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Reward-to-Reduce: An Incentive Mechanism for Economic Demand Response of Colocation Datacenters

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***Abstract*—Even though demand response of datacenters has attracted many studies, there are very limited attempts on an important segment: colocation datacenters. Unlike large-scale (Google-type) datacenters, the colocation operator lacks control over its tenant servers, which entails a special interest in design of incentive mechanisms such that the operator can coordinate tenants to reduce the power usage for demand response. However, most previous studies ignore the role of the Demand Response Provider (DRP), who uses pricing signals as a guide for customer response and as a compensation for their cutting electricity usage. To address this oversight, we propose an incentive mechanism Reward-to-Reduce (R2R) for colocation's economic demand response, which shows an interaction between the DRP compensation to the colocation operator, and the colocation operator reward to tenants. Observing that this interaction contains strategic behaviors, we first formulate a two-stage Stackelberg game, where we show a unique competitive equilibrium of the operator strategy in the second stage, and a nonconvex problem of finding the optimal DRP compensation price in the first stage. We next analyze the second-stage equilibrium using an exact analysis and design an algorithm that can efficiently search the first-stage optimal DRP price with a reduced search space. Since the exact analysis can be impractical due to required tenants' private information, we also propose an approximate approach with limited tenant information. Extensive case studies show (a) the approximate approach can have the same performance as the exact analysis in a wide array of case studies, and (b) the optimal DRP price can be determined effectively, with which the corresponding DRP individual cost is compared with the social cost.**

I. Introduction

Datacenters – with their extremely large power demands (e.g., 91 billion kWh in 2013 in the U.S. [1]) and usage flexibility with many controlling knobs (e.g., workload shedding, migration, cooling) – are considered as ideal contributors to demand response, a program that helps improve power grid reliability [2]. However, while large-scale datacenters (e.g., Google) have received considerable attention for demand response (see survey [3] and the references therein), another important segment of datacenters is largely under-explored: *colocation datacenters* (e.g., Equinix).

Even though there have lately been initial attemps on demand response for colocation datacenters[1] [4]–[6], these efforts are limited, and are not sufficient to capture the importance of colocations: First, colocations provide a universal solution to a wide array of tenants, including top-brand Internet websites (e.g., Twitter and Wikipedia [7], [8]), cloud providers (e.g., Salesforce [9]), content delivery providers (e.g., Akamai [10]), and even the giant business Amazon. Second, the growth of colocations continues to increase sharply: there are more than 1400 colocations in the U.S. alone [11], and the colocation market is expected to grow from \$25 billion in 2014 to \$43 billion in 2018 [12]. In addition to its critical role in datacenter business, colocation is also an ideal participant in demand response: (a) Colocations have extremely large power demands, (e.g., colocations consume 37% of the electricity of all datacenters in U.S. [13]), and (b) colocations are often located in urban areas, e.g., Los Angeles [11], where demand responses are frequently required.

Unlike large-scale (Google type) datacenters, a colocation is a multi-tenant datacenter where multiple tenants house and fully control their servers in a shared building, whereas the colocation operator[2] is mainly responsible for facility support (e.g., power, cooling, etc.). Due to the operator's lack of control over tenant properties, there is growing interest in exploring how the operator incentivizes tenants for the demand response. Most of the recent works on colocation focus on *emergency demand response* [5], [6] with intra-colocation interaction between the operator and its tenants, where the role of the Demand Response Provider (DRP)–a.k.a. Curtailment Service Provider, an authorized intermediary between Independent System Operators (ISO) and customers (e.g., colocations) who functions to deliver demand response capacity– is nullified. Emergency (or reliability) demand response requires a mandatory response (with penalty for non-compliance) for the participants, who are not only compensated for their reduction during emergency events, but are also paid for their availability (i.e., even when no emergent signal is triggered) [14]. Such programs are currently employed by many Independent

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[1]Henceforth, we simply call them colocations.

[2]Henceforth, we simply call them operators.

