \lambda_2^e)$ are achieved by cloud users in the service selection game, in which a large number of cloud users individually determine CSPs, they should buy the cloud service. In the service selection game, there are two conditions: first, cloud users individually minimize the perceived cost, which is expressed as C_1 in (1) if they choose the CB and C_2 in (3) if they choose the PP; second, at the equilibrium point, the cost C_1 is equal to the cost C_2 if both CSPs have a non-zero arrival rate of cloud users. The above two conditions satisfy the two Wardrop's principles (Wardrop, 1952; Roughgarden and Tardos, 2002), that are: the total costs perceived by users on all used services are equal, and the average delay/cost is minimum. Thus, the Wardrop equilibrium is defined as follows.

Definition 4.1. The pair of arrival rates $(\lambda_1^e, \lambda_2^e)$ is a Wardrop equilibrium if and only if there exist $C > 0$ such that:

$$C_i(\lambda_i^e) = C, \quad \text{if } \lambda_i^e > 0, \ i = 1, 2, \tag{5}$$

$$C_i(\lambda_i^e) > C, \quad \text{if } \lambda_i^e = 0, \ i = 1, 2, \tag{6}$$

$$\lambda_1^e + \lambda_2^e = \lambda. \tag{7}$$

At the Wardrop equilibrium, if $C_1 = C_2 = C$, then we have

$$\frac{\alpha}{\mu} + p_1 = \frac{\alpha}{\mu - \lambda_2^e} + p_2. \tag{8}$$

We can compute the equilibrium cloud user request $\lambda_2{}^e$ for the PP as a function of the prices set by both the CB and PP as follows:

$$\lambda_2^e = \frac{(p_1 - p_2)\mu^2}{(p_1 - p_2)\mu + \alpha}, \tag{9}$$

with $0 < \lambda_2^e < \lambda$. Then, the equilibrium cloud user request $\lambda_1{}^e$ sent to the CB is as follows:

$$\lambda_1^e = \lambda - \lambda_2^e. \tag{10}$$

We observe that, if $p_1 \le p_2$, $(\lambda_1^e = \lambda, \lambda_2^e = 0)$ is the trivial and unique Wardrop equilibrium. To that end, we assume that $p_1 > p_2$ for nontrivial results.

4.2. Noncooperative static game

We consider a noncooperative static game (Osborne, 1994) in which the CB and PP compete with each other by setting the price simultaneously to maximize their utilities. Then, given a particular service price p_1 of the CB, the PP will determine the best response service price p_2, and vice versa. Motivated by the concept of the Nash equilibrium, we define the equilibrium prices (p_1^{ns}, p_2^{ns}), from which no CSP trying to maximize its own utility has any incentive to deviate unilaterally. The Nash equilibrium is obtained using the best response function, which is the best strategy of one CSP given the strategies of other CSPs. To achieve a nontrivial Wardrop equilibrium arrival rate, we focus on a region $p_1 > p_2$ while deriving the Nash equilibrium prices. Given a service price p_2, the best response function (or the reaction curve) $BR_1(p_2)$ of the CB can be expressed in terms of p_2 as follows:

$$BR_1(p_2) = \arg \max_{p_{max} \ge p_1 > p_2} U_1(p_1, p_2). \tag{11}$$

Here, we assume that p_{max} is the maximum price CSPs can set for all users. The reason is that, if the price offered by CSPs is too high, users will prefer to build their own internal server cluster rather than to buy cloud services (i.e., VM) from CSPs. Similarly, given a price p_2, the best response function $BR_1(p_2)$ is as follows:

$$BR_2(p_1) = \arg \max_{p_1 > p_2 \ge 0} U_2(p_1, p_2). \tag{12}$$

For the proposed noncooperative static game, the Nash equilibrium is defined as follows.

Definition 4.2. The prices (p_1^{ns}, p_2^{ns}) are a Nash equilibrium price for the proposed noncooperative static game if and only if $(p_1^{ns} = BR_1(p_2^{ns}),\ p_2^{ns} = BR_2(p_1^{ns}))$.

The existence of the Nash equilibrium price implies that two reaction curves $BR_2(p_1)$ and $BR_1(p_2)$ have intersection points. Let us define:

$$a \triangleq \frac{\alpha}{\mu - \lambda}, \quad \rho \triangleq \frac{\lambda}{\mu}, \quad \text{and} \quad d \triangleq \frac{\alpha}{\mu}. \tag{13}$$

According to the second-order condition (Boyd and Vandenberghe, 2004), we characterize the convexity of utility functions $U_1(p_1, p_2)$ and $U_2(p_1, p_2)$ as shown in Lemma 4.1.

Lemma 4.1. *Given a price p_1, $U_2(p_1, p_2)$ is strictly concave in $(0, p_1)$ w.r.t. p_2. Given a price p_2, if $p_2 < d$, $U_1(p_1, p_2)$ is strictly concave; otherwise $U_1(p_1, p_2)$ is convex in (p_2, p_{max}) w.r.t. p_1.*

To find the intersection points of $BR_2(p_1)$ and $BR_1(p_2)$, we solve simultaneously two CSP revenue maximization problems as follows:

$$\arg \max_{p_{max} \ge p_1 \ge 0} U_1(p_1, p_2), \tag{14}$$

$$\arg \max_{p_1 > p_2 \ge 0} U_2(p_1, p_2), \tag{15}$$

where the utilities $U_1(p_1, p_2)$ and $U_2(p_1, p_2)$, in which the equilibrium arrival rates $(\lambda_1^e, \lambda_2^e)$ are defined by the Wardrop equilibrium as (9) and (10), are given as follows:

$$U_1(p_1, p_2) = p_1\lambda_1^e = p_1\left[\lambda - \frac{(p_1 - p_2)\mu^2}{(p_1 - p_2)\mu + \alpha}\right], \tag{16}$$

$$U_2(p_1, p_2) = p_2\lambda_2^e = p_2\frac{(p_1 - p_2)\mu^2}{(p_1 - p_2)\mu + \alpha}. \tag{17}$$

By solving the first-order condition $\frac{\partial U_1}{\partial p_1} = 0$ and $\frac{\partial U_2}{\partial p_2} = 0$ simultaneously, we obtain

$$\begin{cases} p_1 = p_2 - d + \sqrt{(d - p_2)\dfrac{\alpha}{\mu - \lambda}}, \\ p_2 = p_1 - d + \sqrt{d^2 + p_1 d}. \end{cases} \tag{18}$$

Finally, we obtain

$$\begin{cases} p_1^e = \dfrac{d\left(a^2 - 2ad - 2d^2 + a^{3/2}\sqrt{5a + 4d}\right)}{2(a + d)^2}, \\ p_2^e = d - \dfrac{d^2\left(3a + 2d + \sqrt{a}\sqrt{5a + 4d}\right)}{2(a + d)^2}. \end{cases} \tag{19}$$

Combing the conditions from the Wardrop and Nash equilibrium, we have the sufficient condition for the existence of a Nash equilibrium, as summarized in Theorem 4.1.

