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Joint Pricing and Load Balancing for Cognitive Spectrum Access: Non-Cooperation vs Cooperation

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***Abstract*—In the dynamic spectrum access (DSA), pricing is an efficient approach providing economic incentives for operators, whereas load balancing yields congestion-avoidance incentives for secondary users (SUs). Despite complexities of i) the couplings among pricing, load balancing and SUs' spectrum access decision, and ii) the heterogeneity of primary users' traffic and SUs classes/types, we tackle the joint load balancing and pricing problem to maximize operators' revenue in two cognitive radio markets: monopoly and duopoly. For the monopoly market, we first show there exists a unique SUs' equilibrium arrival rate to the monopolist's channels. We then show that the joint problem can be solved efficiently by exploiting its convex structure. For the duopoly market, we first characterize a unique SUs' equilibrium arrival rate to two operators employing different DSA approaches. When two operators are noncooperative, we show that there exists a unique Nash equilibrium for each operator's revenue. When they are cooperative, we show that the social revenue optimization can achieve a unique optimal solution. Using the Nash bargaining framework, we also present a sharing contract that determines the optimal fraction of the social revenue for each operator. In both markets, we propose two algorithms that can find the largest SU class supportable by the operators.**

***Index Terms*—Pricing, Load Balancing, Nash Equilibrium, Dynamic Spectrum Access, Cognitive Radio.**

I. INTRODUCTION

Dynamic spectrum access (DSA) has been introduced to efficiently utilize scarce wireless spectrum that is conventionally controlled via static licensing. Various DSA approaches, including two popular dynamic shared-use and exclusive-use paradigms, have been proposed to enable secondary users (SUs) to flexibly access underutilized legacy spectrum that is used sporadically by primary users (PUs) [1]–[3]. The shared use allows SUs to opportunistically access the (secondary) operators' "interruptible spectrum" without harming the PUs' activities, whereas the exclusive use allows operators to lease parts of a temporarily unused spectrum (i.e. no PUs operations) for SUs' service provisioning.

In these two paradigms, *pricing* is one effective market-based method to distribute spectrum from operators to SUs since it not only provides economics incentives for operators, but also has the low-overhead operation [4]–[6]. However, for a given operator's price, various SU applications (classes) with distinguished physical conditions (types) will have different spectrum access decisions. Therefore, if the heterogeneity of SUs' classes and types is considered, how to design an efficient pricing mechanism to achieve the optimal revenue for operators is one of the market-based challenges.

While pricing provides an economics incentive for operators, *load balancing*, which distributes the SUs' traffic loads to right channels, provides a congestion-avoidance incentive for SUs [7]–[9]. Nevertheless, SUs' congestion is influenced by service times of the operator's channels, which are affected by varying PUs traffic patterns. Hence, if the heterogeneity of PU traffic is considered, how to design a low-complexity mechanism that can distribute SUs traffic evenly on all channels is one of the load balancing challenges.

In this work, by incorporating the heterogeneity of PUs' traffic and SUs' types and conditions, we study a *joint pricing and load balancing* spectrum access control in multi-channel cognitive radio networks (CRNs). This joint problem can be illustrated as a two-level structure in Fig. 1. At the operator level, in every beginning period of all stationary statistics, the operator will decide the corresponding prices and load balancing information to *maximize its revenue*. Based on the information, at the SU level, an arriving SU with a specific application and physical condition will decide whether or not to join the

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operator to maximize its expected utility. We see that how SUs make joining decisions depends on both price and load balancing information of the operator, and how the operator sets its prices and load balancing depends on SUs' joining policy. Hence, there are certain couplings not only between pricing and load balancing, but also between these information and SUs' joining policy. Separately considering only pricing or load balancing in multi-channel CRNs will lead to suboptimal solutions of the operator's revenue optimization. Therefore, by tackling this joint problem, our contributions can be summarized as follows.

- The first network scenario is a monopoly market with one operator employing shared-use DSA. Given the operator's load balancing and prices, we first characterize the SUs' joining policy and show that there exists a unique SUs' equilibrium arrival rate to a parallel M/G/1 queueing system modelling the operator's channels with PU traffic. By integrating this equilibrium constraint into the operator's revenue maximization problem, we then show that this problem can be solved efficiently by a sequential optimization method, which reveals its convex structure. We also propose an algorithm that can find the largest SU class supportable by the operator.
- The second network scenario is a duopoly market with two operators employing shared-use and exclusive-use DSAs, respectively. We first characterize the SUs' joining policy and show the existence of a unique SUs' equilibrium arrival rate to both operators. We then investigate their interactions through two behaviors: non-cooperation and cooperation. In the noncooperative case, we show that there exists a unique Nash equilibrium. In the cooperative case, we show that the social revenue optimization problem can be splitted into two convex problems that can be solved by each operator to achieve a unique optimal solution. Before jointly optimizing the social revenue, both operators can agree on a sharing contract that determines a fraction of the total revenue for each operator. Using the Nash bargaining framework, we show that there exists a unique solution of this sharing fraction. Finally, we propose an algorithm that can find the largest SUs class supportable by both operators.

The rest of this paper is organized as follows. Section II presents related work. We analyze the monopoly and duopoly markets in Section III and Section IV, respectively. Section V provides numerical results and Section VI concludes our work.

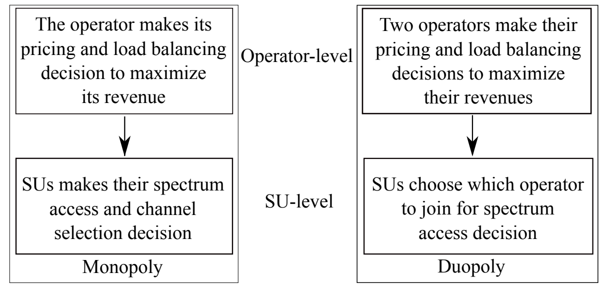

Fig. 1. Two-level structure between operators and SUs.

Due to space limitations, all missing proofs can be found in the technical report available online [10].

II. Related Works

In the literature, pricing and load balancing are two DSA research directions. On the one hand, there are many interests in load balancing spectrum control in multi-channel CRNs [7]–[9]. Using the non-preemptive priority M/G/1 queueing model, [9] proposed a dynamic learning scheme to determine a load balancing strategy that can converge to a Nash equilibrium, which is not necessarily a global optimal point. In contrast, with a preemptive resume priority M/G/1 queueing model, [7] tried to minimize the system time by providing the optimal channel selection solution which, however, relies on the numerical optimization that uses a high-complexity exhaustive search algorithm. Using the M/M/1 queueing model, the recent work [8] suggested a low-complexity optimal load balancing algorithm based on convex optimization theory; however, its channels were restricted to exponential distributions.

On the other hand, pricing methods, which address the DSA economic aspect, have recently received tremendous attention. One of the main interests includes leasing and pricing mechanisms in a three-tier market: spectrum owners, operators and SUs [11]–[13]. The other key direction focuses on the pricing schemes in the two-tier market between operators and SUs, which either characterizes the competition between multiple operators [4]–[6], multiple SUs [14], or interactions between operators and SUs [15], [16]. However, most of these papers characterize SUs' responses via their demand functions, such as bandwidth requirement. There are few papers that consider the pricing impact on SUs' equilibrium behaviors in a *strategic queueing system* as compared with our work in which SUs make their joining decisions strategically based on their perceived queueing delay. Pricing in the strategic queueing system, originated from [17] (see [18] for the survey), can be categorized into the observable [19] and unobservable queueing systems

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[20]–[22], where the latter model is adopted in this work due to its practical meaning in the CR context. Resorting on M/M/1 analysis, while [20] proposed a pricing scheme to maximize a monopolist operator's revenue, [21] accounted for a socially-maximizing pricing mechanism. However, both consider homogeneous SUs with the same class and type, which is an over-simplified model. The recent work [22] investigated not only revenue but also socially-optimal pricing schemes; however, its assumptions are limited to a single-channel case and the same type for all SUs.

By taking heterogeneous SUs' classes and types into account, we overcome many previous simplified assumptions to study the jointly optimal pricing and load balancing with low-complexity mechanisms. This problem is important to multi-channel CRNs, since it can optimally characterize both the operators' economic issue and the SUs' spectrum access behaviors due to their mutual interactions. This mutual dependence will lead to suboptimal solutions of the revenue optimization if we consider only either pricing or load balancing issue.

III. Monopoly

In this section, we first present the system model, how SUs make their joining policy, and SUs' equilibrium with the given price and load balancing of the operator. Based on these information, how the operator maximizes its revenue is studied later.

A. System Model

In this monopoly market, we consider a network that consists of one operator with multiple shared-use channels. A sequence of SUs jobs are assumed to arrive at the network and each SU will make a decision as to either join this operator or balk for its job (cf. Fig.2). The model in this section can be described quantitatively as follows.

1) Shared-use Monopolist Operator: We assume that the operator has a set of channels denoted by $\mathcal{L} = \{1, \dots, L\}$ that are licensed to legacy PUs. Traffic patterns of PUs can be modelled as an ON-OFF renewal process alternating between ON (busy) and OFF (idle) periods. On each channel $l \in \mathcal{L}$, the sojourn times of the ON and OFF periods are modelled by i.i.d. random variables (r.v.) Y_l and Z_l, with probability density functions (pdf) $f_{Y_l}(y)$ and $f_{Z_l}(z)$, respectively. ON and OFF periods are assumed to be independent with SUs' arrival process and service time. This ON-OFF process can be considered a channel model for the SU services. This model captures the idle time period in which the SUs can utilize the channel without causing harmful interference and SUs' service can be interrupted due to incoming PUs' traffic with higher priority.

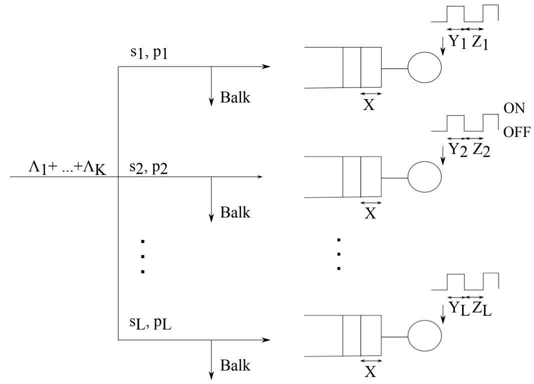

Fig. 2. Class-k SUs arrives with rate Λ_k at a monopoly system with L shared-used channels (with PU traffic as ON/OFF process) and admission price and load balancing vectors, $\vec{p}$ and $\vec{s}$, respectively.

The shared-use operator allows SUs to share the PUs' channels opportunistically to gain revenue. Before each period of all stationary distributions, the operator broadcasts a load balancing vector $\vec{s} = \{s_l\}_{l\in\mathcal{L}}$ and an admission price vector $\vec{p} = \{p_l\}_{l\in\mathcal{L}}$ to all potential SUs. While $\vec{s}$ with $\sum_{l=1}^{L} s_l = 1$ probabilistically guides the SUs in selecting channels, $\vec{p}$ helps SUs decide whether or not to join the network.