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System Operators such as New England or PJM, where the customers' contracts can be established three years in advance [15]. Due to this static contract with an *inelastic* (i.e., strictly matched) demand response capacity, all parameters relating to DRP (e.g., payment, costs) are considered constants; thus, it is conceivable that the role of the DRP is ignored in these works.

On the other hand, there is another type of demand response that receives less attention but is not less important than the emergency counterpart: *economic demand response*. This is a voluntary program such that, when requested by the ISO, customers can reduce their electricity usage during peak periods with high wholesale power prices in exchange for monetary compensation via DRP. This program provides customers the flexible control on an *elastic* demand response capacity in that they can *at will* reduce the electricity usage for payment (e.g., buildings can turn up the temperature on the air conditioning thermostat up to a threshold). However, due to the regulations, consumers are usually charged for their usage based on an average rate, which masks the fluctuation of the wholesale prices. Since consumers have no incentive to reduce their usage without dynamic price signals to indicate peak periods, DRP emerges as a coordinator to help customers react to the compensation price, which can imitate the wholesale price pattern [16]. Obviously, the role of DRP cannot be negligible in an economic demand response model because the DRP is able to strategically deviate from an elastic demand response capacity by setting a lower price for energy reduction procurement. However, even though there are some existing attempts on colocation's economic demand response [4], [17], none of them considers the DRP as an integrated and strategic component in their mechanisms.

Therefore, with an effort to fill this gap, we make a significant departure to the existing literature by designing the first incentive mechanism for colocation's economic demand response that incorporates the DRP role. In summary, our contributions are as follows:

- We propose an incentive mechanism, Reward-to-Reduce (R2R), that uses reward/price to incentivize colocations to reduce energy consumption for economic demand response. The R2R models the *interaction* between the DRP decision on the compensation price for the colocations and the colocation decision on rewarding tenants for the response.
- Since there exist the strategic decisions of both DRP and colocation sides in their interaction, we formulate R2R as a two-stage Stackelberg game, where the DRP has first-move advantage to set its compensation price in the first stage, and the colocation operator uses this DRP compensation price to set the reward in the second stage. Under some mild conditions for the tenant cost, which is bourned by reducing its energy usage, we show an exact analysis such that there exists a unique *competitive equilibrium* in the second stage, where the expected profits of all operators are minimized. Given this competitive equilibrium, the DRP will choose an optimal *market-clearing price* to match the colocation response to the demand response capacity in the first stage, where the DRP cost is minimized. The market-clearing compensation price and its corresponding competitive equilibrium make up the Stackelberg equilibrium as a part of our R2R analysis.
- In the second stage, the exact analysis of the competitive equilibrium requires the operator to know the full distribution of tenants' cost-related parameters, which is expensive in practice. Thus, we also propose an approximate approach that requires limited information, but can provide a comparable performance with the exact analysis with a sufficiently large number of tenants due to Central Limit Theorem (CLT). Interestingly, the extensive case studies show that the approximation method has almost the same performance as the exact analysis even for a small number of tenants, which benefits a wide range of colocation business sizes. On the other hand, in the first stage, finding the market-clearing price is generally non-convex. Thus, we design an algorithm that reduces the problem's search space to improve the search speed. Numerous case studies also show that the optimal compensation price can be effectively found and the performance of the DRP individual cost is compared with the social cost to explore how much social cost is suffered due to an individual strategy of the DRP.

The rest of this paper is organized as follows. In Section II, we review the related work. Section III presents the system model, including the R2R procedure and the interaction between the DRP and colocations via the Stackelberg game formulation. Section IV and Section V give the analysis of the colocation and DRP decisions with corresponding illustrative case studies, respectively. Finally, Section VII concludes our work.

II. Related Work

Demand response of datacenters has been studied using various proposed methods for different types of demand response programs, ranging from the price response of datacenters to the grid operator [18] for economic demand responses to controlling the IT (e.g., turning servers on/off) and non-IT (e.g., cooling) knobs for ancillary and/or emergency demand responses [3], [19]–[23]. While most of the mentioned results focus on large-scale datacenters (e.g., Google), their approaches cannot be directly applied to colocations with the lack of operator's control over tenant facilities.