Theorem 4.1. *If there exists a pair* (p_1^*, p_2^*) *such that:*

$$p_{max} > p_1^* > p_2^* > 0, \tag{20}$$

$$\lambda > \lambda_2^e = \frac{(p_1^* - p_2^*)\mu^2}{(p_1^* - p_2^*)\mu + \alpha} > 0, \tag{21}$$

$$U_1(p_1^*, p_2^*) \ge U_1(p_1, p_2^*), \quad \forall\, p_{max} \ge p_1 > p_2^*, \tag{22}$$

$$U_2(p_1^*, p_2^*) \ge U_2(p_1^*, p_2), \quad \forall\, p_1^* > p_2 > 0, \tag{23}$$

then (p_1^*, p_2^*) *is an interior Nash equilibrium.*

Proof. Condition (20) guarantees that (p_1^*, p_2^*) in the feasible region. Condition (21) is obtained directly by the Wardrop equilibrium. Conditions (22) and (23) guarantee that (p_1^*, p_2^*) is an interior Nash equilibrium. □

Having results from Theorem 4.1, the sufficient condition, which guarantees (p_1^e, p_2^e) be an interior Nash equilibrium, is stated in Theorem 4.2 as follows.

Theorem 4.2. *If* (p_1^e, p_2^e) *satisfies:*

$$\min(d(1/(1-\rho)^2 - 1), p_{max}) > p_1^e > p_2^e > 0, \tag{24}$$

then (p_1^e, p_2^e) *is an interior Nash equilibrium.*

Proof. Since $p_1^e > 0$ and $p_2^e > 0$, we have $p_1^e - p_2^e = \sqrt{d^2 + p_1^e d} - d > 0$, which implies $p_1^e > p_2^e$. Thus, the condition in (20) is satisfied. To guarantee the condition in (21), we require the inequality $d(1/(1-\rho)^2 - 1) > p_1^e$, which is obtained by substituting (p_1^e, p_2^e) in (19) into (21). We prove that (p_1^e, p_2^e) satisfies the condition of the Nash equilibrium (22) and (23) as follows.

Since we have $d > p_2^e > 0$, by using Lemma 4.1, $U_1(p_1, p_2^e)$ is a strictly concave function in (p_2^e, p_{max}) w.r.t. p_1. We can solve problem (14) by finding the root of $\frac{\partial U_1}{\partial p_1} = 0$. Since ${p_1}^e$ is the root of $\frac{\partial U_1}{\partial p_1} = 0$, ${p_1}^e$ maximizes $U_1(p_1, p_2^e)$ in (p_2^e, p_{max}) w.r.t. p_1. Then, condition (22) is verified.

From Lemma 4.1, because $U_2(p_1^e, p_2)$ is a strictly concave function in $(0, p_1^e)$ w.r.t. p_2 and the root of $\frac{\partial U_2}{\partial p_2} = 0$ is ${p_2}^e$, then ${p_2}^e$ maximizes $U_2(p_1^e, p_2)$ in $(0, p_1^e)$ w.r.t. p_2. Therefore, condition (23) is verified.

Then, (p_1^e, p_2^e) satisfies all condition in Theorem 4.1. The proof is completed. □

We will investigate the uniqueness of the Nash equilibrium together with the convergence of an iterative algorithm (Algorithm 1) based on the best response dynamics, which is commonly used to study Nash equilibrium stability. With starting prices (p_1^0, p_2^0), the algorithm iterates until the convergence condition ($\vartheta <$ a predefined threshold th) is satisfied.

Algorithm 1. Iterative algorithm.

1. Initialize parameters: a value of $(p_1(0), p_2(0))$, t = 1;
2: *while* $\vartheta \ge th$:
3: $p_2(t) = BR_2(p_1(t-1))$,
4: $p_1(t) = BR_1(p_2(t-1))$,
5: $\vartheta = |p_2(t) - p_2(t-1)| + |p_1(t) - p_1(t-1)|$,
6: $t = t + 1$,
7: *end while*

By imposing a condition on p_{max}, we can guarantee both the uniqueness of the Nash equilibrium and convergence of Algorithm 1 to this equilibrium by using contraction mapping theorem (Bertsekas and Tsitsiklis, 1989; Lee et al., 2007). Contraction mapping theorem in Bertsekas and Tsitsiklis (1989) is as follows.

Theorem 4.3. *Let* Φ *be a complete metric Euclidean space and* $f: \Phi \to \Phi$ *be a mapping. Suppose there is a constant* $\kappa \in [0, 1)$ *such that* $\|f(x) - f(y)\| \le \kappa \|f(x) - f(y)\|, \forall\, x, y \in \Phi$, *where* $\|\cdot\|$ *is some norm. Such an f is called a contraction. Then f has a unique fixed point* $p^* \in \Phi$. *Furthermore, the sequence* $p(t) = f(p(t-1))$ *converges to a unique fixed point.*

Then, the convergence of Algorithm 1 and uniqueness of the Nash equilibrium are characterized in Theorem 4.4 as follows.

Theorem 4.4. *Let* (p_1^e, p_2^e) *be the Nash equilibrium. Then, Algorithm 1 converges to the Nash equilibrium price* (p_1^e, p_2^e), *which is unique if* $p_1^e < p_{max} < d - a/16$.

Proof. If $p_1^e < p_{max} < d - a/16$, both $U_1(p_1)$ and $U_2(p_2)$ are strictly concave in the feasible region. Based on Algorithm 1, we have iterations to update $(p_1(t), p_2(t))$ by using the first order conditions $\frac{U_2}{\partial p_2} = 0$ and $\frac{U_1}{\partial p_1} = 0$ as follows:

$$p_1(t) = BR_1(p_2(t-1)) = p_2(t-1) - d + \sqrt{(d - p_2(t-1))a}, \tag{25}$$

$$p_2(t) = BR_2(p_1(t-1)) = p_1(t-1) + d - \sqrt{d^2 + p_1(t-1)d}. \tag{26}$$

We show that the updating rule from $(p_1(t-1), p_2(t-1))$ to $(p_1(t), p_2(t))$ is a contraction mapping, which proves that the iterations of Algorithm 1 converge to a unique fixed point (i.e., the Nash equilibrium price (p_1^e, p_2^e)).

The update iterations in Algorithm 1 can be written as

$$p(t) = F(p(t-1)), \quad t = 1, 2, \ldots \tag{27}$$

where $p(t) \in \Re_+^2$ is the price vector $(p_1(t), p_2(t))$ and $F: \Re_+^2 \to \Re_+^2$ is a mapping from $\Re_+^2$ into itself. Based on the property of matrix norm, we have

$$\|F(x) - F(y)\| \le \left\| \frac{\partial F}{\partial p} \right\| \cdot \|x - y\|. \tag{28}$$