2) SUs: We assume that there is a set of classes of SUs denoted by $\mathcal{K} = \{1, \dots, K\}$ in the network where class-k SUs arrive at the network according to a Poisson process with a potential rate Λ_k, $\forall k \in \mathcal{K}$. Each class-$k$ SU carries a distinct job (e.g. a packet, session, or connection) upon arrival and its job is associated with a specific delay-sensitive application characterized by a value θ_k. For example, multimedia applications with stringent delay requirements will have high values of θ_k. Without loss of generality, we assume $0 < \theta_1 < \dots < \theta_K$[1]. The requested time to complete a SU's job is represented by a r.v. X with pdf $f_X(x)$. This r.v. is assumed to be independent of the arrival process. We denote the expectation of any r.v. X by $\overline{X}$.

We further assume that each SU belonging to a type α has a monetary evaluation value αr about the operator's service, where r is the operator's intrinsic quality (e.g. the coverage area and/or the aggregated channels capacity) and α is a random variable characterizing the *heterogeneity* of SUs under the same operator's quality which depends on independent SUs' physical conditions such as fast fading, location, random moving, etc. The common technique that can be used to estimate α is conjoint analysis [23]. We assume that α follows a

[1] If two classes m and n have $\theta_m = \theta_n$, we can merge them into one class with potential arrival rate $\Lambda_m + \Lambda_n$.

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uniform distribution on $[0, 1]$, which is widely used in the literature [24]–[26], with their cumulative distribution function $F(\cdot)$. One of the main reasons to assume α with uniform distribution is for the analysis tractability.

When a potential SU arrives at the network, it makes a decision: either join the network or balk. The utility of any balking SU is set to zero. For given $\vec{s}$ and $\vec{p}$ from the operator, the utility of a class-k type-α SU that joins channel l is modelled by

$$U_{k,l}(\alpha, s_l, p_l) = \alpha r - \theta_k d_l(s_l) - p_l, \quad \forall k, l. \quad (1)$$

This utility function is perceived by a class-k type-α SU as the difference between net benefit $\alpha r - p_l$ and a delay disutility $\theta_k d_l(s_l)$ representing the delay cost of the SU's job with delay $d_l(s_l)$ in channel l and a delay cost per unit time θ_k. We note that this utility is generalized to capture the heterogeneity of both user types (i.e. α) and classes (i.e. θ_k). In the literature, most utility functions either have the same value r [22] or θ_k [27] for all users. A variant form is $\alpha q_l - p_l$ [24], [28], where the QoS function q_l presents an inverse effect of our congestion function d_l.

3) Steady-State Queueing Delay: Since many SUs may attempt to share the same licensed channel $l \in \mathcal{L}$, congestion can occur, which will affect the delay $d_l(s_l)$ of each SU job. An arriving SU at this channel will be informed of its job's delay in a queue containing other SU jobs that also wish to use that licensed channel. Therefore, the operator is assumed to maintain a parallel queueing system of L M/G/1 queues (cf. Fig. 2) whose service time of each queue l, denoted by a random variable χ_l, has a general distribution dictated by $f_X(x)$, $f_{Y_l}(y)$ and $f_{Z_l}(z)$. We denote $\overline{T}_l(\lambda_l)$ the mean steady-state queueing delay (i.e. waiting time + service time) induced by an effective arrival rate λ_l. Denoting the first and second moments of channel l's service time by $\bar{\chi}_l$ and $\overline{\chi_l^2}$, respectively, we have the extended-value mean queueing delay defined as follows according to the Pollaczek-Khinchin formula [29]

$$\overline{T}_l(\lambda_l) = \begin{cases} \frac{\lambda_l \overline{\chi_l^2}}{2(1-\lambda_l \bar{\chi}_l)} + \bar{\chi}_l, & \text{if } \lambda_l < 1/\bar{\chi}_l; \\ \infty, & \text{otherwise.} \end{cases} \quad (2)$$

This definition can eliminate the explicit condition $\lambda_l < 1/\bar{\chi}_l$ in our arguments hereafter. It turns out that by deriving $\bar{\chi}_l$ and $\overline{\chi_l^2}$, we can complete the queueing delay model. Without loss of generality, we also assume $0 < \bar{\chi}_1 < \ \ldots \ < \bar{\chi}_L$ [2].

We assume that a SU can use its spectrum sensing and handoff capabilities to detect and protect the PUs. Spectrum sensing is used to inform the SU whether the channel is busy or idle. When the channel is sensed to be idle, the SU job can be in service. When the channel is sensed to be busy, the spectrum handoff interrupts the current SU's service, returns the channel to the PUs, and resumes the SU's service when the PU leaves. Using the renewal theory to handle multiple interruptions due to spectrum handoffs, we have derived $\bar{\chi}_l$ and $\overline{\chi_l^2}$ in [22] as follows

$$\bar{\chi}_l = \overline{X}\left(1 + \frac{\overline{Y_l}}{\overline{Z_l}}\right), \quad (3)$$

$$\overline{\chi_l^2} = \overline{X^2}\left(1 + 2\frac{\overline{Y_l}}{\overline{Z_l}}\right) + \overline{Y_l^2}\left(\frac{\overline{X}}{\overline{Z_l}} + \overline{g(X)}\right), \quad (4)$$

where the Laplace transform of $g(X \mid X = x)$ is

$$g^*(s) = \frac{2}{s^2 \overline{Z_l}} \frac{f^*_{Z_l}(s)}{1 - f^*_{Z_l}(s)}, \quad (5)$$

and $f^*_{Z_l}(s)$ is the Laplace transform of $f_{Z_l}(z)$. Let λ_k denote the effective arrival rate of class-k SUs into the system. Due to the load balancing control $\vec{s}$, the effective arrival rate into channel l is $\lambda_l(s_l) = s_l \sum_{k=1}^{K} \lambda_k$. Then, the utility function in (1) can be rewritten

$$U_{k,l}(\alpha, s_l, p_l) = \alpha r - \theta_k \overline{T}_l(\lambda_l(s_l)) - p_l, \quad \forall k, l. \quad (6)$$

B. SUs' Decision Policy and Equilbrium

We assume that the SUs are rational decision-makers in that they only join the network when their utilities are positive (Individual Rationality). Therefore, we have:

Definition 1. A class-k type-α SU in channel l with its utility $U_{k,l}(\alpha, s_l, p_l)$ will follow a joining decision policy such that

- it joins the channel l if $U_{k,l}(\alpha, s_l, p_l) > 0$, which requires $\alpha > \alpha_{k,l}(\lambda_l(s_l))$, where

$$\alpha_{k,l}(\lambda_l(s_l)) := \frac{\theta_k \overline{T}_l(\lambda_l(s_l)) + p_l}{r}; \quad (7)$$

- it balks, otherwise.

[2]If two channels m and n have the same first moment value, we can order them according to the second moment value. If they have the same first and second moments, we can merge them into a virtual channel v with $\bar{\chi}_v = \bar{\chi}_m = \bar{\chi}_n$ such that for given s_v and p_v from the operator, each of the channels m and n will have equally $s_v/2$ and $p_v/2$.

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Then, the effective arrival rate of class-k SU into channel l, defined by $\lambda_{k,l} := s_l \lambda_k$, is as follows

$$\begin{aligned}\lambda_{k,l} &= s_l \Lambda_k \Pr\left[U_{k,l}(\alpha, s_l, p_l) > 0\right] \\ &= s_l \Lambda_k \Pr\left[\alpha > \alpha_{k,l}\left(\lambda_l(s_l)\right)\right] \\ &= s_l \Lambda_k \left(1 - F\left(\alpha_{k,l}\left(\textstyle\sum_{k=1}^K \lambda_{k,l}\right)\right)\right). \end{aligned} \tag{8}$$

Proposition 1. *For given $\vec{p}$ and $\vec{s}$ of a shared-use monopolist operator, there exists a unique equilibrium arrival rate $\lambda_{k,l}^{\mathsf{eq}}$ of the class-k SU into channel l such that*

1) *if $p_l \geq r - \theta_k \bar{X}_l$, then*

$$\lambda_{k,l}^{\mathsf{eq}} = 0, \tag{9}$$

2) *if $p_l < r - \theta_k \bar{X}_l$, then*

$$\lambda_{k,l}^{\mathsf{eq}} = s_l \Lambda_k \left(1 - \alpha_{k,l}\left(\textstyle\sum_{k=1}^K \lambda_{k,l}^{\mathsf{eq}}\right)\right) > 0. \tag{10}$$

C. Operator's Revenue Maximization

In this subsection, we formulate the revenue maximization problem and present a sequential optimization method, based on which we can achieve the optimal solution and algorithm.

1) Problem Formulation: At this stage, the operator temporarily assumes that there exists a unique SUs' equilibrium $\lambda_{k,l}^{\mathsf{eq}} > 0$, $\forall k, l$. Based on this knowledge, the operator's objective is to maximize its revenue, which can be formulated as the following optimization problem

$$\begin{aligned} \underset{\vec{s}, \vec{p}}{\text{maximize}} \quad & \textstyle\sum_{l=1}^L p_l \lambda_l^{\mathsf{eq}} \\ \text{subject to} \quad & \lambda_{k,l}^{\mathsf{eq}} = s_l \Lambda_k \left(1 - \alpha_{k,l}\left(\textstyle\sum_{k=1}^K \lambda_{k,l}^{\mathsf{eq}}\right)\right), \ \forall k, l, \\ & \textstyle\sum_{l=1}^L s_l = 1, \\ & 0 \leq s_l \leq 1, \ p_l \geq 0, \quad \forall l. \end{aligned} \tag{11}$$

The first constraint is the SUs' equilibrium knowledge from Proposition 1, whereas the second constraint is the load balancing constraint and the third constraint is the operational space of $\vec{s}$ and $\vec{p}$. This problem is a non-convex optimization problem, which is difficult to solve.

From the first constraint of (11), the equilibrium arrival rate into channel l can be obtained as

$$\lambda_l^{\mathsf{eq}} = \textstyle\sum_{k=1}^K \lambda_{k,l}^{\mathsf{eq}} = s_l \left(\Lambda - \dfrac{\Omega \bar{T}_l\left(\lambda_l^{\mathsf{eq}}\right) + \Lambda p_l}{r}\right), \tag{12}$$

where $\Lambda := \sum_{k=1}^K \Lambda_k$ and $\Omega := \sum_{k=1}^K \Lambda_k \theta_k$. From (12), we have

$$p_l(s_l, \lambda_l^{\mathsf{eq}}) = r - \frac{r}{\Lambda}\frac{\lambda_l^{\mathsf{eq}}}{s_l} - \frac{\Omega}{\Lambda}\bar{T}_l\left(\lambda_l^{\mathsf{eq}}\right). \tag{13}$$

Eliminating the first constraint of problem (11) by substituting (13) into the objective function, we obtain an equivalent optimization problem as follows

$$\begin{aligned} \underset{\vec{s}, \overrightarrow{\lambda^{\mathsf{eq}}} = \{\lambda_l^{\mathsf{eq}}\}_{l \in \mathcal{L}}}{\text{maximize}} \quad & \textstyle\sum_{l=1}^L r\lambda_l^{\mathsf{eq}} - \dfrac{r}{\Lambda}\dfrac{(\lambda_l^{\mathsf{eq}})^2}{s_l} - \dfrac{\Omega}{\Lambda}\lambda_l^{\mathsf{eq}} \bar{T}_l\left(\lambda_l^{\mathsf{eq}}\right) \\ \text{subject to} \quad & \textstyle\sum_{l=1}^L s_l = 1, \\ & 0 \leq s_l \leq 1, \ \forall l. \end{aligned} \tag{14}$$

We observe that problem (14) reveals a structure that can be solved efficiently by using a sequential optimization technique as follows.