Encouragingly, studies on colocation demand response have recently grown in importance. The early study on colocation's economic demand response is [4]. Nevertheless, its mechanism is simple and relies on the tenants' best-efforts, which can entail an untruthful strategy from tenants. The next study [5] proposes a randomized auction mechanism for emergency demand response, which guarantees a 2-approximation of social welfare cost and is approximately truthful. However, both studies uses combinatorial bidding-based approaches, which are NP-hard, to obtain the optimal solutions. Moreover, both are based on a reverse auction with tenants' voluntary bids, which can lead to an unexpected number of participating tenants since tenants are usually not proactive with regard to

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usage reduction. Hence, an upfront reward by the operator, which is used in R2R, is expected to increase tenant participation.

On the other hand, in both [6] and [17], which studies emergency demand response, the proposed mechanisms allow the operator to first announce the payment/reward rules, then each tenant makes a bid to imply its reduction and the corresponding payment. While [6] uses a supply function bidding method that suffers from the social loss due to a particular "parameterized" function, [17] is based on an efficient proportional allocation scheme that aligns the tenant bid to the socially optimal performance. However, in these approaches, tenants need to calculate and reveal their complex bidding schemes, which might leak their private costs. None of the above works explicitly accounts for the role of DRP in their schemes.

III. System Model

In this section, we first overview the proposed R2R mechanism. We next elaborate the model of each component in the proposed mechanism. Finally, we investigate the interaction between these components using a Stackelberg game formulation.

A. R2R: *Overview*

We consider one DRP that provides curtailment services for a set of I colocations. Each colocation $i \in I$ provides services for a set of tenants N_i. Henceforth, we use I and N_i to denote the sets and their corresponding cardinality without any confusion. Based on the economic demand response program, we propose a mechanism that rewards incentives for colocation in order to reduce energy consumption, namely Reward-to-Reduce (R2R). During a considered demand response timeslot (30 minutes to hours), the overview of R2R is as follows.

R2R **Procedure:**

Stage 1: The DRP receives a demand response target D, then determines p and d, where p is the compensation price paid for every unit of the demand response capacity d procured from all colocations.

Stage 2: At each colocation $i \in I$, given the price p, each operator determines q_i and r_i, where q_i is the operator i's energy reduction response,[3] and r_i is the reward paid for every unit of its tenants' reduced energy.

i) At each tenant $n \in N_i$ of colocation i, given a reward r_i, tenant n decides its energy reduction supply $S_n(r_i)$[4].

ii) If the aggregate tenant supply of the colocation $\sum_{n \in N_i} S_n(r_i)$ is less than the operator response q_i, the operator will use a backup generator to supplement its response deficit.

We see that there are strong couplings between three parties in R2R: DRP, the (colocation) operator, and tenants. The first coupling is between the DRP and operator where the DRP needs to know the operator commitment in order to make decision on its compensation price p and demand response capacity d. The second coupling is between the operator and others, where the operator has its response q_i as a function of price p and relies on the tenant supply $S_n(r_i)$ to make decisions on the reward rate r_i. The last coupling is between tenants and its operator where tenants decisions on shedding energy depends on the reward price r_i.

In practice, the role of DRP is to aggregate the responses of its customers (e.g., datacenters) in order to make the compensation payment. Therefore, there is an inter-dependence between DRP and its customers. However, it is still not clear that how this dependence will affect to the decisions of each other. Especially when the customer is a colocation datacenter, this dependence is further complicated since there are two rational components inside a colocation: the operator and its tenants. While the operator wants its tenants to reduce the energy for demand response, the tenants has no incentive to shed the energy due to its fixed power subscription payment. Therefore, the R2R mechanism aims to not only bridge the split-incentive between colocation operator and tenants, but also characterize the equilibrium behavior of all parties' dependencies.