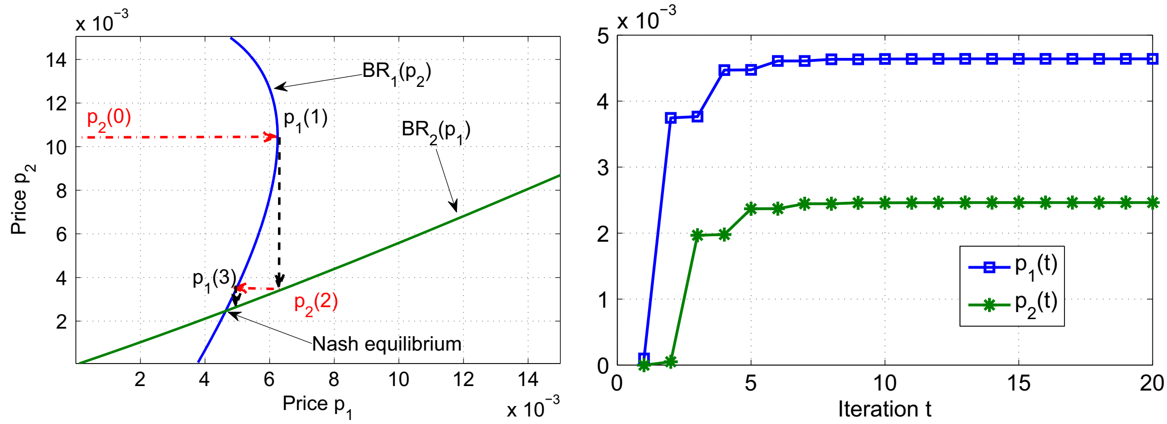
To prove that the mapping (27) is a contraction, we show that there exists a constant $\kappa \in [0, 1)$ such that the Jacobian $\|\frac{\partial F}{\partial p}\| \le \kappa$ for a certain norm. The Jacobian $\frac{\partial F}{\partial p}$ is defined as follows:

$$\mathbf{J} = \begin{bmatrix} 0 & 1 - \dfrac{a}{2\sqrt{a(d - p_2)}} \\ 1 - \dfrac{d}{2\sqrt{d^2 + dp_1}} & 0 \end{bmatrix} \tag{29}$$

Using $\|\cdot\|_\infty$, we have

$$\|\,\mathbf{J}\,\|_\infty = \max\left(\left| 1 - \frac{a}{2\sqrt{a(d - p_2)}} \right|, \left| 1 - \frac{d}{2\sqrt{d^2 + dp_1}} \right| \right).$$

We can see that $\left| 1 - \frac{d}{2\sqrt{d^2 + dp_1}} \right| < 1, \quad \forall\, p_1 > 0$. To guarantee $\left| 1 - \frac{a}{2\sqrt{a(d - p_2)}} \right| < 1, \forall\, p_2 \in (0, p_{max})$, we have to require $p_{max} < d - a/16$. Therefore, if $p_1^e < p_{max} < d - a/16$, we have



(a) Illustration of Algorithm 1. (b) Convergence speed of Algorithm 1.

Fig. 4. Illustration of convergence speed of Algorithm 1 with parameters: $\mu = 30$, $\lambda = 10$, $\alpha = 0.5$, $p_{max} = 15.\ 10^{-3}$. These parameters imply the Nash equilibrium point $(p_1^e, p_2^e) = (4, 6.\ 10^{-3}, 2, 4.\ 10^{-3})$.

$\|\mathbf{J}\|_\infty < 1$. Then, there exists a positive constant $\kappa \in [0, 1)$ such that $\|\frac{\partial F}{\partial p}\|_\infty \le \kappa$. Thus, we have shown that

$$\|F(x) - F(y)\|_\infty \le \kappa \|x - y\|_\infty, \tag{30}$$

which proves that F is a contraction. Using Theorem 4.3, Algorithm 1 converges to the unique fixed point, which is the Nash equilibrium price (p_1^e, p_2^e). □

Numerical example: We use an example to illustrate how prices converge iteratively. Fig. 4(a) shows an example of the best response function $BR_2(p_1)$ and $BR_1(p_2)$ when two reaction curves have one intersection point that is the unique Nash equilibrium point (p_1^{ns}, p_2^{ns}). Fig. 4(a) also illustrates the iterative algorithm. At first, given a starting price $p_2(0)$, the CB reacts by setting the best response price $p_1(1)$ according to (11). In the next period, given the price $p_1(1)$, the PP reacts by setting the best response price $p_2(2)$ according to (12). This process continues until the convergence condition is satisfied. Fig. 4(b) shows that the iterative algorithm can reach the convergent point quickly and smoothly.

5. Service selection game of cloud users

In this section, we introduce the evolutionary game in order to study the dynamic behaviors of cloud users who decide which service (from the PP or CB) to use based on the observed attributes of the system (i.e., delay and prices). First, we provide some preliminaries of the evolutionary game. Then, we formulate the service selection of the cloud user as an evolutionary game, in which we apply replicator dynamics to study the dynamic behaviors of cloud users. We provide convergence analysis of replicator dynamics in the deterministic model and the stationary distribution vector in the stochastic model. We also implement the service selection algorithms through two approaches.

5.1. Preliminaries of evolutionary game

In this subsection, we briefly introduce theoretic concepts of evolutionary games and replicator dynamics (Sandholm, 2010) which were recently used in a network selection game (Niyato and Hossain, 2009; Niyato et al., 2009; Elias et al., 2013; Chen and Huang, 2013).

An evolutionary game is an extension of the formulation of a noncooperative game by introducing the concept of population. This population is a group of individuals (i.e., players) who are evolutionarily identical in that they have the same strategy set and experience the same expected payoffs from using the same strategies. Like noncooperative games, the individuals from one population may choose strategies against individuals in another population. When the game is repeated, the population of players evolves over time since the population is able to reproduce (i.e., replicate) itself through the process of mutation and selection. The goal of an evolutionary game is to determine the equilibrium point for the game of the population. At this evolutionary equilibrium, none of the individuals wants to change its strategy since its payoff (or cost) is equal to the average payoff of the population.

An evolutionary game is defined by players, population, strategies and payoff. Consider an evolutionary game where each player follows one of pure strategy s_i from a finite set of strategies, $i = 1, \ldots, I$. Let n_i denote the number of individuals choosing strategy s_i, and $N = \sum_{i=1}^{I} n_i$ denote the total population size. The proportion of individuals choosing strategy s_i is $x_i = \frac{n_i}{N}$, and this value is referred to as the population share. The population state can be denoted by the vector $x = [x_1, \ldots, x_i, \ldots, x_I]$. The payoff of players in the same population who play strategy s_i is denoted by $\pi_i = \pi_i(x)$. The game is repeated in periods $t = 1, 2, \ldots$.

The replicator dynamics can be defined as follows:

$$\dot{x}_i(t) = x_i(t)\sigma[\pi_i(t) - \bar{\pi}(t)], \quad i = 1, \ldots, I, \tag{31}$$

where $\pi_i(t)$ is the payoff (or cost) of the individuals choosing strategy s_i at time t, $\bar{\pi}(t)$ is the average payoff of the entire population, and σ is the rate of strategy adaptation. $\dot{x}_i(t)$ describes the derivative of the population state $x_i(t)$ with respect to time. The replicator dynamics can measure the essence of selection (e.g., proportion of individuals who choose different strategies), given a particular point in time. Based on the replicator dynamics, the evolutionary equilibrium is defined as the set of fixed points at which the replicator dynamics are stable.