2) Sequential Optimization: First, by fixing $\overrightarrow{\lambda^{\mathsf{eq}}}$, problem (14) is equivalent to

$$\begin{aligned} \underset{\vec{s}}{\text{maximize}} \quad & \textstyle\sum_{l=1}^L -\dfrac{r}{\Lambda}\dfrac{(\lambda_l^{\mathsf{eq}})^2}{s_l} \\ \text{subject to} \quad & \textstyle\sum_{l=1}^L s_l = 1, \ \forall l, \\ & 0 \leq s_l \leq 1, \ \forall l. \end{aligned} \tag{15}$$

It can be seen that (15) is a convex problem. Using the necessary and sufficient KKT condition, the solution of (15) can be obtained as follows

$$s_l^* = \frac{\lambda_l^{\mathsf{eq}}}{\sum_{l=1}^L \lambda_l^{\mathsf{eq}}}, \quad \forall l. \tag{16}$$

Substituting (16) back into (14) and introducing an auxiliary variable $\lambda_{\mathsf{tot}}^{\mathsf{eq}} = \sum_{l=1}^L \lambda_l^{\mathsf{eq}}$, we have an equivalent problem of (14) as follows

$$\begin{aligned} \text{maximize} \quad & r\lambda_{\mathsf{tot}}^{\mathsf{eq}} - \frac{r}{\Lambda}\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}\right)^2 - \frac{\Omega}{\Lambda}\textstyle\sum_{l=1}^L \lambda_l^{\mathsf{eq}} \bar{T}_l\left(\lambda_l^{\mathsf{eq}}\right) \\ \text{subject to} \quad & \textstyle\sum_{l=1}^L \lambda_l^{\mathsf{eq}} = \lambda_{\mathsf{tot}}^{\mathsf{eq}}, \\ \text{variables} \quad & \lambda_{\mathsf{tot}}^{\mathsf{eq}}, \overrightarrow{\lambda^{\mathsf{eq}}} \geq 0. \end{aligned} \tag{17}$$

Lemma 1. *Problem* (17) *is a convex optimization problem.*

The Lagrangian of this problem is $L\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}, \overrightarrow{\lambda^{\mathsf{eq}}}, \mu\right) = L_{\mathsf{tot}}\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}, \mu\right) + \sum_{l=1}^L L_l\left(\lambda_l^{\mathsf{eq}}, \mu\right)$, where

$$L_{\mathsf{tot}}\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}, \mu\right) = r\lambda_{\mathsf{tot}}^{\mathsf{eq}} - \frac{r}{\Lambda}\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}\right)^2 + \mu\lambda_{\mathsf{tot}}^{\mathsf{eq}}, \tag{18}$$

$$L_l\left(\lambda_l^{\mathsf{eq}}, \mu\right) = -\frac{\Omega}{\Lambda}\lambda_l^{\mathsf{eq}} \bar{T}_l\left(\lambda_l^{\mathsf{eq}}\right) - \mu\lambda_l^{\mathsf{eq}}, \ \forall l. \tag{19}$$

It is easy to see that $L_{\mathsf{tot}}\left(\lambda_{\mathsf{tot}}^{\mathsf{eq}}, \mu\right)$ is a strictly concave function of $\lambda_{\mathsf{tot}}^{\mathsf{eq}}$ for a given μ, and we also have $L_l\left(\lambda_l^{\mathsf{eq}}, \mu\right)$ is a strictly concave function of λ_l^{eq}, $\forall l$, for a given μ from Lemma 1. Using the first-order condition, we can obtain the unique optimal solutions for a given

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μ as follows

$$\lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu) = \frac{\Lambda(r+\mu)}{2r}, \tag{20}$$

$$\lambda_{l}^{\mathsf{eq}}(\mu) = \left[\Phi_l(\mu)\right]^+, \ \forall l, \tag{21}$$

where $[.]^+ := \max\{0, .\}$ and

$$\Phi_l(\mu) := \frac{2\left(\mu\Lambda + \Omega\bar{\chi}_l\right)}{-\zeta_l(\mu) - \sqrt{\Omega\overline{\chi_l^2}\zeta_l(\mu)}}, \tag{22}$$

with $\zeta_l(\mu) = \Omega\left(\overline{\chi_l^2} - 2\bar{\chi}_l^2\right) - 2\Lambda\bar{\chi}_l\mu$. We have the following property.

Lemma 2. *$\Phi_l(\mu)$ is continuous, strictly decreasing, positive on $\left(-\infty, -\frac{\Omega}{\Lambda}\bar{\chi}_l\right)$, and non-positive on $\left[-\frac{\Omega}{\Lambda}\bar{\chi}_l, -\frac{\Omega}{\Lambda}\left(\bar{\chi}_l - \frac{\overline{\chi_l^2}}{2\bar{\chi}_l}\right)\right)$, $\forall l$.*

3) Optimal Solutions: We can achieve the optimal solution $\lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu^*)$ and $\lambda_{l}^{\mathsf{eq}}(\mu^*)$, $\forall l$, by finding the optimal dual variable μ^* that satisfies the first constraint $\sum_{l=1}^{L}\lambda_{l}^{\mathsf{eq}}(\mu) = \lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu)$ of problem (17). Thus, we have the following result.

Lemma 3. *If*

$$\tau^{\mathsf{mo}} := r > \frac{\Omega}{\Lambda}\bar{\chi}_1, \tag{23}$$

there exist a unique solution $\mu^* \in \left(-\tau^{\mathsf{mo}}, -\frac{\Omega}{\Lambda}\bar{\chi}_1\right)$ of $\sum_{l=1}^{L}\left[\Phi_l(\mu)\right]^+ = \lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu)$ and a corresponding channel index $1 \le L^* \le L$ such that $\Phi_l(\mu^*) > 0$, $\forall l \le L^*$, and $\Phi_l(\mu^*) = 0$, $\forall l > L^*$. If $\tau^{\mathsf{mo}} \le \frac{\Omega}{\Lambda}\bar{\chi}_1$, $\left[\Phi_l(\mu)\right]^+ = \lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu) = 0$, $\forall l$.

We further illustrate this lemma numerically in Fig. 6 in Section V. With this unique μ^*, we obtain the unique solution $\lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu^*)$ and $\lambda_{l}^{\mathsf{eq}}(\mu^*)$, $\forall l$, as per (20) and (21), which is also the global unique optimal solution of problem (17) since μ^*, $\lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu^*)$ and $\lambda_{l}^{\mathsf{eq}}(\mu^*)$ satisfy the necessary and sufficient KKT condition [30]. Substituting these values into (16) and (13), we can achieve s_l^* and p_l^*, $\forall l$.

Proposition 2. *With μ^* and L^* from Lemma 3, the load balancing and pricing optimal solutions of operator's revenue maximization problem are unique as follows*

$$s_l^* = \frac{\Phi_l(\mu^*)}{\lambda_{\mathsf{tot}}^{\mathsf{eq}}(\mu^*)}, \quad \forall l \le L^*, \tag{24}$$

$$p_l^* = \left[p_l(s_l^*, \Phi_l(\mu^*))\right]^+ \textit{ from } (13), \forall l \le L^*, \tag{25}$$

$$s_l^* = 0 \textit{ and } p_l^* = 0, \quad \forall l > L^*. \tag{26}$$

4) Algorithm: The optimal solution of the operator's revenue maximization problem provided by Proposition 2 is based on the assumption that $\lambda_{k,l}^{\mathsf{eq}} > 0$, $\forall k, l$, which is not always true. Therefore, we propose Algorithm 1 to search for a class $K^* \le K$ such that the optimal solutions in Proposition 2 corresponds to $\lambda_{k,l}^{\mathsf{eq}} > 0$, $\forall\ l \le L^*, k \le K^*$. We can consider K^* the largest class that can be supportable by the operator. We have the following property on which Algorithm 1 relies.

Lemma 4. *Defining $\Lambda(k) := \sum_{j=1}^{k}\Lambda_j$ and $\Omega(k) := \sum_{j=1}^{k}\Lambda_j\theta_j$, $\frac{\Omega(k)}{\Lambda(k)}$ is increasing in $k \in \mathcal{K}$.*

Proof: Since $\Omega(k+1) = \Omega(k) + \Lambda_{k+1}\theta_{k+1}$, $\Lambda(k+1) = \Lambda(k) + \Lambda_{k+1}$, we have

$$\frac{\Omega(k+1)}{\Lambda(k+1)} - \frac{\Omega(k)}{\Lambda(k)} = \frac{\Lambda_{k+1}\left(\Lambda(k)\theta_{k+1} - \Omega(k)\right)}{\Lambda(k)\left(\Lambda(k) + \theta_{k+1}\right)} > 0.$$

■

Algorithm 1 Optimal Pricing and Load-Balancing in the Monopoly Market

1: The operator collects $\bar{\chi}_l, \overline{\chi_l^2}, \forall l$ and $\Lambda_k, \theta_k, \forall k$
2: $i \leftarrow \arg\max_{k\in\mathcal{K}}\left\{r > \frac{\Omega(k)}{\Lambda(k)}\bar{\chi}_1\right\}$
3: update $p_l^*(i)$, $s_l^*(i)$ and $L^*(i)$ by Proposition 2 with $\Lambda(i)$, $\Omega(i)$
4: **while** $p_l^*(i) \ge r - \theta_i\bar{\chi}_l$ for some $l \le L^*(i)$ **do**
5:     $i \leftarrow i - 1$
6:     repeat step 3
7: **end while**
8: $K^* \leftarrow i$, $p_l^* \leftarrow p_l^*(K^*)$, $s_l^* \leftarrow s_l^*(K^*)$, $\forall l$
9: Operator broadcasts p_l^*, s_l^* and $\bar{T}_l\left(\lambda_l^{\mathsf{eq}}(\mu^*)\right)$ $\forall l$, and all SUs join the network by Definition 1.

Proposition 3. *If*

$$r > \theta_1\bar{\chi}_1, \tag{27}$$

Algorithm 1 always returns a class $K^* \ge 1$.

Proof: We prove by contradiction. Assuming that Algorithm 1 cannot return any optimal class, which means in the worst case with a single class $k = 1$, we have $p_l^*(1) \ge r - \theta_1\bar{\chi}_l > 0$ for some $1 \le l \le L^*(i)$ with condition (27), which means $\lambda_{1,l}^{\mathsf{eq}} = 0$ according to (9), implying $s_l^*(1) = 0$ by (16), leading to $p_l^*(1) = 0$ for theses ls according to (26), which is a contradiction. ■

Remark 1. i) Since $\theta_1\bar{\chi}_1$ is the smallest cost that can be experienced by a potential SU (without queueing delay and with zero price), condition (27) precludes a trivial scenario where no SU has any incentive to join the operator. ii) In line 1 of Algorithm 1, all parameters can be estimated by the existing method [31]

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and through some feedback mechanisms from the SUs. The algorithm then determines the largest supportable class i (line 2), where we can always find such a class $i \geq 1$ with condition (27). According to Lemma 4, condition (23) is always satisfied with $\frac{\Omega(j)}{\Lambda(j)}$, $\forall j \leq i$. Thus, by Proposition 2, we always obtain $p_l^*(j)$, $s_l^*(j)$ and $L^*(j)$, $\forall j \leq i$ in line 3 of Algorithm 1. Hence, the algorithm keeps lowering this largest class until, if it is possible, there is a class K^* and the corresponding L^* satisfying condition $p_l^* < r - \theta_k \overline{X}_l$, $\forall l \leq L^*, k \leq K^*$ (lines 3 to 8 of Algorithm 1), which induces $\lambda_{k,l}^{\mathsf{eq}} > 0$, $\forall l \leq L^*, k \leq K^*$ according to Proposition 1. iii) In line 3, Algorithm 1 needs to compute μ^* satisfying Lemma 3. This μ^* can be found using a bisection method with a constant complexity[3]. Hence, Algorithm 1 has a complexity $O(K)$. vi) The system can operate on sequential time slots where Algorithm 1 runs repeatedly in each slot with fixed channel distributions such that all incoming SUs can receive broadcast information (line 9) from the operator in any time slot.