As report in [8], while many dedicated datacenters (e.g., Facebook, Google) are highly validated for their "green" image, colocations are very "dirty" in their energy portfolios. Because of global presences and scales, colocations play a vital role in building a green computing and communication industry. Hence, one of the motivations for the operator's decision sequence in R2R is to improve the "green" factor of the colocations by limiting the use of diesel backup generator, which is notorious for environmentally unfriendly.

Next, we will provide the system model of the R2R mechanism, starting from the tenant supply and operator response of the colocations in the second stage and ending at the DRP pricing and response in the first stage.

B. R2R: *Colocation Model in the Second Stage*

Tenant Supply. Consider a tenant $n \in N_i$ of a colocation $i \in I$. Given the reward r_i from the operator, a rational tenant n will consider an amount of energy reduction e_n to save its operational cost[5]. However, a tenant n will incur a cost $C_n(e_n)$ when reducing energy consumption. This cost function is used to model typical tenant costs such as wear-and-tear, performance degradation [6], [24], etc. We also assume that $C_n(e_n)$ is positive, convex, and strictly increasing[6], which is standard in the literature [6], [18].

Therefore, tenant n will rationally choose the optimal e_n^* that maximizes its surplus as follows:

$$\underset{e_n \geq 0}{\text{maximize}} \qquad u_n(e_n) = r_i e_n - C_n(e_n). \tag{1}$$

[3]Henceforth, we briefly call this operator response.

[4]Henceforth, we briefly call this tenant supply.

[5]Even though, in practice, tenants can use various techniques to reduce their energy (such as workload shifting, turning servers on/off), we do not consider any specific technique to keep the model simple.

[6]This reflects a conventional assumption that, for every energy unit decreased, the unit cost of the tenant is increased.

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By solving problem (1), we can obtain a unique tenant supply defined as

$$S_n(r_i) := e_n^* = C_n'^{-1}(r_i), \tag{2}$$

where $C_n'^{-1}(\cdot)$ is the inverse function of the derivative of $C_n(\cdot)$. In this work, we choose the following cost function that satisfies all of its assumed properties

$$C_n(e_n) = \omega_n \, e_n^{\alpha_i}, \tag{3}$$

where ω_n is the *per-unit cost* of tenant n reduction, and each tenant of colocation i has the same α_i, reflecting its *sensitivity* to energy reduction. In practice, tenants can use the history cost data to infer the unit cost ω_n. On the other hand, α_i depends on specific applications hosted by the tenants. For example, tenant with delay-sensitive applications (e.g. computing-search, online gaming, etc.) will have high α_i. On the other hand, tenants with delay-tolerant applications (e.g. backup tasks, MapReduce jobs, etc.) can have low α_i. From (2), the tenant supply is

$$S_n(r_i) = \left(\frac{r_i}{\omega_n}\right)^{\frac{1}{\alpha_i - 1}} \alpha_i^{\frac{1}{1-\alpha_i}}, \tag{4}$$

which depends on the ratio between the per-unit reward and cost.

We next show the connection between α_i and the elasticity of the tenant supply, which is defined as a measurement of the responsiveness of a firm supply to the price fluctuation in microeconomics [25] as

$$\zeta_n := \frac{r_i S_n'(r_i)}{S_n(r_i)} = \frac{1}{\alpha_i - 1}. \tag{5}$$

There are many interesting connections between the tenant's sensitivity α_i and price elasticity ζ_n: (a) Tenant's price elasticity only depends on α_i; (b) Tenants with high α_i are more sensitive to the energy reduction cost according to (3), which means their supply is less responsive to a change in the reward, i.e., low ζ_n [25]. Therefore, in this work, we choose $\alpha_i \geq 2$ such that $\zeta_n \leq 1$ in order to enable the diminishing return for tenant supply to prevent the tenant supply becoming infinite when the reward r_i is sufficiently large. We note that $\alpha_i = 2$ is widely used in the literature (see [18] and references therein).