5.2. Formulation of evolutionary game

We consider an evolutionary game in which the players (i.e., cloud users) choose by a strategy set denoted by $S = \{s_1, s_2\}$: s_1 means that the player chooses the service from the CB, and s_2 means that the player chooses the service from the PP. N denotes the total number of cloud users that join the market, and n_i denotes the number of individuals choosing strategy s_i. The corresponding arrival rate of users joining the CSP is $\lambda_i = \frac{n_i}{N}\lambda$, $i = 1, 2$. The proportion of individuals choosing the CB service x_1 is equal to $x_1 = \frac{\lambda_1}{\lambda}$, and the proportion of individuals choosing the PP service is

$x_2 = \frac{\lambda_2}{\lambda} = \frac{\lambda - \lambda_1}{\lambda}$.

The cost of the individuals choosing the CB service is $C_1 = [\frac{\alpha}{\mu} + p_1]$, and the cost of the individuals choosing the PP service is $C_2 = [\frac{\alpha}{\mu - \lambda_2} + p_2]$. Thus, the average cost of the entire population $\bar{C}$ is equal to

$$\bar{C} = \frac{\lambda_1}{\lambda} C_1 + \frac{\lambda_2}{\lambda} C_2. \tag{32}$$

5.3. Replicator dynamics of the service selection game

The service selection game is repeated, and in each period, the user observes the cost of other users in the same area. Then, in the next period, the user adopts a strategy that gives a lower cost than other strategies. For the infinite populations assumption, replicator dynamics is useful to analyze how players can "learn" about their environment and investigate the convergence speed of strategy adaptation to reach a stable solution in the game (Sandholm, 2010; Elias et al., 2013; Niyato and Hossain, 2009).

Here, the aim of each cloud user is to minimize his cost. Hence, we can formalize the service selection game as follows:

$$\dot{x}_1(t) = x_1(t)\sigma[\bar{C}(t) - C_1(t)], \tag{33}$$

where $\dot{x}_1(t)$ represents the derivative of x_1 with respect to time. A similar equation can be written for a cloud user choosing the PP. Thus we can express the replicator dynamics for such cloud users as follows:

$$\dot{x}_2(t) = x_2(t)\sigma[\bar{C}(t) - C_2(t)]. \tag{34}$$

Based on the replicator dynamics of the users, the number of users choosing service from either the PP or CB increases if their cost is below the average cost. We can observe that, if we have $\dot{x}_1(0) + \dot{x}_2(0) = 0$ at the starting time, then at every time t, we obtain $\dot{x}_1(t) + \dot{x}_2(t) = 0$.

5.4. Convergence analysis of replicator dynamics

In Sandholm (2010), the author shows that Wardrop equilibria are the stationary points of (33) and (34). In this subsection, we prove that the unique non-trivial fixed point of such dynamics coincides with the Wardrop equilibrium point of the cloud user service selection game already determined in Section 4.1.

5.4.1. Evolutionary equilibrium

We consider the evolutionary equilibrium as the solution to this service selection game. An evolutionary equilibrium is a fixed point of the replicator dynamics. At this fixed point, which can be obtained numerically, costs of all users in the population are identical. In other words, since the rate of strategy adaptation is zero (i.e., $\dot{x}_i = 0$), there is no user who deviates to gain a lower cost. To find the fixed point x_i^*, $i = 1, 2$ of the replicator dynamics, we solve $\dot{x}_1 = \dot{x}_2 = 0$. The solution of $\dot{x}_1 = \dot{x}_2 = 0$ coincides with one of $C_1 - C_2 = 0$, which is the Wardrop equilibrium, as presented in (8) in Section 4.1.

5.4.2. Stability analysis

We analyze the stability of the replicator dynamics given by (33) and (34). However, we present the analysis for (34), and the same analysis can be conducted for (33). From (34), we can represent the dynamics of value x_2 as follows:

$$\begin{aligned} x_2(t+1) &= x_2(t) + x_2(t)\sigma[\bar{C}(t) - C_2(t)] \\ &= x_2(t) + x_2(t)[1 - x_2(t)]q\left(r - \frac{1}{s - x_2(t)}\right) = G(x_2(t)), \end{aligned} \tag{35}$$

where

$$q \triangleq \frac{\alpha\sigma}{\lambda}, \quad r \triangleq \frac{\lambda(d + p_1 - p_2)}{\alpha}, \quad \text{and } s \triangleq \frac{\mu}{\alpha}, \tag{36}$$

and $G(x) = x + x[1 - x]q(r - \frac{1}{s - x})$, G: $[0, 1] \to [0, 1]$ is a mapping from $[0, 1]$ onto itself. The above dynamics has the fixed point $x_2^* = \frac{\lambda_2^e}{\lambda}$, in which ${\lambda_2}^e$ is the Wardrop equilibrium already derived by (9) in Section 4.1. Define

$$x_2^g \triangleq r - \left(\frac{s(s-1)}{r}\right)^{1/3}, \tag{37}$$

$$\mathrm{F} \triangleq G'(x_2^g) = 1 + q(1 - 2x_2^g)\left(r + \frac{1}{x_2^g - s}\right) + \frac{q(x_2^g - 1)x_2^g}{(s - x_2^g)^2}, \tag{38}$$

$$\zeta \triangleq \max_{x \in [0,1]} |G'(x)|. \tag{39}$$

To investigate the convergence of the replicator dynamics in (35) to the fixed point x_2^*, we start by establishing the following auxiliary lemma.

Lemma 5.1.

$$\zeta = \begin{cases} \max[|\mathrm{F}|, |G'(1)|], & \text{if } x_2^g \in (0, 1) \\ |G'(1)| = \left|1 - q\left(r - \dfrac{1}{s - 1}\right)\right|, & \text{otherwise} \end{cases} \tag{40}$$

Proof. The proof of the first part is straightforward by verifying the second-order condition $G''(x)$. ${x_2}^g$ is a unique root of $G''(x) = 0$. □

Using the result from Lemma 5.1, we characterize the convergence of the replicator dynamics in (35) using the following theorem.

Theorem 5.1. *If $\zeta < 1$, then the replicator dynamics depicted in (34) converges to the nontrivial fixed point x_2^* for any initial state $0 < x_2(0) < 1$.*

Proof. If $\zeta < 1$, then there exists a positive constant $\xi \in [0, 1)$ such that $\|G'(x)\|_\infty \le \xi$. Therefore, we have

$$\|G(u) - G(v)\|_\infty \le \|G'(x)\|_\infty \cdot \|u - v\|_\infty \le \xi\|u - v\|_\infty. \tag{41}$$

Thus, we have shown that the replicator dynamics $x_2(t) \to x_2(t+1)$ in (35) is a contraction. Using Theorem 4.3, the replicator dynamics converges to the unique fixed point, which is the Wardrop equilibrium x_2^*. □

5.5. Stochastic evolutionary game

The replicator dynamics in (32) are deterministic and do not include the noise. The noise is generated from a small proportion of irrational users who may choose to use service with a lower performance and higher price because of incorrect cost information. In this scenario, we use a Markov chain to analyze the long term behavior in the cloud user society of the stochastic evolutionary game (Sandholm, 2010; Niyato and Hossain, 2009).

We assume that there is a finite population of N cloud users, n_2 is the proportion of users choosing the PP and $n_1 = N - n_2$ is the proportion of users choosing the CB. Then, the finite state space S of the Markov chain is $\{\chi_2 | 0 \le \chi_2 \le N\}$, where χ_2 is a random variable denoting the number of cloud users choosing the PP. The number of cloud users choosing the CB $\chi_1 = N - \chi_2$.