IV. Duopoly

In this section, we first present the system model and how SUs choose which operator to join and their equilibrium with the given price and load balancing of two operators. Based on these information, how these two operators noncooperatively and cooperatively, respectively, maximize their revenues are investigated later.

A. System Model

We assume that there are two wireless network operators providing different DSA models. The first operator, denoted by $\mathcal{O}_1$, uses the shared-use model, whereas the second operator, denoted by $\mathcal{O}_2$, employs the exclusive-use model. A sequence of SUs' jobs is assumed to arrive at the network and each SU will make a decision as to which operator to join for its job (cf. Fig. 3). The model in this section can be described quantitatively as follows.

1) Shared-use Operator ($\mathcal{O}_1$): The model of this operator is similar to Section III-A1. However, given $\vec{s}$ and $\vec{p}$ from $\mathcal{O}_1$, SUs will see the service of $\mathcal{O}_1$'s channels in the average sense: the average delay $\sum_{l=1}^{L} s_l \overline{T}_l(\lambda_{1,l})$ and average price $\sum_{l=1}^{L} s_l p_l$. Therefore, $\mathcal{O}_1$ can simplify the pricing structure by only setting a single price p_1 instead of $\vec{p}$ to replace for $\sum_{l=1}^{L} s_l p_l$. Hence, the utility of a class-k type-α SU with $\mathcal{O}_1$ is

$$U_{1,k}(\alpha, \vec{s}, p_1) = \alpha r_1 - \theta_k \sum_{l=1}^{L} s_l \overline{T}_l(\lambda_{1,l}) - p_1, \quad (28)$$

where r_1 is the intrinsic quality of $\mathcal{O}_1$'s channels and $\lambda_{1,l}$ is the effective arrival rate into channel l of $\mathcal{O}_1$.

[3]It depends on chosen starting points and a tolerance value [32].

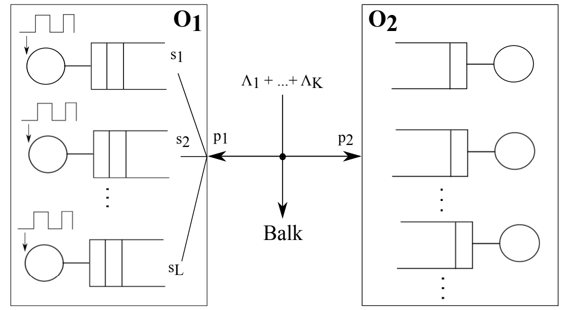

Fig. 3. A duopoly between shared-use and exclusive-use operators.

2) Exclusive-use Operator ($\mathcal{O}_2$): The operator $\mathcal{O}_2$ is assumed to obtain (i.e. via leasing) the part of the spectrum which is temporarily unused by the spectrum owner. This spectrum chunk is divided into multiple bands that have the same bandwidth as that of $\mathcal{O}_1$'s channels. Since there is no PU traffic on these bands, SU services are not interrupted in this case.

Whenever an arriving SU decides to join $\mathcal{O}_2$, the operator allocates a dedicated channel for the SU. We assume that $\mathcal{O}_2$ always has enough dedicated channels to serve the SUs[4]. Therefore, we can consider $\mathcal{O}_2$ to be a M/G/∞ queueing system where queueing delays of all SUs are equal to $\overline{X}$. From (1), the utility of a class-k type-α SU with $\mathcal{O}_2$ is

$$U_{2,k}(\alpha, p_2) = \alpha r_2 - \theta_k \overline{X} - p_2, \quad \forall k, \quad (29)$$

where r_2 is the intrinsic quality of $\mathcal{O}_2$ channels. Since $\mathcal{O}_1$ with interruptible service always has higher delay cost than $\mathcal{O}_2$'s dedicated channels, $\mathcal{O}_1$ needs to have better intrinsic quality in order to survive in the market. Hence, we assume that $r_1 > r_2$. An example is that $\mathcal{O}_2$ is an incumbent, whereas $\mathcal{O}_1$ is an entrant with a wider coverage area.

B. SUs' Decision Policy and Equilbrium

We denote the type of class-k *critical users* of $\mathcal{O}_1$ and $\mathcal{O}_2$ by $\alpha_{1,k}$ and $\alpha_{2,k}$ such that $U_{1,k}(\alpha_{1,k}, \vec{s}, p_1) = 0$ and $U_{2,k}(\alpha_{2,k}, p_2) = 0$, respectively. Since $\lambda_{1,l} = s_l \sum_{k=1}^{K} \lambda_{1,k}$, we have

$$\alpha_{1,k}(\overrightarrow{\lambda_{1,k}}) = \frac{\theta_k \sum_{l=1}^{L} s_l \overline{T}_l \left(s_l \sum_{k=1}^{K} \lambda_{1,k} \right) + p_1}{r_1}, \quad (30)$$

$$\alpha_{2,k} = \frac{\theta_k \overline{X} + p_2}{r_2}, \quad (31)$$

where $\overrightarrow{\lambda_{1,k}} = \{\lambda_{1,k}\}_{k \in \mathcal{K}}$ is the vector of effective arrival rate into $\mathcal{O}_1$ of K classes of SUs. We also denote

[4]We can relax this assumption by borrowing/leasing more channels from other homogeneous operators when $\mathcal{O}_2$ lacks the dedicated channels [2], [33].

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the type of class-k *indifferent user* by $\tilde{\alpha}_k$ such that $U_{1,k}(\tilde{\alpha}_k, \vec{s}, p_1) = U_{2,k}(\tilde{\alpha}_k, p_2)$. Then we have

$$\tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}}) = \frac{r_1\, \alpha_{1,k}(\overrightarrow{\lambda_{1,k}}) - r_2\, \alpha_{2,k}}{r_1 - r_2}. \tag{32}$$

Since SUs are rational decision-makers, they choose to join $\mathcal{O}_i$ if their utilities with $\mathcal{O}_i$ are not only positive (Individual Rationality) but also higher than those of the other operator (Incentive Compatibility). Hence, we have

Definition 2. A class-k type-α SU that has $U_{1,k}(\alpha, \vec{s}, p_1)$ with $\mathcal{O}_1$ and $U_{2,k}(\alpha, p_2)$ with $\mathcal{O}_2$ will follow a joining decision policy such that

- it joins $\mathcal{O}_1$ if $U_{1,k}(\alpha, \vec{s}, p_1) > 0$ and $U_{1,k}(\alpha, \vec{s}, p_1) > U_{2,k}(\alpha, p_2)$, which requires
$$\alpha > \alpha_{1,k}(\overrightarrow{\lambda_{1,k}}) \text{ and } \alpha > \tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}}), \tag{33}$$
- it joins $\mathcal{O}_2$ if $U_{2,k}(\alpha, p_2) > 0$ and $U_{2,k}(\alpha, p_2) \geq U_{1,k}(\alpha, \vec{s}, p_1)$, which requires
$$\alpha > \alpha_{2,k} \text{ and } \alpha < \tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}}), \tag{34}$$
- it balks if $U_{1,k}(\alpha, \vec{s}, p_1) \leq 0$ and $U_{2,k}(\alpha, p_2) \leq 0$, which requires
$$\alpha \leq \alpha_{1,k}(\overrightarrow{\lambda_{1,k}}) \text{ and } \alpha \leq \alpha_{2,k}. \tag{35}$$

With SUs' joining policy defined as above, the effective arrival rate of class-k SUs into $\mathcal{O}_1$ and $\mathcal{O}_2$, respectively, are as follows:

$$\begin{aligned} \lambda_{1,k} &= \Lambda_k \Pr\left[\alpha > \alpha_{1,k}(\overrightarrow{\lambda_{1,k}}) \text{ and } \alpha > \tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}})\right] \\ &= \Lambda_k \int_{\max\left\{\tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}}),\, \alpha_{1,k}(\overrightarrow{\lambda_{1,k}})\right\}}^{1} dF(\alpha), \end{aligned} \tag{36}$$

$$\begin{aligned} \lambda_{2,k} &= \Lambda_k \Pr\left[\alpha_{2,k} < \alpha < \tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}})\right] \\ &= \Lambda_k \int_{\alpha_{2,k}}^{\tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}})} dF(\alpha). \end{aligned} \tag{37}$$

Based on (36) and (37), we have the following result.

Proposition 4. *For a given $\vec{s}$, p_1 of $\mathcal{O}_1$ and p_2 of $\mathcal{O}_2$ in a duopoly market, there exists a unique pair of equilibrium arrival rates $\lambda_{1,k}^{\mathsf{eq}}$ and $\lambda_{2,k}^{\mathsf{eq}}$ of the class-k SUs $\forall k \in \mathcal{K}$ into $\mathcal{O}_1$ and $\mathcal{O}_2$, respectively, such that*

1) *if $p_2 \geq \beta_k^{\mathsf{up}}(\vec{s}, p_1)$, then*
$$\lambda_{1,k}^{\mathsf{eq}} = \Lambda_k\Big(1 - F(\alpha_{1,k}(\overrightarrow{\lambda_{1,k}^{\mathsf{eq}}}))\Big), \tag{38}$$
$$\lambda_{2,k}^{\mathsf{eq}} = 0, \tag{39}$$

2) *if $\beta_k^{\mathsf{lo}}(\vec{s}, p_1) < p_2 < \beta_k^{\mathsf{up}}(\vec{s}, p_1)$, then*
$$\lambda_{1,k}^{\mathsf{eq}} = \Lambda_k(1 - \tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}^{\mathsf{eq}}})) > 0, \tag{40}$$
$$\lambda_{2,k}^{\mathsf{eq}} = \Lambda_k(\tilde{\alpha}_k(\overrightarrow{\lambda_{1,k}^{\mathsf{eq}}}) - \alpha_{2,k}) > 0, \tag{41}$$

3) *if $p_2 \leq \beta_k^{\mathsf{lo}}(\vec{s}, p_1)$, then*
$$\lambda_{1,k}^{\mathsf{eq}} = 0, \tag{42}$$
$$\lambda_{2,k}^{\mathsf{eq}} = (1 - F(\alpha_{2,k})), \tag{43}$$

where

$$\beta_k^{\mathsf{up}}(\vec{s}, p_1) = \theta_k\Big(r_2 \sum\nolimits_{l=1}^{L} s_l \bar{X}_l - r_1 \overline{X}\Big) + \frac{r_2}{r_1} p_1,$$
$$\beta_k^{\mathsf{lo}}(\vec{s}, p_1) = \theta_k\Big(\sum\nolimits_{l=1}^{L} s_l \bar{X}_l - \overline{X}\Big) + r_2 - r_1 + p_1.$$

We see that there is an interaction between $\mathcal{O}_1$ and $\mathcal{O}_2$'s decisions on $(\vec{s}, p_1)$ and p_2, respectively. If p_2 is greater than a value $\beta_k^{\mathsf{up}}(\vec{s}, p_1)$, $\mathcal{O}_1$ becomes a monopolist as case (1). In contrast, if p_2 is less than a value $\beta_k^{\mathsf{lo}}(\vec{s}, p_1)$, $\mathcal{O}_2$ becomes a monopolist as case (3). Therefore, we can consider $\beta_k^{\mathsf{lo}}(\vec{s}, p_1)$ and $\beta_k^{\mathsf{up}}(\vec{s}, p_1)$ the lower and upper thresholds, respectively, for the operational range of p_2 in order to have duopoly coexistence between $\mathcal{O}_1$ and $\mathcal{O}_2$ in case (2).