On the other hand, in order to determine the response and reward (c.f. Stage 2 of R2R), the operator i needs to know its tenant supply. However, since the tenant's per-unit cost is time-varying (i.e., the workload to tenants is dynamic [26]) and/or private (i.e., knowing per-unit cost can infer the workload pattern of tenants' customers), tenants are often not ready to disclose their per-unit cost to the operator. Therefore, in order to capture the uncertainty of the operator in tenant supply, we rewrite the tenant supply as follows:

$$S_n(r_i) = \omega_n^{\frac{1}{1-\alpha_i}} s_i(r_i), \forall n \in N_i, \tag{6}$$

where $s_i(r_i) := \left(\frac{r_i}{\alpha_i}\right)^{\frac{1}{\alpha_i - 1}}$. Then, by defining $\bar{\omega}_n := \omega_n^{\frac{1}{1-\alpha_i}}$ and considering $\bar{\omega}_n$ as a random variable (R.V.), we model the tenant supply $S_n(r_i) = \bar{\omega}_n s_i(r_i)$ as an R.V. to the operator i. We further assume that $\bar{\omega}_n$, $\forall n \in N_i$, is i.i.d with mean μ_i and variance σ_i^2. Define a new R.V.

$$W_i := \sum_{n=1}^{N_i} \bar{\omega}_n \tag{7}$$

with distribution function $F_i(\cdot)$, density function $f_i(\cdot)$, and support in the non-negative interval $[W_i^l, W_i^u]$. Then, the total tenant supply of colocation i is

$$\sum\nolimits_{n=1}^{N_i} S_n(r_i) = W_i \, s_i(r_i). \tag{8}$$

The total tenant supply model in (8) reminds us of the well-known multiplicative supply model in economics literature [27], from which we restate the explanation: "One interpretation of this model is that the shape of the supply curve is deterministic (i.e., $s_i(r_i)$) while the scaling parameter (i.e., W_i) representing the size of market is random." Based on the tenant supply, we have the following result.

Proposition 1. *Given $\alpha_i \geq 2$ and a reward $r_i \geq 0$, the expected aggregated tenant surplus is non-negative.*

Proof: Since $\mathbb{E}[W_i] \geq 0$ and $s_i(r_i) \geq 0$ with $r_i \geq 0$, we have

$$\begin{aligned} \mathbb{E}\left[\sum\nolimits_{n=1}^{N_i} u_n(r_i)\right] &= \mathbb{E}[W_i] \left(r_i s_i(r_i) - s_i(r_i)^{\alpha_i}\right) \\ &= \mathbb{E}[W_i] \left[\alpha_i \left(\frac{r_i}{\alpha_i}\right)^{\frac{1}{\alpha_i - 1}+1} - s_i(r_i)^{\alpha_i}\right] \\ &= \mathbb{E}[W_i] \, s_i(r_i)^{\alpha_i} (\alpha_i - 1) \geq 0. \end{aligned} \tag{9}$$

■

Operator Procurement and Reward. Based on the estimation of its tenant supply and the given DRP compensation price, the operator i will decide its response and reward by maximizing the expected profit as the following problem:

$(\mathbf{P_{opr}})$:

$$\begin{aligned} \max_{q_i, r_i \geq 0} \quad & \Pi_i^{op}(q_i, r_i | p) := p q_i - \mathbb{E}\left[r_i \sum_{n=1}^{N_i} S_n(r_i) + \beta \, [Y_i]^+\right] \\ \text{s.t.} \quad & Y_i = q_i - \sum\nolimits_{n=1}^{N_i} S_n(r_i), \end{aligned} \tag{10}$$

where Y_i is an R.V. that denotes the response deficit when the total tenant supply is less than the response q_i, and β is the backup generation (e.g., diesel) per-unit cost. We see that the operator profit $\Pi_i^{op}(q_i, r_i|p)$ comes from its revenue $p\,q_i$ (received from the DRP) minus costs, which includes the incentive cost for all tenants $r_i \sum_{n=1}^{N_i} S_n(r_i)$ and backup cost due to the deficit supplement $\beta\,[Y_i]^+$. Since all colocations of the DRP locate in the same region, we assume they receive the same β. The linear cost of backup generation captures the fuel cost and comes from the fact that for a given constant power produced by the generator (e.g., from 10kW to 2MW), it would cost an approximate constant amount of diesel/gas per unit time. Furthermore, this model is also widely used in the literature [5].