Let us revise the replicator dynamics in (34) for users choosing the PP as $\dot{x}_2(t) = x_2(t)\sigma[\bar{C}(t) - C_2(t)]$. In the replicator dynamics, the costs $C_2(t)$ and $\bar{C}(t)$ of a user are varied by the proportion of users $\dot{x}_2(t)$ that is equivalent to the number of user n_2 at time t. Thus, we can present the cost of users as a function of the number of cloud users, i.e., $C_2(n_2)$ and $\bar{C}(n_2)$.

The deterministic replicator dynamics in (32) are based on the assumption: all users are rational users. If the cost offered by the PP is less the average cost (i.e., $\bar{C}(n_2) \geq C_2(n_2)$), rational users will choose the PP. Different from deterministic replicator dynamics, in the stochastic evolutionary game, there is a small number of irrational users. This means that, if the cost offered by the PP is greater than the average cost (i.e., $\bar{C}(n_2) < C_2(n_2)$), then irrational users choosing the PP keep selecting the PP with probability ε, which is noise. The elements of the state transition matrix $\mathbf{Q}$ can be calculated directly as follows:

$$q_{n_2,n_2'} = \begin{cases} n_2(\bar{C}(n_2) - C_2(n_2')), & \text{if } \bar{C}(n_2') \geq C_2(n_2'), \\ n_2\varepsilon, & \text{otherwise}, \end{cases} \tag{42}$$

with $n_2 \neq n_2'$, and the element $q_{n_2,n_2'}$ indicates the rate of state transition when the number of users choosing the PP changes from n_2 to n_2'. Then we have

$$q_{n_2,n_2} = -\sum_{n_2 \neq n_2'} q_{n_2,n_2'}. \tag{43}$$

In the stochastic evolutionary game, the objective is to identify the stochastically stable states when the noise is relatively small (e.g., $\varepsilon = 10^{-4}$). Thus, the state transition matrix $\mathbf{Q}$ of the Markov chain can be defined as follows:

$$\mathbf{Q} = \begin{bmatrix} q_{0,0} & q_{0,1} & & & \\ q_{1,0} & q_{1,1} & q_{1,2} & & \\ \cdot & \vdots & \vdots & \cdot & \\ & q_{N-1,N-2} & q_{N-1,N-1} & q_{N-1,N} & \\ & & q_{N,N-1} & q_{N,N} & \end{bmatrix}. \tag{44}$$

The stationary distribution of this chain can be found by solving $\Pi\mathbf{Q} = 0$ subject to the constraint that elements of vector Π must sum to 1. Vector Π is the steady-state probability of the Markov chain and is defined as

$$\Pi = [\Pi_0, \Pi_1, \dots, \Pi_{n_2}, \dots, \Pi_N], \tag{45}$$

where Π_{n_2} indicates the probability of n_2 number of users choosing the PP in a long time span. We will observe the stationary distribution vector Π in the numerical results later in this paper.

5.6. Implementations of the service selection algorithm

We present two approaches for dynamic evolutionary game-based service selection by each individual user in the heterogeneous market in cloud computing. The first approach is based on population evolution, in which cost information of cloud users using different CSPs is exchanged between groups of cloud users. The second approach is based on reinforcement learning and more specifically the application of Q-learning, in which each user observes the average delay and prices from the chosen service, explores different services and selects which services are good based on past observation. Both approaches attract cloud users to the equilibrium of service selection game. However, the service selection algorithm based on population evolution utilizes information from all users to achieve fast convergence using an information exchanged mechanism. On the other hand, in the reinforcement learning-based approach, the cloud users learn the performances and prices of different cloud providers through interaction so as to make the optimal decision for service selection.

5.6.1. Population evolution approach

This approach is based on population evolution in which cost information of cloud users using different providers is exchanged between two groups of cloud users (e.g., by a third party who collect cost information of all cloud users or through an information exchanging mechanism). The service-selection decision of each user is based on its current cost and the average cost of all users. This service-selection algorithm based on the population evolution approach can be described in Population Evolution Algorithms (Algorithm 2).

Algorithm 2. Population Evolution Algorithm.

1: Initializing: all cloud users choose randomly the service from the CB or PP, t=1.
2: *repeat for all cloud users*:
3: A user computes cost $C_i(t)$ (i=1,2) from the average delay and price. This cost information is informed to the other user group.
4: Based on the exchanged cost information, the average cost $\bar{C}(t) = \frac{\lambda_1(t)C_1(t) + \lambda_2(t)C_2(t)}{\lambda}$ is calculated and informed to all user groups.
5: **if** $C_i(t) < \bar{C}(t)$ **then**
6: **if** $rand(\cdot) < \sigma(\bar{C}(t) - C_i(t))/\bar{C}(t)$ **then**
7: Choose service j, where $j \neq i$.
8: **else**
9: Keep service i.
10: $t = t + 1$, go back to step 2.

5.6.2. Reinforcement learning approach

The cost information exchange requires overhead and may provide out-of-date information due to delay in exchanging information. In the reinforcement learning approach, each user has to learn and adapt its service-selection decision independently. The Q-learning is applied in evolutionary game in Niyato and Hossain (2009), Tuyls and Nowé (2005), and Panait et al. (2008), which presented the relation between Q-learning and the replicator dynamics. Thus, we use the Q-learning algorithm (Sutton and Barto, 1998) to implement the service-selection algorithm. In this Q-learning game formulation, the agent (i.e., the cloud user) has a set of actions denoted by $A = \{a_1, a_2\}$: a_1 means that the agent chooses the service from the CB, and a_2 means that the agent chooses the service from the PP. The reward of the agent that chooses the service of CSP i is $-C_i(t)$ at time t. To select the best service, the Q-learning algorithm uses the Q-value (i.e., $Q_i(t)$) to compare the expected cost of the available service selections without requiring complete cost information exchange. The service-selection algorithm is described in Algorithm 3.

Algorithm 3. Q-Learning algorithm.

1: Initializing: Q-value $Q_i(1) = 0$ associated with service $i = 1, 2$ (value 1 denotes the service from the CB, and value 2 denotes the service from the PP) for all cloud users in all groups (CB users and PP users).
2: *repeat for all cloud users*:
3: **if** $rand(\cdot) < \gamma$ **then**
4: Choose the service i randomly [exploration step].
5: **else**
6: Choose the service $i^* = \arg\max_i Q_i(t)$ [exploitation step].
7: Cloud user computes cost $C_i(t)$.
8: Cloud user updates $Q_i(t+1) = (1-\xi)Q_i(t) + \xi(-C_i(t) + \beta \max_i Q_i(t))$.
9: $t = t + 1$, go back to step 2.

In the Q-Learning algorithms, ξ denotes the learning rate and β is a discount factor, which controls the speed of adjustment of the Q-value. In the Q-learning, there are two phases: exploration and exploitation. At step 4, the Q-learning algorithm performs the exploration phase with γ-greedy selection, which controls the randomness of exploration. This means that there is a small probability γ of users that, rather than take the best action, will uniformly select an action from the set of actions. This exploration phase guarantees that good CSP will not be missed during the learning procedure. The exploitation is represented in step 6, in which the cloud users learn to take the optimal actions by comparing the Q-value of CSPs. The trade-off between exploration and exploitation is one of the great challenges of Q-learning. The interpretation of exploitation–exploration is the selection–mutation concept in the evolutionary game approach.