Cases (1) and (3) correspond to the shared-use and exclusive-use monopolist analyzed in Section III and [22], respectively. Henceforth, we will focus only on the duopoly market of case (2). If the condition of case (2) is satisfied $\forall k$, the total equilibrium arrival rates to $\mathcal{O}_1$ and $\mathcal{O}_2$ are, respectively, as follows:

$$\lambda_1^{\mathsf{eq}} = \sum\nolimits_{k=1}^{K} \lambda_{1,k}^{\mathsf{eq}} = \Lambda - \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right), \tag{44}$$
$$\lambda_2^{\mathsf{eq}} = \sum\nolimits_{k=1}^{K} \lambda_{2,k}^{\mathsf{eq}} = \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right) - \alpha_2, \tag{45}$$

where $\alpha_1(\lambda_1^{\mathsf{eq}}) = \sum_{k=1}^{K} \Lambda_k \alpha_{1,k}$, $\alpha_2 = \sum_{k=1}^{K} \Lambda_k \alpha_{2,k}$, and $\tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right) = \sum_{k=1}^{K} \Lambda_k \tilde{\alpha}_k$.

C. Duopoly: NonCooperative Operators

In this subsection, we study the noncooperative case as a one-shot game and characterize the Nash equilibrium of this game.

1) Game Formulation: Based on the SUs' equilibrium arrival rates in (44) and (45), the operators will compete with each other to maximize their revenues, which can be modelled as the following one-shot game:

- Players: $\mathcal{O}_1$ and $\mathcal{O}_2$,
- Strategies: $\mathcal{O}_1$ determines $\vec{s}$ and p_1; $\mathcal{O}_2$ determines p_2,
- Payoff functions: $\pi_1(\vec{s}, p_1; p_2) = \lambda_1^{\mathsf{eq}} p_1$ and $\pi_2(p_2; \vec{s}, p_1) = \lambda_2^{\mathsf{eq}} p_2$.

In order to find the Nash equilibria of this game, we investigate the operators' best responses first.

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2) $\mathcal{O}_1$'s best response: The following optimization problem captures $\mathcal{O}_1$'s best response

$$\begin{aligned} \underset{\vec{s},\, p_1}{\text{maximize}} \quad & \lambda_1^{\mathsf{eq}} p_1 \\ \text{subject to} \quad & \lambda_1^{\mathsf{eq}} = \Lambda - \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right), \\ & \sum\nolimits_{l=1}^{L} s_l = 1, \\ & 0 \le s_l \le 1,\ \forall l. \end{aligned} \tag{46}$$

The first constraint is equivalent to

$$\begin{aligned} p_1\left(\lambda_1^{\mathsf{eq}}, \vec{s};\, p_2\right) = p_2 - \frac{r_1 - r_2}{\Lambda}\lambda_1^{\mathsf{eq}} + r_1 - r_2 \\ + \frac{\Omega}{\Lambda}\left(\overline{X} - \sum\nolimits_{l=1}^{L} s_l \overline{T}_l\left(s_l \lambda_1^{\mathsf{eq}}\right)\right). \end{aligned} \tag{47}$$

Eliminating the first constraint by substituting (47) into the objective function of problem (46) and introducing the new variables

$$\lambda_{1,l}^{\mathsf{eq}} := s_l \lambda_1^{\mathsf{eq}},\ \forall l, \tag{48}$$

we obtain the following equivalent optimization problem

$$\begin{aligned} \text{max.} \quad & \left(\frac{\Omega\overline{X}}{\Lambda} + p_2 + r_1 - r_2\right)\lambda_1^{\mathsf{eq}} - \frac{(r_1 - r_2)}{\Lambda}\left(\lambda_1^{\mathsf{eq}}\right)^2 \\ & - \frac{\Omega}{\Lambda}\sum\nolimits_{l=1}^{L} \lambda_{1,l}^{\mathsf{eq}} \overline{T}_l\left(\lambda_{1,l}^{\mathsf{eq}}\right) \\ \text{s.t.} \quad & \sum\nolimits_{l=1}^{L} \lambda_{1,l}^{\mathsf{eq}} = \lambda_1^{\mathsf{eq}}, \\ & \lambda_{1,l}^{\mathsf{eq}} \ge 0,\ \forall l, \\ \text{var.} \quad & \lambda_1^{\mathsf{eq}}, \overrightarrow{\lambda_{1,l}^{\mathsf{eq}}} = \{\lambda_{1,l}^{\mathsf{eq}}\}_{l \in \mathcal{L}}. \end{aligned} \tag{49}$$

We see that the problem (49) has the same convex structure as problem (17). Similarly, using the first-order necessary and sufficient condition, we have

$$\lambda_1^{\mathsf{eq}}\left(p_2;\, \nu\right) = \frac{\overline{X}\Omega + \Lambda(\nu + p_2 + r_1 - r_2)}{2(r_1 - r_2)} \tag{50}$$

$$\lambda_{1,l}^{\mathsf{eq}}\left(\nu\right) = \left[\Phi_l\left(\nu\right)\right]^+, \quad \forall l, \tag{51}$$

where ν is a dual variable associated with the first constraint. Then, $\mathcal{O}_1$'s best response with a given ν is defined as follows

$$BR_1(p_2;\, \nu) := \left\{\vec{s}\left(p_2;\, \nu\right),\, p_1\left(p_2;\, \nu\right)\right\}, \tag{52}$$

where

$$s_l\left(p_2;\, \nu\right) = \frac{\Phi_l(\nu)}{\lambda_1^{\mathsf{eq}}\left(p_2;\, \nu\right)},\ \forall l,\ \text{and} \tag{53}$$

$$p_1\left(p_2;\, \nu\right) := p_1\left(\lambda_1^{\mathsf{eq}}\left(p_2;\, \nu\right), \vec{s}\left(p_2;\, \nu\right)\right) \tag{54}$$

from (48) and (47), respectively.

Then we can find ν^*, which is the solution of the following equation

$$\sum\nolimits_{l=1}^{L} \left[\Phi_l\left(\nu\right)\right]^+ = \lambda_1^{\mathsf{eq}}\left(p_2;\, \nu\right), \tag{55}$$

such that $\lambda_1^{\mathsf{eq}}\left(p_2;\, \nu^*\right)$ and $\lambda_{1,l}^{\mathsf{eq}}\left(\nu^*\right) = \left[\Phi_l\left(\nu^*\right)\right]^+, \forall l$ are the unique optimal solutions of problem (49). Therefore, $\mathcal{O}_1$'s best response, which is the optimal solution of (46), is $BR_1(p_2;\, \nu^*)$.

3) $\mathcal{O}_2$'s best response: The following optimization problem captures $\mathcal{O}_2$'s best response

$$\begin{aligned} \underset{p_2 \ge 0}{\text{maximize}} \quad & \lambda_2^{\mathsf{eq}} p_2 \\ \text{subject to} \quad & \lambda_2^{\mathsf{eq}} = \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right) - \alpha_2. \end{aligned} \tag{56}$$

Eliminating the first constraint by substituting into the objective function and using the first-order condition, we obtain the best response of $\mathcal{O}_2$

$$\begin{aligned} & BR_2(\vec{s}, p_1) := p_2(\vec{s}, p_1) \\ & = \frac{\Omega}{2\Lambda}\left(\frac{r_2}{r_1}\sum\nolimits_{l=1}^{L} s_l \overline{T}_l\left(s_l \lambda_1^{\mathsf{eq}}\right) - \overline{X}\right) + \frac{r_2}{2r_1}p_1. \end{aligned} \tag{57}$$

4) Nash Equilibrium: Based on the best responses of $\mathcal{O}_1$ and $\mathcal{O}_2$, we can find the Nash equilibria of this game, denoted by $(\overrightarrow{s^{\mathsf{na}}}, p_1^{\mathsf{na}})$ and p_2^{na}, through the intersections of two best responses (52) and (57). Specifically, for a given ν, any pair of $(\overrightarrow{s^{\mathsf{na}}}, p_1^{\mathsf{na}})$ and p_2^{na} must satisfy

$$(\overrightarrow{s^{\mathsf{na}}}, p_1^{\mathsf{na}}) = BR_1(p_2^{\mathsf{na}};\nu), \tag{58}$$

$$p_2^{\mathsf{na}} = BR_2(\overrightarrow{s^{\mathsf{na}}}, p_1^{\mathsf{na}}). \tag{59}$$

Substituting (58) into (59), we have

$$p_2^{\mathsf{na}} = BR_2(BR_1(p_2^{\mathsf{na}};\nu)), \tag{60}$$

Substituting (52) into (57) to solve (60), we see that there exists a p_2^{na} for a given ν as follows

$$p_2^{\mathsf{na}}(\nu) = \frac{-r_2\nu + r_2(r_1 - r_2)}{(4r_1 - r_2)} - \frac{\Omega}{\Lambda}\frac{\overline{X}(2r_1 - r_2)}{(4r_1 - r_2)}. \tag{61}$$

From (61), we see that p_2^{na} only depends on dual variable ν of problem (49). Therefore, if we can find a condition such that a unique ν^{na} exists, then we would have a corresponding unique $p_2^{\mathsf{na}}(\nu^{\mathsf{na}})$.

By substituting (61) into (50), we have

$$\lambda_1^{\mathsf{eq}}(p_2^{\mathsf{na}}(\nu);\, \nu) = \frac{\Lambda(2r_1 - r_2)\nu + r_1(\overline{X}\Omega + 2\Lambda(r_1 - r_2))}{(r_1 - r_2)\left(4r_1 - r_2\right)}. \tag{62}$$

Denoting $\lambda_1^{\mathsf{eq}}(\nu) = \lambda_1^{\mathsf{eq}}(p_2^{\mathsf{na}}(\nu);\, \nu)$, we see that $\lambda_1^{\mathsf{eq}}(\nu)$ must satisfy the constraint (55). We have the following result, which can be proved similarly as Lemma 3.

Lemma 5. *If*

$$\tau^{\mathsf{na}} := \frac{r_1(\overline{X}\Omega + 2\Lambda(r_1 - r_2))}{\Lambda(2r_1 - r_2)} > \frac{\Omega}{\Lambda}\overline{X}_1, \tag{63}$$

there exists a unique solution $\nu^{\mathsf{na}} \in \left(-\tau^{\mathsf{na}}, -\frac{\Omega}{\Lambda}\overline{X}_1\right)$ *of* $\sum_{l=1}^{L}\left[\Phi_l\left(\nu\right)\right]^+ = \lambda_1^{\mathsf{eq}}(\nu)$ *and a corresponding channel*

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index $1 \leq L^* \leq L$ *such that* $\Phi_l(\nu^{\mathsf{na}}) > 0,\ \forall l \leq L^*$, *and* $\Phi_l(\nu^{\mathsf{na}}) = 0,\ \forall l > L^*$. *If* $\tau^{\mathsf{na}} \leq \frac{\Omega}{\Lambda}\bar{\chi}_1$, $[\Phi_l(\nu)]^+ = \lambda_1^{\mathsf{eq}}(\nu) = 0,\ \forall l$.