We note that, in $(\mathbf{P_{opr}})$, the operator attempts to reduce tenants' energy before resorting to the backup generator. This strategy mitigates both electricity and diesel usage, which provides a double effect on carbon footprint alleviation since electricity and diesel usage are notorious for high carbon emission [28]. With the current trend of green certificate pursuit of datacenters (e.g., LEED program [29]), the operators are focusing on exploiting more "green" renewable energy (e.g., solar, wind), and using less "dirty" high-carbon energy (e.g., char coal-based electricity, diesel).

Since each DRP of an ISO can serve a large number of colocations (e.g., there are almost 200 colocations in California [11]), we can ignore the effect of each colocation predicting its decision impact to change the DRP compensation price. Therefore, with a given DRP price, we have the following definition.

Definition 1. Given p, a strategy profile $(r_i^*(p), q_i^*(p))_{i\in I}$[7] is a competitive equilibrium at the second stage of R2R if it satisfies

$$\Pi_i^{op}(q_i^*, r_i^*|p) \geq \Pi_i^{op}(\bar{q}_i, \bar{r}_i|p), \forall i \in I, \bar{q}_i, \bar{r}_i \geq 0. \quad (11)$$

C. R2R*: DRP Model in the First Stage*

After receiving the response functions of its colocations, the DRP will decide its price p and procurement d by solving the following problem:

$$(\mathbf{P_{drp}}): \quad \min_{(p,d)\geq 0} \quad \Pi^{lse}(p,d) := p\sum_{i=1}^{I} q_i(p) + \lambda\big(d - D\big)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{I} q_i(p) = d. \quad (12)$$

In $(\mathbf{P_{drp}})$, the DRP minimizes the demand response cost, including the payment for colocations $p\sum_{i=1}^{I} q_i(p)$ and the penalty cost $\lambda\big(d-D\big)^2$ due to the deviation of its procurement d from the target D, where λ is the penalty rate.

We note that, in economic demand response, D is considered as a *soft* target that allows the DRP to rationally procure d deviated from D. In case of emergency demand response, D is a strictly *hard* target that must be matched by an exact procurement [5], [6], [17]. The DRP compensation price p^* is also called the market-clearing price that matches the aggregate operator response to the demand response capacity (c.f. constraint (12)).

D. R2R*: A Two-stage Stackleberg Game*

Since the interaction between the DRP and colocations contains the strategic decisions of both sides, we model the interaction as a two-stage Stackelberg game in the following setting:

- *Players*: the DRP and its serving colocations.
- *Strategies*: the DRP is the leader who announces the compensation price p and demand response capacity d. Each colocation is a follower who decides its response q_i and reward r_i.
- *Outcome*: a Stackelberg equilibrium, including the optimal solution (p^*, d^*) of $(\mathbf{P_{opr}})$ in the first stage, and the corresponding competitive equilibrium $\{r_i^*(p^*), q_i^*(p^*)\}_{i\in I}$ in the second stage, such that the optimal demand response capacity d^* is procured by the market-clearing price p^*: $\sum_{i=1}^{I} q_i^*(p^*) = d^*$.

In order to determine the outcome of this game, we use the backward induction method with the second-stage operator decision in Section IV and the first-stage DRP decision in Section V.

[7]For presentation brevity, sometimes we omit the argument p of $r_i^*(p), q_i^*(p)$.