6. Extension: multiple public providers and cloud brokers scenario

The study of the duopoly scenario can be extended to a general scenario with multiple N CBs and M PPs, where each CSP (both CB and PPs are known as CSPs) competes with the others.

The noncooperative static game between N CBs and M PPs can be considered similar to that in Section 4.2. Each CSP i ($i = 1, \ldots, N + M$) in a total of $N + M$ CSPs has the utility function defined as $U_i(P) = p_i\lambda_i$, where p_i and λ_i are a service price and user arrival rate at the CSP i, respectively. $\mathbf{P} = [p_1, \ldots, p_{N+M}]$ and $\Lambda = [\lambda_1, \ldots, \lambda_{N+M}]$ are the price vector and arrival rate vector of CSPs, respectively. The vector Λ^e is a Wardrop equilibrium, if and only if there exist $C > 0$ such that:

$$C_i(\lambda_i^e) = C, \quad \text{if } \lambda_i^e > 0, \ \forall\, i, \tag{46}$$

$$C_i(\lambda_i^e) > C, \quad \text{if } \lambda_i^e = 0, \ \forall\, i, \tag{47}$$

$$\sum_{i=1}^{N+M} \lambda_i = \lambda. \tag{48}$$

Anselmi et al. (2011) and Roughgarden and Tardos (2002) have shown that finding a Λ^e is equivalent to finding a solution of following convex optimization problem:

$$\arg\min_{\lambda_i \geq 0} \sum_{i=1}^{N+M} \left[\int_0^{\lambda_i} C_i(x)\, dx \right], \tag{49}$$

$$s.\,t. \quad \sum_{i=1}^{N+M} \lambda_i = \lambda. \tag{50}$$

Thus, we can find the vector Λ^e is a Wardrop equilibrium using a certain standard convex tool in Boyd and Vandenberghe (2004).

We consider the Nash equilibrium prices as a solution of the noncooperative static game among CSPs. Then, the Nash equilibrium prices are obtained using the best response function of each of CSP. The best response function can be defined by extending that in Section 4.2 as follows:

$$BR_i(P_{-i}) = \arg\max_{p_i \geq 0} U_i(p_i, \mathbf{P}_{-i}), \tag{51}$$

where $\mathbf{P}_{-i}$ denotes the vectors of service price of all CSPs except CSP i (i.e., $\mathbf{P}_{-i} = [\ldots, p_j, \ldots]$ for $j \neq i$). The Nash equilibrium price vector $\mathbf{P}^{ns} = [\ldots, p_i^{ns}, \ldots]$ of the price competition between CSPs can be obtained from the condition

$$p_i^{ns} = BR_i(P_{-i}), \quad \forall\, i. \tag{52}$$

The service selection game is similar to that in Section 5. We denote the population state by the vector $\mathbf{X} = [x_1, \ldots, x_{N+M}]$. Then, the replicator dynamics is defined as follows:

$$\dot{x}_i(t) = x_i(t)\sigma[\bar{C}(t) - C_i(t)], \quad \forall\, i, \tag{53}$$

where $C_i(t)$ and $\bar{C}(t)$ are the cost of the individuals choosing CSP i service and the average cost of the entire population at time t, respectively. Then, the population equilibrium is defined as the stable fixed point of the replicator dynamics (Sandholm, 2010). When a population of users evolves over time, it will converge to the population equilibrium, which can be obtained by solving

$$\dot{x}_i = 0, \quad \forall\, i. \tag{54}$$

However, studying the multiple N CBs and M PPs scenario is difficult, even to find the solution numerically. The difficulty is on how to define the equilibrium arrival rate at Eq. (46). We need to define the arrival vector Λ^e which contains the cloud user rates at N CBs and M PPs. However, there will exist $N + M$ variables. Therefore finding the Nash equilibrium solution in such a case is not trivial as solving Duopoly scenario with one variable. However, once additional assumptions are made, it may be possible to solve for the equilibrium point. We suppose that N CBs belong to a Primary CB (PCB) (e.g., Google Cloud) and the PCB sets the same admission price p for all N CBs to reduce the number of variables. Thus, the analysis presented in Sections 4 and 5 can be similarly extended to the multiple PPs and CBs scenarios at the expense of the increased complexity.

7. Performance evaluation

In this section, we analyze and discuss the numerical results obtained from solving pricing and service selection games in different scenarios. At first, we measure the sensitivity of CSP utilities and prices, as well as cloud user equilibrium arrival rate and costs. Then, we evaluate the convergence to the evolutionary equilibrium of the service selection game with the population algorithm and reinforcement learning algorithm. We also study the impact of delay in the Population Evolution Algorithms and noise in the stochastic evolutionary game. The design of our simulator is based on a time-slotted synchronous model with all events generated and processed in their respective time slots. Our simulator is developed using MATLAB.

7.1. Pricing competition numerical results

7.1.1. Pricing competition in a duopoly market

We first consider the impact of service rate μ in a duopoly heterogeneous CSP market. Since the proposed game model is used to achieve the Nash equilibrium prices, we will investigate the effects of resource capacities (i.e., μ) on equilibrium prices. Table 1 shows the comparison of the utilities, equilibrium prices

Table 1
Comparison of the utilities, equilibrium prices and user arrival rates of CSPs vs. service rate μ with the parameters as follows: $\alpha = 0.5$, $p_{max} = 0.015$, total cloud user arrival rate $\lambda = 10$.

μ	U_1	U_2	λ_1	λ_2	p_1	p_2	Cost C
30	0.0303	0.0085	6.53	3.46	0.00463	0.00360	0.0213
35	0.0215	0.0059	6.55	3.44	0.00329	0.00173	0.0175
40	0.0161	0.0044	6.56	3.43	0.00245	0.00128	0.0149
45	0.0125	0.0033	6.58	3.41	0.00190	0.00098	0.0012
50	0.0100	0.0026	6.58	3.41	0.00151	0.00078	0.0115

Table 2
Comparison of the utilities, equilibrium prices and user arrival rates of CSPs vs. arrival rate λ with the parameters as follows: $\alpha = 0.5$, $p_{max} = 0.015$, service rate $\mu = 50$.

λ	U_1	U_2	λ_1	λ_2	p_1	p_2	Cost C
05	0.0023	0.0006	3.13	1.68	0.00070	0.00036	0.0107
10	0.0100	0.0026	6.58	3.41	0.00151	0.00078	0.0115
15	0.0240	0.0066	9.82	5.17	0.00244	0.00128	0.0124
20	0.0457	0.0132	13.00	7.00	0.00351	0.00189	0.0135

and user arrival rates of CSPs when the service rate μ varies. We observe that the equilibrium utilities of the CB (U_1) are always higher than these of the PP (U_2) in this scenario. As we can see, the equilibrium prices of both CB and PP decrease when the service rate increases. Our results in Table 1 show that, when service rate changes, the user arrival rates of CSPs remain nearly constant. Both the equilibrium arrival rate and the equilibrium price of the CB are higher than those of the PP, demonstrating that utilities of the CB (U_1) are always higher than those of the PP (U_2). The cost C of cloud users decreases when the service rate increases; however, the service rate μ has a greater affect on the equilibrium prices and CSP utilities than the user cost and arrival rates.