Then, we can obtain a unique $p_2^{\mathsf{na}} = [p_2^{\mathsf{na}}(\nu^{\mathsf{na}})]^+$ from (61). Substituting ν^{na} and p_2^{na} into (58), we can obtain a unique $\left(\overrightarrow{s^{\mathsf{na}}}, p_1^{\mathsf{na}}\right)$. Therefore, we have the following result.

Proposition 5. *With* ν^{na} *and* L^* *from Lemma 5, there exists a unique Nash Equilibrium in a noncooperative duopoly market as follows*

$$p_2^{\mathsf{na}} = [p_2^{\mathsf{na}}(\nu^{\mathsf{na}})]^+ \ \textit{from}\ (61), \tag{64}$$

$$s_l^{\mathsf{na}} = \frac{\Phi_l(\nu^{\mathsf{na}})}{\lambda_1^{\mathsf{eq}}(\nu^{\mathsf{na}})},\ \forall l \leq L^* \ \textit{and}\ s_l^{\mathsf{na}} = 0,\ \forall l > L^*, \tag{65}$$

$$p_1^{\mathsf{na}} = [p_1(\nu^{\mathsf{na}};\ p_2^{\mathsf{na}})]^+ \ \textit{from}\ (54). \tag{66}$$

D. Duopoly: Cooperative Operators

In this subsection, we study the cooperative case where both shared-use and exclusive-use operators jointly maximize the social revenue based on a sharing contract agreement.

1) Social Revenue Maximization: Based on the knowledge of SUs' equilibrium in the duopoly coexistence case, the social revenue can be defined as the total revenue $\pi(\vec{s}, p_1, p_2) := \lambda_1^{\mathsf{eq}} p_1 + \lambda_2^{\mathsf{eq}} p_2$ that both operators can achieve with a given setting $\vec{s}, p_1$ and p_2. Therefore, the social revenue optimization can be formulated as follows

$$\begin{aligned} \underset{\vec{s},\, p_1,\, p_2}{\text{maximize}} \quad & \lambda_1^{\mathsf{eq}} p_1 + \lambda_2^{\mathsf{eq}} p_2 \\ \text{subject to} \quad & \lambda_1^{\mathsf{eq}} = \Lambda - \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right), \\ & \lambda_2^{\mathsf{eq}} = \tilde{\alpha}\left(\lambda_1^{\mathsf{eq}}\right) - \alpha_2, \\ & \sum_{l=1}^{L} s_l = 1, \\ & 0 \leq s_l \leq 1,\ \forall l. \end{aligned} \tag{67}$$

Problem (67) is a non-convex problem, which is difficult to solve efficiently. Fortunately, we can split it into separate subproblems which can be solved efficiently as follows.

2) Separate Subproblems: The first constraint of (67), which can be expressed according to (47), is eliminated by substituting (47) into p_1 of the objective function. The second constraint can also be eliminated by substituting it into λ_2^{eq} of the objective function. When a new variable $\lambda_{1,l}^{\mathsf{eq}} = s_l \lambda_1^{\mathsf{eq}}$ is further introduced, the original problem (67) can decomposed into two separate optimization problems. The first problem is

$$\begin{aligned} \underset{\lambda_1^{\mathsf{eq}}, \overrightarrow{\lambda_{1,l}^{\mathsf{eq}}}}{\text{maximize}} \quad & \left(\frac{\Omega}{\Lambda}\overline{X} + r_1 - r_2\right)\lambda_1^{\mathsf{eq}} - \frac{(r_1 - r_2)}{\Lambda}\left(\lambda_1^{\mathsf{eq}}\right)^2 \\ & - \frac{\Omega}{\Lambda}\sum_{l=1}^{L} \lambda_{1,l}^{\mathsf{eq}}\, \bar{T}_l\left(\lambda_{1,l}^{\mathsf{eq}}\right) \\ \text{subject to} \quad & \sum_{l=1}^{L} \lambda_{1,l}^{\mathsf{eq}} = \lambda_1^{\mathsf{eq}}, \\ & \lambda_{1,l}^{\mathsf{eq}} \geq 0,\ \forall l, \end{aligned} \tag{68}$$

and the second problem is

$$\underset{p_2 \geq 0}{\text{maximize}} \quad \frac{\Lambda r_2 - \Omega\overline{X}}{r_2} p_2 - \frac{\Lambda}{r_2} p_2^2. \tag{69}$$

Problem (69) is a single-variable quadratic optimization, which is easy to solve. Problem (68) has the same convex structure as problem (17), which can be solved similarly. Denoting a given dual variable of this problem by ξ, we have the following result due to the first-order condition

$$\lambda_1^{\mathsf{eq}}(\xi) = \frac{\Lambda\xi + \overline{X}\Omega + \Lambda(r_1 - r_2)}{2(r_1 - r_2)}, \tag{70}$$

$$\lambda_{1,l}^{\mathsf{eq}}(\xi) = [\Phi_l(\xi)]^+, \quad \forall l. \tag{71}$$

We have the following result, which can be proved similarly as Lemma 3.

Lemma 6. *If*

$$\tau^{\mathsf{co}} := \frac{\Omega}{\Lambda}\overline{X} + r_1 - r_2 > \frac{\Omega}{\Lambda}\bar{\chi}_1, \tag{72}$$

there exists a unique solution $\xi^* \in \left(-\tau^{\mathsf{co}}, -\frac{\Omega}{\Lambda}\bar{\chi}_1\right)$ *of* $\sum_{l=1}^{L}[\Phi_l(\xi)]^+ = \lambda_1^{\mathsf{eq}}(\xi)$ *and a corresponding channel index* $1 \leq L^* \leq L$ *such that* $\Phi_l(\xi^*) > 0,\ \forall l \leq L^*$, *and* $\Phi_l(\xi^*) = 0,\ \forall l > L^*$. *If* $\tau^{\mathsf{co}} \leq \frac{\Omega}{\Lambda}\bar{\chi}_1$, $[\Phi_l(\xi)]^+ = \lambda_1^{\mathsf{eq}}(\xi) = 0,\ \forall l$.

3) Optimal Solutions: It is easy to obtain the optimal solution of (69), denoted by p_2^{co}, by the first-order condition. The optimal solution of (68) can also be obtained similarly as that of (17) through ξ^* in Lemma 6. By substituting these optimal solutions of (68) and (69) into (47), we obtain the $\mathcal{O}_1$'s optimal price, denoted by p_1^{co}. Hence, we have the following result.

Proposition 6. *With* ξ^* *and* L^* *from Lemma 6, there exists a unique optimal solution of the social revenue maximizing* (67)*, denoted by* $\left(\overrightarrow{s^{\mathsf{co}}}, p_1^{\mathsf{co}}; p_2^{\mathsf{co}}\right)$*, in the cooperation duopoly market as follows*

$$p_2^{\mathsf{co}} = \left[\frac{r_2}{2} - \frac{\Omega}{\Lambda}\frac{X}{2}\right]^+, \tag{73}$$

$$s_l^{\mathsf{co}} = \frac{\Phi_l(\xi^*)}{\lambda_1^{\mathsf{eq}}(\xi^*)},\ \forall l \leq L^* \ \textit{and}\ s_l^{\mathsf{co}} = 0,\ \forall l > L^*, \tag{74}$$

$$p_1^{\mathsf{co}} = [p_1(\xi^*;\ p_2^{\mathsf{co}})]^+ \ \textit{from}\ (54). \tag{75}$$

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4) Sharing Contract: After jointly achieving the optimal social revenue value $\pi^{\mathsf{co}} := \pi(\overrightarrow{s^{\mathsf{co}}}, p_1^{\mathsf{co}}, p_2^{\mathsf{co}})$, two operators will decide the fraction of π^{co} that each of them will receive based on a sharing contract γ. Suppose that $\mathcal{O}_1$ receives its revenue $\pi_1^{\mathsf{co}} = \gamma\pi^{\mathsf{co}}$, then $\mathcal{O}_2$ receives its share $\pi_2^{\mathsf{co}} = (1-\gamma)\pi^{\mathsf{co}}$. The problem becomes how to find a value γ^* that satisfies both operators. Based on the Nash bargaining theory [34], a proper γ^* can be a solution of the following optimization problem

$$\begin{aligned} \underset{\gamma\in[0,1]}{\text{maximize}} \quad & (\gamma\pi^{\mathsf{co}} - \pi_1^{\mathsf{na}})^{w_1}((1-\gamma)\pi^{\mathsf{co}} - \pi_2^{\mathsf{na}})^{w_2} \\ \text{subject to} \quad & \gamma\pi^{\mathsf{co}} \geq \pi_1^{\mathsf{na}}, \\ & (1-\gamma)\pi^{\mathsf{co}} \geq \pi_2^{\mathsf{na}}, \end{aligned} \tag{76}$$

where w_1 and w_2 are weight values representing the bargaining power of $\mathcal{O}_1$ and $\mathcal{O}_2$, respectively, and $(\pi_1^{\mathsf{na}}, \pi_2^{\mathsf{na}})$, the revenues of $\mathcal{O}_1$ and $\mathcal{O}_2$ at the Nash equilibrium, is a disagreement point. The first and second constraints capture the cooperation incentive in that cooperative sharing revenues must be at least equal to the revenues obtained in the noncooperative scenario. Denoting the set of all possible revenues that two operators can achieve by S, we have

$$S = \{(\pi_1^{\mathsf{co}}, \pi_2^{\mathsf{co}}) \mid \pi_1^{\mathsf{co}} + \pi_2^{\mathsf{co}} = \pi^{\mathsf{co}}, \pi_1^{\mathsf{co}} \geq 0, \pi_2^{\mathsf{co}} \geq 0\}, \tag{77}$$

which is a convex set. In addition, the feasibility of problem (76) guarantees a following unique solution.

Proposition 7. *Problem* (76) *has a unique optimal solution such that* (a)

1) if $\pi^{\mathsf{co}} = \pi_1^{\mathsf{na}} + \pi_2^{\mathsf{na}}$, then $\gamma^* = \frac{\pi_1^{\mathsf{na}}}{\pi^{\mathsf{co}}}$,
2) if $\pi^{\mathsf{co}} > \pi_1^{\mathsf{na}} + \pi_2^{\mathsf{na}}$, then

$$\gamma^* = \frac{\pi_1^{\mathsf{na}}}{\pi^{\mathsf{co}}} + \frac{w_1}{w_1 + w_2}\frac{\pi^{\mathsf{co}} - (\pi_1^{\mathsf{na}} + \pi_2^{\mathsf{na}})}{\pi^{\mathsf{co}}}. \tag{78}$$

E. Duopoly: Algorithms

The optimal solutions provided by Propositions 5 and 6 are based on the assumption $(\lambda_{1,k}^{\mathsf{eq}}, \lambda_{2,k}^{\mathsf{eq}}) > 0$, $\forall k \in \mathcal{K}$, which is not always true. Therefore, similar to the monopoly case, we propose Algorithm 2 for the duopoly market to search for the largest class $K^* \leq K$ that can be supportable by the operators such that the optimal solutions in Propositions 5 and 6 correspond to $(\lambda_{1,k}^{\mathsf{eq}}, \lambda_{2,k}^{\mathsf{eq}}) > 0$, $\forall k \leq K^*$.