IV. Operator Decisions in the Second Stage

In this section, we first provide the optimal decisions of operators on the reward rate and energy procurement and shows that there exists a unique competitive equilibrium in the second stage of R2R. Then we illustrate these results through a numerical case study.

A. Operator Optimal Reward and Procurement

In order to solve the operator's problem, we first change the variable $z_i := q_i/s_i(r_i)$. Then the operator profit in (10) is rewritten as

$$\begin{aligned}
&\Pi_i^{op}(z_i, r_i|\, p) \\
&= ps_i(r_i)z_i - r_i s_i(r_i)\mathbb{E}\left[W_i\right] - \beta s_i(r_i)\mathbb{E}\left[z_i - W_i\right]^+ \\
&= \Psi(r_i) - \Xi(z_i, r_i), \quad (13)
\end{aligned}$$

where

$$\Psi(r_i) = (p - r_i)s_i(r_i)\mathbb{E}\left[W_i\right] \quad (14)$$

$$\Xi(z_i, r_i) = ps_i(r_i)\big(\mathbb{E}\left[W_i\right] - z_i\big) + \beta s_i(r_i)\mathbb{E}\left[z_i - W_i\right]^+. \quad (15)$$

There are different interpretations of the rewritten operator profit in (13), which can lead to different solution methods. From the view point of a stochastic programming with the recourse model [30], the operator first makes a "here-and-now" decision r_i with the current profit $\Psi(r_i)$ before a realization of the random W_i is known. After W_i is disclosed, the operator then chooses a recourse action z_i that minimizes the recourse cost $\Xi(z_i, r_i)$. From the viewpoint of the classical newsvendor problem, the operator will maximize its riskless-profit $\Psi(r_i)$, which is the expected profit that would occur without uncertainty, and minimizes the expected loss $\Xi(z_i, r_i)$ due to the uncertainty of W_i. In this context, the expected loss includes two trade-off components: the opportunity revenue and the expected overage (i.e., diesel) cost in the first and second terms on the right side of (15). Obviously, if the operator chooses a high z_i (e.g. $z_i > \mathbb{E}\left[W_i\right]$), it will earn an opportunity revenue but also bear a high overage cost, and vice versa.

The typical solution methods for stochastic programming with the recourse action are scenario construction or statistical inference [30], which are not the exact analysis approaches that we target to obtain a closed-form operator response function. Therefore, using the standard approach of the classical newsvendor problem [27], we obtain the unique solution of

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problem ($\mathbf{P_{opr}}$) for each colocation i, inducing the unique competitive equilibrium as follows.

Theorem 1. *For any given $p \geq 0$, there exists a unique competitive equilibrium $(q_i^*, r_i^*)_{i\in I}$ in the second stage of* R2R *such that*

(a) If $p \geq \beta$:

$$\begin{cases} z_i^* &= F_i^{-1}(1) = W_i^u, \\ r_i^* &= \frac{\beta}{\alpha_i}; \end{cases}$$

(b) If $p \leq \beta$:

$$\begin{cases} z_i^* &= F_i^{-1}\left(\frac{p}{\beta}\right), \\ r_i^* &= \frac{p}{\alpha_i} \cdot \frac{\mathbb{E}\left[W_i \middle| W_i \leq z_i^*\right]}{\mathbb{E}[W_i]}, \end{cases} \tag{16}$$

where F_i^{-1} is the quantile function of F_i. As a result, $q_i^* = z_i^* \, s_i(r_i^*)$.

Proof: Please see Appendix A ∎

We remark on some properties of Theorem 1.