We then study how the total arrival rate, λ, affects the Nash equilibrium prices. Table 2 shows how the utilities, equilibrium prices and user arrival rates of CSPs react when the total arrival rate λ increases. The CSP utilities and equilibrium price of the CB are always higher than those of the PP. The results from Table 2 show that, the arrival rate λ has a greater affect on the CSP equilibrium prices and utilities than the user cost and arrival rates.

Because of the fluctuation of arrival rate λ at the cloud market, we should update the pricing according to the total arrival rate λ. We assume that the pricing can be updated effectively during one day, which is the period in which CSPs and cloud users obtain the expected delay and arrival rates. We use realistic traces for the average incoming request rate λ per hour in one day. All of them are scaled in a range of $[0, 20]$, as shown in Fig. 5. The data is a one-year trace from 2007 of Microsoft Research (MSR), and it is taken from 6 RAID volumes at MSR Cambridge (Lin et al., 2013). Figs. 5 and 6 show the equilibrium arrival rates and prices during 60 days. The fluctuation of equilibrium arrival rates and prices is similar to that of the total arrival rate, which shows the importance of the total arrival rate.

7.1.2. Pricing competition in a multiple CSP market

This subsection presents some interesting observations when multiple CSPs are competing for the same poll of cloud users. We conduct our evaluation in a scenario with one CB and two PPs,

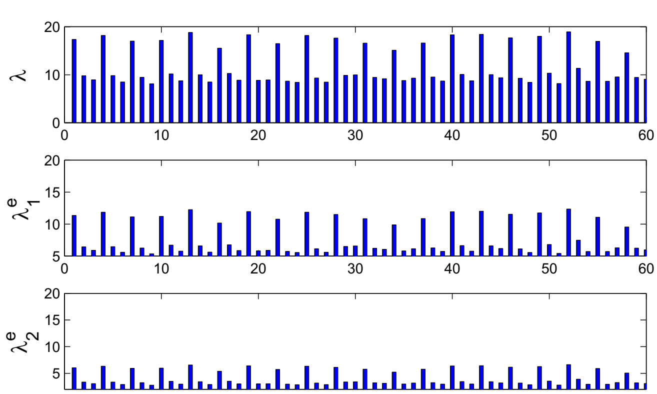


Fig. 5. The total arrival rate and equilibrium arrival rates over 60 days. Equilibrium cost.

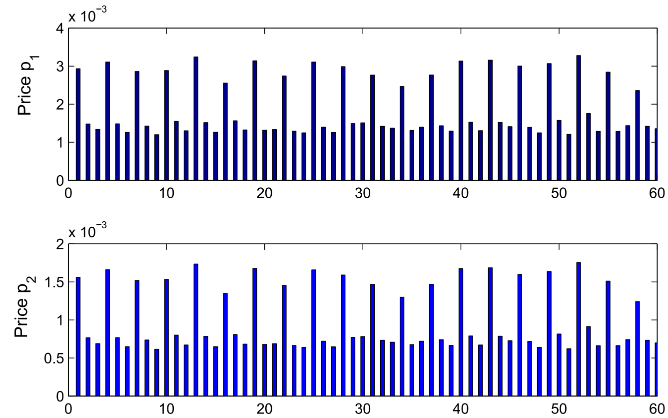


Fig. 6. The equilibrium prices over 60 days.

with resource capacities of $\mu_1 = 30$, $\mu_2 = 35$ and $\mu_3 = 40$, respectively. Table 3 shows the comparison of the utilities, equilibrium prices and user arrival rates of CSPs when the arrival rate λ varies. It is interesting that the variety of the resource capacities has a strong effect on the equilibrium prices as well as the other parameters of the market. We see that, when the resource capacity $\mu_2 = 35$ is less than that of $\mu_3 = 40$, the utilities, arrival rates and equilibrium rates of the PP 2 are always lower than those of the PP 2. However, with respect to the CB, there is a different observation between this scenario and the duopoly scenario; that is, the utilities of the CB (U_1) are not always the highest. When arrival rate λ is set to 10, the utility U_1 is the lowest among three CSPs. However, when the arrival rate λ increases, the market share (i.e., the arrival rate λ_1) of the CB also increases. As a result, the utility U_1 increases with the arrival rate λ_1 and the equilibrium price p_1. As we can see, a larger arrival rate leads to greater utility for all CSPs, which further shows the importance of the arrival rate.

7.2. Evolutionary game numerical results

7.2.1. Convergence to the evolutionary equilibrium

We consider a heterogeneous market CSP cloud computing with a random initial proportion of users choosing each CSP and the parameters as follows: $\alpha = 0.5$, total cloud user arrival rate $\lambda = 70$ and service rates $\mu = 100$, $\gamma = 0.1$, $\xi = 0.1$, and $\beta = 0.2$. Fig. 7 shows the convergence properties of the service-selection algorithms based on the population evolution and the Q-learning approaches. The former algorithm converges to the equilibrium within less than ten iterations (i.e., with a cloud user cost 0.009). In contrast, the latter one requires a larger number of iterations to reach the equilibrium. From the implementation viewpoint, the Q-learning algorithm is more practical due to the fact that a procedure of gathering, processing, and exchanging cost information of cloud users may not be available in practice. However, the Population Evolution Algorithm utilizes the average cost information and so uses less iterations to converge than the Q-learning algorithm in which a user independently selects a service using only its local cost information obtained through exploration.

7.2.2. Impact of delay in Population Evolution Algorithms

When a user makes the decision of service selection, current information at a certain time t about the average cost (i.e., $\bar{C}$ in Algorithm 3) may not be available. Therefore, a user must rely on historical information, which again, may be delayed for a certain period. This delay can occur due to the information exchange latency among users. Thus, we assume that a user makes a service selection at time t based on the information from time $t - \tau$ (i.e., a delay of τ units of time). In this case, the replicator dynamics can

Table 3
Comparison of the utilities, equilibrium prices and user arrival rates of CSPs vs. arrival rate λ with the parameters as follows: $\alpha = 0.5$, $p_{max} = 0.015$, service rate $\mu_1 = 30$, $\mu_2 = 35$ and $\mu_3 = 40$.