Proposition 8. *If*

$$r_1 - \theta_1\bar{X}_1 > r_2 - \theta_1\overline{X} \tag{79}$$

$$\textit{and} \quad \frac{r_2}{r_1} > \max\left\{\frac{\overline{X}}{\bar{X}_1 - 1/2\theta_1}, \frac{\overline{X}}{\bar{X}_1}\right\}, \tag{80}$$

Algorithm 2 Optimal Pricing and Load-Balancing in the Duopoly Market

1: Operators collect $\bar{X}_l, \overline{X_l^2}, \forall l$ and $\Lambda_k, \theta_k, \forall k$
2: $i \leftarrow \arg\max_{k\in\mathcal{K}}\left\{r_1 - r_2 > \frac{\Omega(k)}{\Lambda(k)}(\bar{X}_1 - \overline{X})\right\}$
3: Update $\overrightarrow{s^{\mathsf{co}}}(i), p_1^{\mathsf{co}}(i)$ and $p_2^{\mathsf{co}}(i)$ by Prop. 6 with $\Lambda(i)$, $\Omega(i)$
4: **while** $p_2^{\mathsf{co}}(i) \leq \beta_i^{\mathsf{lo}}\left(\overrightarrow{s^{\mathsf{co}}}(i), p_1^{\mathsf{co}}(i)\right)$ **do**
5: $\quad i \leftarrow i - 1$
6: $\quad$ repeat step 3
7: **end while**
8: $K^* \leftarrow i$, $p_1^{\mathsf{co}} \leftarrow p_1^{\mathsf{co}}(K^*)$, $\overrightarrow{s^{\mathsf{co}}} \leftarrow \overrightarrow{s^{\mathsf{co}}}(K^*)$ and $p_2^{\mathsf{co}} \leftarrow p_2^{\mathsf{co}}(K^*)$
9: Operators broadcasts p_1^{co}, $\sum_{l=1}^{L} s_l^{\mathsf{co}}\bar{T}_l\left(s_l^{\mathsf{co}}\lambda_1^{\mathsf{eq}}(\xi^*)\right)$, p_2^{co} and all SUs join the network by Definition 2.
10: *% Steps 1 to 9 are applied to cooperative duopoly; for the noncooperative duopoly, all of steps are the same except replacing every* $p_1^{\mathsf{co}}, \overrightarrow{s^{\mathsf{co}}}, \xi^*, p_2^{\mathsf{co}}$ *by* $p_1^{\mathsf{na}}, \overrightarrow{s^{\mathsf{na}}}, \nu^*, p_2^{\mathsf{na}}$ *and Prop. 6 by Prop. 5 in line 3. %*

Algorithm 2 always returns a class $K^* \geq 1$ *such that* $(\lambda_{1,k}^{\mathsf{eq}}, \lambda_{2,k}^{\mathsf{eq}})_{1\leq k\leq K^*} > 0$ *for both cooperation and noncooperation.*

Remark 2. It is clear that if (79) is violated, no SUs has any incentive to join $\mathcal{O}_1$. Therefore, while condition (79) says that r_1 must be larger than r_2 an amount at least $\theta_1(\bar{X}_1 - \overline{X})$ for $\mathcal{O}_1$ to have the market share instead of being eliminated by $\mathcal{O}_2$, (80) provides an upper bound on r_1 to sufficiently guarantee for both noncooperative and cooperative duopoly coexistence.

We express the intuition of Algorithm 2 in the cooperative case since the other case can follow the same lines of argument. In line 2, with condition (79) we can always find such a class $i \geq 1$. According to Lemma 4, condition (72) is always satisfied with $\frac{\Omega(j)}{\Lambda(j)}$, $\forall j \leq i$. Thus, by Proposition 6 we can always obtain $\overrightarrow{s^{\mathsf{co}}}(j), p_1^{\mathsf{co}}(j)$ and $p_2^{\mathsf{co}}(j)$, $\forall j \leq i$ in line 3. Based on the observation that $p_2^{\mathsf{co}}(i)$ is decreasing by Lemma 4 and Proposition 6, the algorithm keeps lowering this largest class until, if it is possible, there is a class K^* satisfying condition $p_2^{\mathsf{co}}(K^*) > \beta_{K^*}^{\mathsf{lo}}(\overrightarrow{s^{\mathsf{co}}}(K^*), p_1^{\mathsf{co}}(K^*))$ (lines 3 to 8 in Algorithm 2), which guarantees $(\lambda_{1,k}^{\mathsf{eq}}, \lambda_{2,k}^{\mathsf{eq}}) > 0$, $\forall k \leq K^*$ by Proposition 8. Similar to Algorithm 1, Algorithm 2 has the complexity $O(K)$.

V. Numerical Results

In this section, we apply the analytical results to numerically illustrate the operators' optimal solutions by Algorithms 1 and 2 corresponding to the monopoly and duopoly scenarios, respectively.

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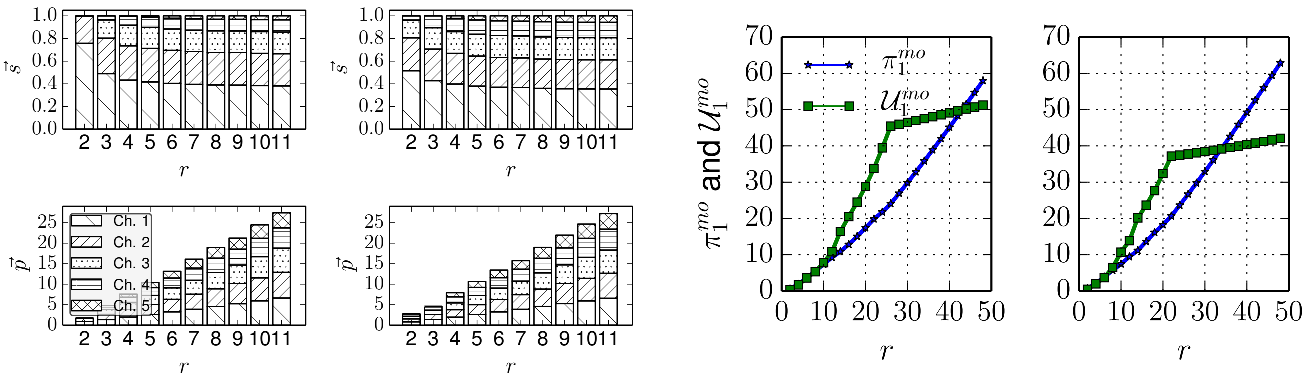

(a) (b)

Fig. 4. (a) The optimal load balancing and pricing solutions of the monopolist in the setting ExpErl (left column plots) and UniExp (right column plots), (b) The monopoly revenue and the aggregate SUs' utilities with the setting ExpErl (left plot) and UniExp (right plot).

First, we consider 15 classes of SUs, represented by $(\theta_1, \theta_2, \ldots, \theta_{15}) = (0.2, 0.4, \ldots 3)$. Furthermore, Λ_k follows a uniform distribution on $[0, 3]$ for $k = 1, \ldots, 15$.

Second, we consider a shared-use operator with five channels. In the first setting termed ExpErl, X has the exponential distribution with $f_X(x) = \mu_X e^{-\mu_X x}$, whereas Y and Z have the Erlang distributions with $f_{Y_l}(y) = \mu_{\text{on}}^2 y e^{-\mu_{\text{on}} y}$ and $f_{Z_l}(z) = \mu_{\text{off}}^2 z e^{-\mu_{\text{off}} z}$, $\forall l$, respectively. We set μ_X to 1 (i.e. $\overline{X} = 1$) and the PU activities from channel 1 to 5 are set to $(\mu_{\text{on}}, \mu_{\text{off}}) = (1.5, 0.5)$, $(1.2, 0.8)$, $(1.0, 1.0)$, $(0.8, 1.2)$ and $(0.5, 1.5)$. In the second setting termed UniExp, X is uniformly distributed on $[0.1, 1.9]$ (i.e. $\overline{X} = 1$), whereas Y and Z have exponential distributions with $f_{Y_l}(y) = \mu_{\text{on}} e^{-\mu_{\text{on}} y}$ and $f_{Z_l}(z) = \mu_{\text{off}} e^{-\mu_{\text{off}} z}$, $\forall l$, respectively, where the PU activities from channel 1 to 5 are $(\mu_{\text{on}}, \mu_{\text{off}}) = (1.4, 0.6)$, $(1.3, 0.7)$, $(1.0, 1.0)$, $(0.7, 1.3)$ and $(0.4, 1.6)$. The PU channels in both settings model the increasing PU occupancy, i.e., light to heavy PU traffic. From (3) and (4), $\overline{\chi}_l$ and $\overline{\chi_l^2}$ of ExpErl are (1.25, 1.54, 2.0, 2.85, 5.0) and (3.52, 5.73, 10.33, 22.37, 72.38), respectively; and those of UniExp are (1.33, 1.66, 2.0, 2.5, 4.0) and (2.7, 4.64, 7.08, 11.69, 32.32), respectively, for $l = 1, \ldots, 5$. We can see that for all channels, the second moments of case ExpErl are greater than those of UniExp.

Finally, for the exclusive-use operator, we simply set $\overline{X} = 1$ to illustrate the same bandwidth for all channels of $\mathcal{O}_1$ and $\mathcal{O}_2$.

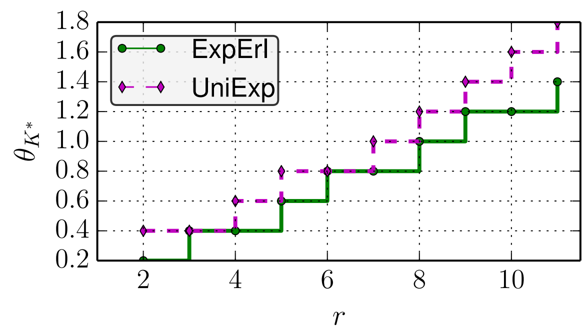

Fig. 5. The largest class value θ_{K^*} that can be supportable by $\mathcal{O}_1$ using Algorithm 1.

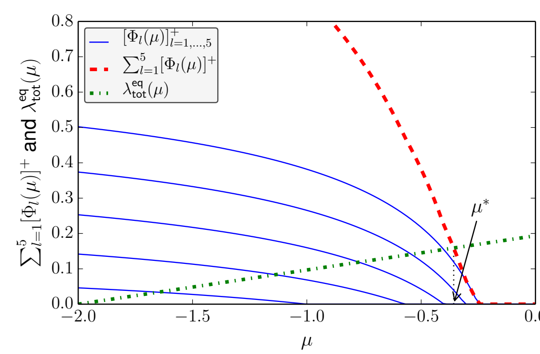

Fig. 6. Numerical illustration of Lemma 3 in the setting ExpErl with $r = 2$. The lines of $[\Phi_l(\mu)]^+$ are continuous and decreasing to 0 at $-\frac{\Omega}{\Lambda}\overline{\chi}_l$, $l = 1, \ldots, 5$, respectively. The functions $\sum_1^5 [\Phi_l(\mu)]^+$ and $\lambda_{\text{tot}}^{\text{eq}}(\mu)$ intersect at a unique μ^* that corresponds to $L^* = 2$.

A. Monopoly

Fig. 4a shows a sample of the optimal load balancing and pricing solutions of the monopolist in both settings ExpErl and UniExp when r is varied. We can see that when r is small, only those channels ls with low value $\overline{\chi}_l$ and $\overline{\chi_l^2}$ are activated with $s_l > 0$, $p_l > 0$; and the setting ExpErl, with greater channels' variability (second moments), has less activated channels than those of UniExp (e.g. when $r = 2$, only channels 1 and 2 of

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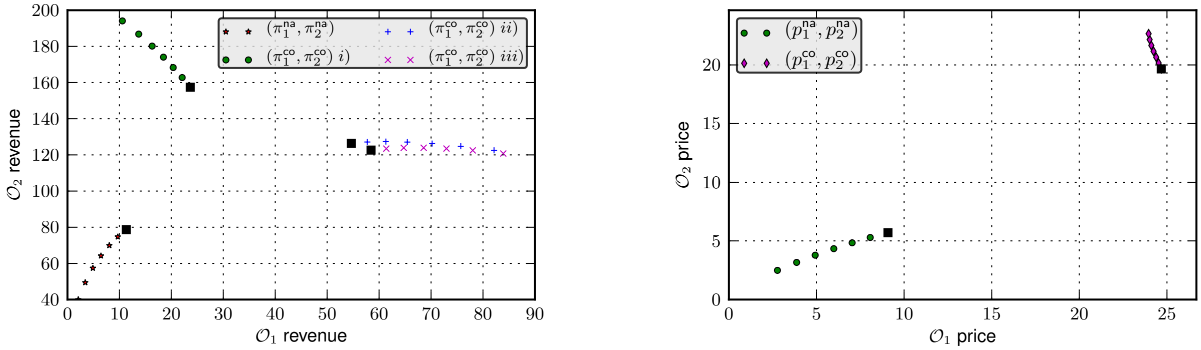
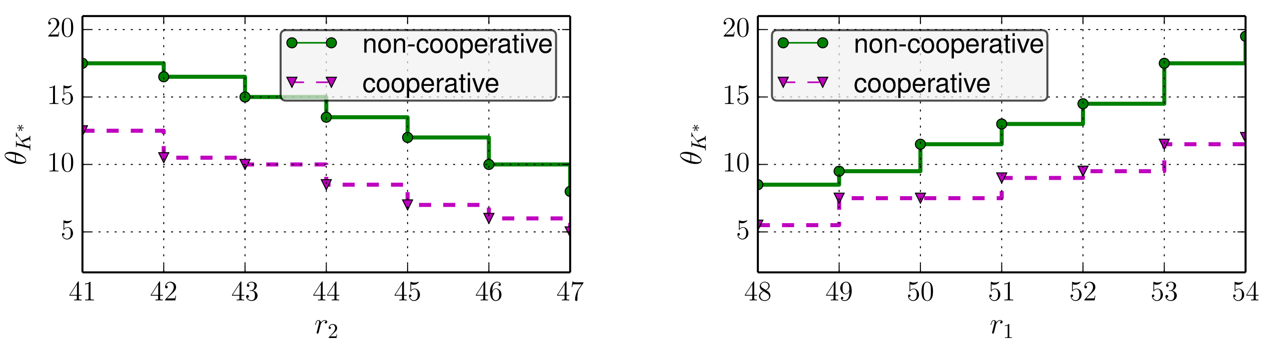


(a) (b)

Fig. 7. Duopoly performance with 15 SUs' classes: (a) Non-cooperative revenues $(\pi_1^{\text{na}}, \pi_2^{\text{na}})$ and cooperative revenues $(\pi_1^{\text{co}}, \pi_2^{\text{co}})$ with three different weight settings (w_1, w_2): i) $\left(\lambda_1^{\text{eq}}(\xi^*)\, p_1^{\text{co}}, \lambda_2^{\text{eq}}(\xi^*)\, p_2^{\text{co}}\right)$, ii) $\left(\frac{r_1}{p_1^{\text{co}}+\sum_{l=1}^{L} s_l^{\text{na}}\bar{T}_l\left(s_l^{\text{na}}\lambda_1^{\text{eq}}\right)}, \frac{r_2}{p_2^{\text{co}}+\bar{X}}\right)$, and iii) $\left(\frac{p_1^{\text{co}}}{r_1-\sum_{l=1}^{L} s_l^{\text{na}}\bar{T}_l\left(s_l^{\text{na}}\lambda_1^{\text{eq}}\right)}, \frac{p_2^{\text{co}}}{r_2-\bar{X}}\right)$, (b) Optimal Pricing.



(a) (b)

Fig. 8. The largest class K^* of Algorithm 2 with 40 SUs' classes: (a) r_1 is fixed at 50, r_2 varies from 41 to 47, (b) r_2 is fixed at 45, r_1 varies from 48 to 54.

ExpErl and channels 1, 2 and 3 of UniExp are active). For these activated channels, it is clear that the channel with low value $\bar{X}_l$ and $\overline{X_l^2}$ will have high value s_l. When r increases, the number of activated channels and the optimal prices also increase in both settings. Furthermore, we observe that the load balancing solution converges to a fixed distribution when r keeps increasing. In Fig. 4b, we further examine the relationship between the monopoly revenue, π_1^{mo}, and the corresponding aggregate utility of SUs, $\mathcal{U}_1^{\text{mo}} := \sum_k \sum_l \int_{\alpha_{k,l}(\lambda_l^{\text{eq}})}^{1} U_{k,l}(\alpha)d\alpha$, via the operator's quality r. When r increases, we observe that while π_1^{mo} increases with a slightly increasing slope, $\mathcal{U}_1^{\text{mo}}$ first increases with a sharply increasing slope up to a critical value of $r = 26$, then changes to a linearly increasing state. It can be explained that: First, not only K^* increases (i.e. more classes are supported), but also the utility of each class $k \leq K^*$ increases when r is increased to the critical value 26, where the maximum number of classes $K = 15$ is achieved. Second, when r is increased past this critical value, while the majority of the utilities of low classes (i.e. small k) continue increasing, some higher classes start to decrease their utilities because the increase of value r does not compensate for their high cost $\theta_k\bar{T}_l(\lambda_l(s_l)) + p_l^*$ due to high values of θ_k and p_l^*.

Fig. 5 shows the largest class θ_{K^*} that can be supportable using Algorithm 1. In both settings, we can see that the higher value r, the higher class-K^* SUs that can be admitted into the monopoly network. We also observe that the UniExp setting with lower channels' variability can support higher class SUs than ExpErl does.

Fig. 6 illustrates Lemma 3 in the setting ExpErl when $r = 2$. With the unique value μ^*, we can see that $L^* = 2$ where $[\Phi_l(\mu)]^+ = 0$ for $l = 3, 4, 5$ and $[\Phi_l(\mu)]^+ > 0$ for $l = 1, 2$, which leads to $s_l > 0$, $l = 1, 2$, in the top left graph of Fig. 4a with $r = 2$.

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TABLE I
MONOPOLY AND DUOPOLY REVENUE COMPARISON

(r_1, r_2)	$(\pi_1^{\mathsf{mo}}, \pi_2^{\mathsf{mo}})$	$(\pi_1^{\mathsf{na}}, \pi_2^{\mathsf{na}})$	$(\pi_1^{\mathsf{co}}, \pi_2^{\mathsf{co}})$ setting i)	$(\pi_1^{\mathsf{co}}, \pi_2^{\mathsf{co}})$ setting ii)
(30, 28)	(31, 303)	(1, 31)	(5, 126)	(49, 82)
(31, 29)	(32, 315)	(1, 32)	(5, 132)	(52, 85)
(32, 30)	(34, 326)	(1, 32)	(5, 137)	(54, 88)
(33, 31)	(35, 337)	(1, 32)	(5, 142)	(56, 91)
(34, 32)	(37, 349)	(1, 33)	(6, 147)	(59, 93)
(35, 33)	(38, 349)	(2, 44)	(8, 159)	(60, 106)

B. Duopoly

The duopoly performance is presented in Fig. 7, where r_1 is fixed at 50 and r_2 is varied from 41 to 47 where the starting points are marked by black squares. We note that these values of r_1 and r_2 satisfy (79) and (80). Fig. 7a shows the revenue comparison between non-cooperation and cooperation. In the case of cooperation, we consider three different weight settings (w_1, w_2) to characterize the effect of sharing contract: setting (i) corresponds to the revenues without sharing contract, setting (ii) relates to the intrinsic quality per cost and setting (iii) can be considered as price per QoS. We can see that the cooperation always gains more revenues for both operators than the non-cooperation; especially when r_2 is large, the gain is significant. When r_2 increases, the Nash equilibrium of both operator's revenues decrease, since they competitively reduce their low prices to attract more SUs in Fig. 7b. Fig. 7b also shows that while $\mathcal{O}_2$ increases its price, $\mathcal{O}_1$ decreases its price to cooperatively maximize their social revenue. In this case, Fig. 7a shows that the social revenue of setting (i) keeps decreasing $\mathcal{O}_1$'s revenue and increasing $\mathcal{O}_2$'s revenue, which is clearly not favored by $\mathcal{O}_1$; whereas settings (ii) and (iii) drive their social revenue in a similar direction that can satisfy both operators. We also observe that all classes are supported by both operators with all values r_2 (i.e. $\theta_{K^*} = 3$). We continue to compare the revenue gain between monopoly and duopoly in Table I, where we increase the pair of (r_1, r_2) such that their difference is a fixed value and satisfies Proposition 8. Since sharing the market means losing revenue, we clearly see that the sum of monopoly revenues is larger than that of the duopoly in all cases.

To illustrate the effect of the largest class K^* admission, we change the setting to 40 classes of SUs with $(\theta_1, \theta_2, \ldots, \theta_{40}) = (0.5, 1, \ldots 20)$ and Λ_k follows a uniform distribution on $[0, 20]$ for $k = 1, \ldots, 40$. With this new setting, Fig. 8 shows the largest class value θ_{K^*} that can be supportable by $\mathcal{O}_1$ and $\mathcal{O}_2$ using Algorithm 2. At line 2 of Algorithm 2, we see that θ_{K^*} depends on $r_1 - r_2$, given fixed channels' distributions. Hence, θ_{K^*} decreases when $r_1 - r_2$ decreases in Fig. 8a and θ_{K^*} increases when $r_1 - r_2$ increases in Fig. 8b. Furthermore, the noncooperation always maintains a higher index K^* than that of the cooperation case since noncooperative operators try to attract as many SUs' classes as possible leading to their price reduction, whereas the cooperative ones conservatively reduce the largest supportable class K^* to increase their prices to maximize their total revenue.

VI. CONCLUSION

Traditionally, pricing and load balancing are designed separately for dynamic spectrum access control. We perform a joint optimization in two network markets: monopoly and duopoly. In both scenarios, we first address the heterogeneous multi-class SUs' equilibrium behavior as a constraint in the operators' revenue optimization problem that can be decomposed into smaller problems revealing their convex structures. Based on that, we next provide the unique optimal pricing and load balancing solutions not only for the monopolist's revenue but also for the duopoly's Nash equilibrium and social revenues. We finally propose two algorithms to find the largest supportable SUs' class in both scenarios. Numerical results are provided to validate our analysis and show that by cooperation, both operators can enhance their revenues significantly when compared with noncooperation. Our model can be extended to the cases where both operators are shared-use or both are exclusive. However, for an oligopoly case with any finite operators, the model becomes complicated and needs a different approach for tractable analysis.

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