λ	U_1	U_2	U_3	λ_1	λ_2	λ_3	p_1	p_2	p_3	Cost C
10	0.0004	0.0040	0.0137	1.41	2.90	5.68	0.00031	0.00140	0.00241	0.0169
15	0.0055	0.0187	0.0068	4.84	3.66	6.49	0.00115	0.00186	0.00289	0.0178
20	0.0169	0.0105	0.0250	8.24	4.43	7.31	0.00205	0.00237	0.00342	0.0187
25	0.0355	0.0153	0.0328	11.62	5.21	8.15	0.00305	0.00293	0.00402	0.0197

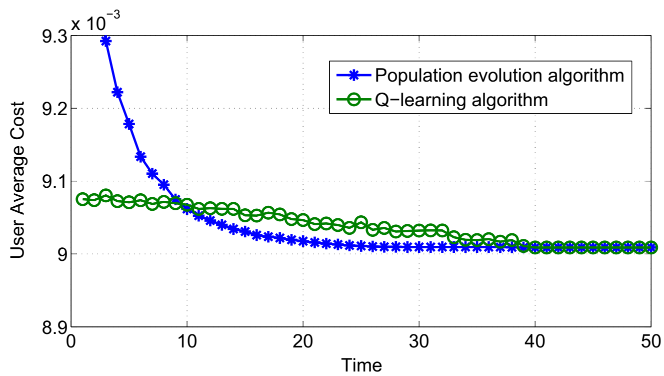


Fig. 7. Convergence of the service-selection algorithms to the evolutionary equilibrium cost.

be modified as follows:

$$\dot{x}_i(t) = x_i(t-\tau)\sigma[\bar{C}(t-\tau) - C_i(t-\tau)], \quad i = 1, 2. \tag{55}$$

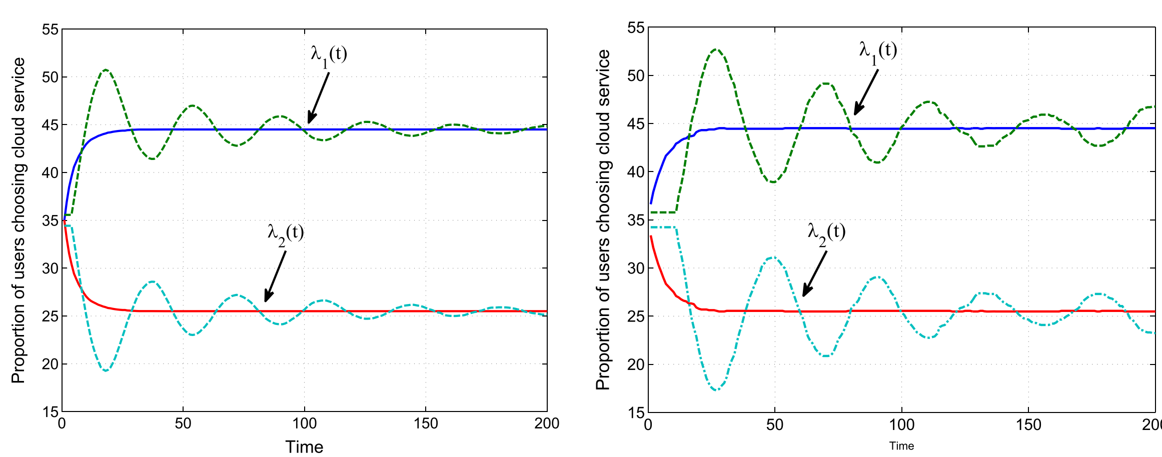
The convergence of the Population Evolution Algorithms with different values of τ is shown in Fig. 8. We investigate the impact of τ on the dynamics of strategy adaptation. When $\tau \geq 1$, we observe a fluctuating dynamics of strategy adaptation. For a small value $\tau = 3$, the difference between dynamics of strategy adaptation without delay and that with delay is negligible as time increases. The larger is the delay, the greater is the fluctuation. If $\tau > 10$, the dynamics of strategy adaptation of users never reaches the evolutionary equilibrium, as presented in Fig. 8(b), because the decisions of users tend to be inaccurate when information is out-of-date.

7.2.3. *Impact of noise in the stochastic evolutionary game*

The stationary distribution vector Π of PP users obtained from the stochastic model is shown in Fig. 9. The stationary distribution depend largely on the population size N of the cloud users. In particular, when the number of users N is small, it its more likely that a user will select irrationally. Consequently, the stationary distribution probabilities for the different states corresponding to the evolutionary equilibria become more nonuniform as the bell-shape probability mass function becomes larger, as shown in Fig. 9 (a). The top of the bell-shape in Fig. 9 is close to the number of PP users (i.e., 350/1000 or 35/100 users) in the deterministic model (i.e., replicator dynamics).

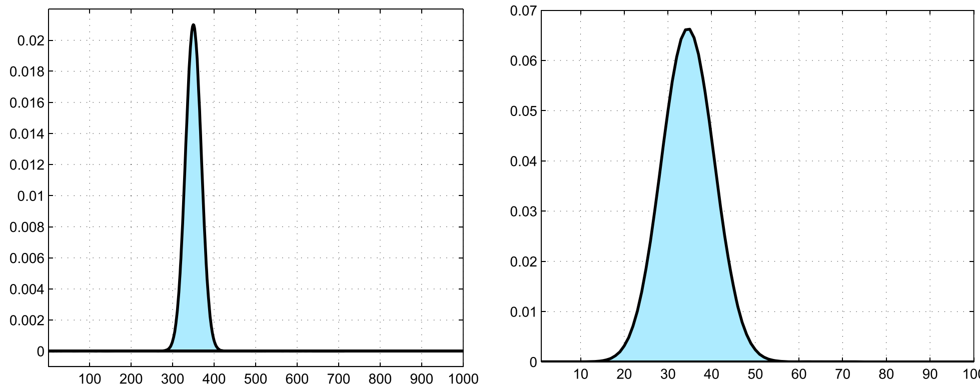
8. Conclusion

In this paper, we have studied the price competition in a heterogeneous CSPs market with two stages of competition. In pricing competition between the CSP in stage I, we have derived the equilibrium prices in the noncooperative static game. We provide the sufficient conditions for the existence and uniqueness of the Nash equilibrium and convergence of the iterative algorithm. At the same time, we study the dynamic of cloud users in the service selection game using the evolutionary game model in stage II. We used the Wardrop equilibrium concept and replicator dynamics to compute the equilibrium and characterized its convergence properties in the service selection game. We also proposed two approaches to implement the evolution of cloud users to attract them to the equilibrium. Performance evaluation demonstrates that our game model can represent the main characteristics of pricing and service selection of the heterogeneous CSP market in cloud computing. Numerical results show that, in a duopoly market, the arrival rate or resource capacity has a stronger effect on the CSP side (i.e., equilibrium prices and utilities) than cloud user side (i.e., user cost and arrival rates). The CB always has higher utility in the duopoly scenario; however, it may be not true in the multiple CSPs scenario because the market share of the CB depends on the total arrival rate. Our algorithms represent a first step toward designing practical mechanisms to price resources in operational IaaS cloud providers and cloud users, and are shown to



(a) Delay time $\tau = 3$. (b) Delay time $\tau = 10$.

Fig. 8. The convergence of Population Evolution Algorithms with different values of τ. The solid line is Population Evolution Algorithms without delay (i.e., $\tau = 0$).



(a) Population size $N = 1000$. (b) Population size $N = 100$.

Fig. 9. Stationary distribution of PP users Π obtained from the stochastic model with noise level $\varepsilon = 10^{-4}$ and different population size N.

converge quickly.

In this paper, there are some limitations that are interesting to consider in the future works. For example, we have considered one service, but there are many cloud services in the practical cloud market. The game between providers indeed can be extended as a dynamic game and try to, e.g., show that dynamics converge to an equilibrium. The multiple PPs and CBs scenarios are needed to further investigate to reduce the complexity of the solution. Additionally, we have not considered service-level agreement issues that are also important for cloud users. To focus on the price competition only, we ignore the operating costs (i.e., the costs to maintain the resource capacity μ of the CSPs), which is a function of service rate μ. Thus, the join provider pricing and resource capacity optimization are interesting objectives.

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