Remarks:

1) Even though there is a difference between the classical newsvendor problem (with the penalty cost for shortage if the stocking quantity is lower than the demand) and the operator response problem ($\mathbf{P_{opr}}$) (no penalty if the response is less than the tenant supply), the same technique of the newsvendor problem is used for solving ($\mathbf{P_{opr}}$): (a) By introducing an auxiliary variable $z_i = q_i/s(r_i)$, we reduce the complexity of dealing with both variables r_i and q_i inside $\mathbb{E}[Y]^+$ of the operator profit in ($\mathbf{P_{opr}}$) to a single variable z_i inside $\mathbb{E}[z_i - W_i]^+$ of the transformed operator profit in (13). (b) Using a sequential optimization approach, for a fixed r_i, we first obtain the optimal z_i as a function of r_i due to the strict concavity in z_i of $\Pi_i^{op}(z_i, r_i|\, p)$, then we substitute this $z_i(r)$ back into $\Pi_i^{op}(z_i(r_i), r_i|\, p)$ to find a unique r_i based on the first-order condition.
2) Similar to the classical newsvender problem [27] that defines z_i and q_i as the stocking factor and number of stocked units, respectively, in our settings q_i can be considered the energy procurement from tenants with procurement factor z_i. We can see that the optimal procurement q_i^* depends on the z_i^*, which is the result of a quantile function of price ratio p/β. This quantile structure means that the optimal procurement factor is the largest value such that the operator's over-procurement (deficit supplement) probability is less than p/β. Since the quantile function is non-decreasing, the decrease of compensation price p leads to smaller procurement q_i^*. When $p > \beta$, the procurement factor is inelastic, which is the maximum supply of all tenants.
3) Intuitively, we expect $r_i^* \leq \beta$ because otherwise the operator just uses the backup generator that is less costly than the reward. Theorem 1 shows a stronger result than the intuition: $r_i^* \leq \beta/\alpha_i$, i.e., the optimal reward is inversely proportional to tenant sensitivity α_i.
4) From Theorem 1, when $p \geq \beta$, it is easy for the operator to decide its optimal reward and procurement without knowing the statistical distribution of W_i. In contrast, when $p < \beta$, the operator needs to know the distribution of W_i in order to compute z_i^*, r_i^*, and q_i^* . However, since tenant cost evaluation is personal, and tenants might not know their exact distribution. Using the CLT knowing only μ_i, σ_i and N_i, the operator can *approximate* its optimal solution as characterized by the following result.

Corollary 1. *When $p \leq \beta$, using the CLT results in*

$$z_i^* = \Phi^{-1}(p/\beta)\sqrt{N_i}\sigma_i + N_i\mu_i, \tag{17}$$

$$r_i^* = \frac{\beta}{\alpha_i} \cdot \frac{\int_{x_{lo}}^{x_{up}} \left(\sqrt{N_i}\sigma_i\, x + N_i\mu_i\right) d\Phi(x)}{N_i\, \mu_i}, \tag{18}$$

$$q_i^* = z_i^*\, s_i(r_i^*), \tag{19}$$

where $x_{lo} = \frac{W_i^{lo} - N_i\mu_i}{\sqrt{N_i}\sigma_i}$, $x_{up} = \Phi^{-1}(p/\beta)$, and $\Phi^{-1}(\cdot)$ is the quantile of the standard normal distribution.

Proof: From Theorem 1, when $p \leq \beta$,

$$z_i^* = F_i^{-1}\left(\frac{p}{\beta}\right) \implies p/\beta = F_i(z_i^*) = \Phi\left(\frac{z_i^* - N_i\mu_i}{\sigma_i\sqrt{N_i}}\right), \tag{20}$$

which implies (17). The last equality of (20) results from the CLT since W_i is the sum of N_i i.i.d. R.V.s.

Similarly, we have $\mathbb{E}[W_i] = N_i\mu_i$, and

$$\mathbb{E}\left[W_i \mid W_i \leq z_i^*\right] = \frac{\int_{W_i^l}^{z_i^*} w\, dF_i(w)}{F_i(z_i^*) = p/\beta}. \tag{21}$$

By changing the variable and using the CLT, we have

$$x = \frac{w - N_i\mu_i}{\sigma_i\sqrt{N_i}} \implies F_i(w) = \Phi\left(\frac{w - N_i\mu_i}{\sigma_i\sqrt{N_i}}\right) = \Phi(x). \tag{22}$$

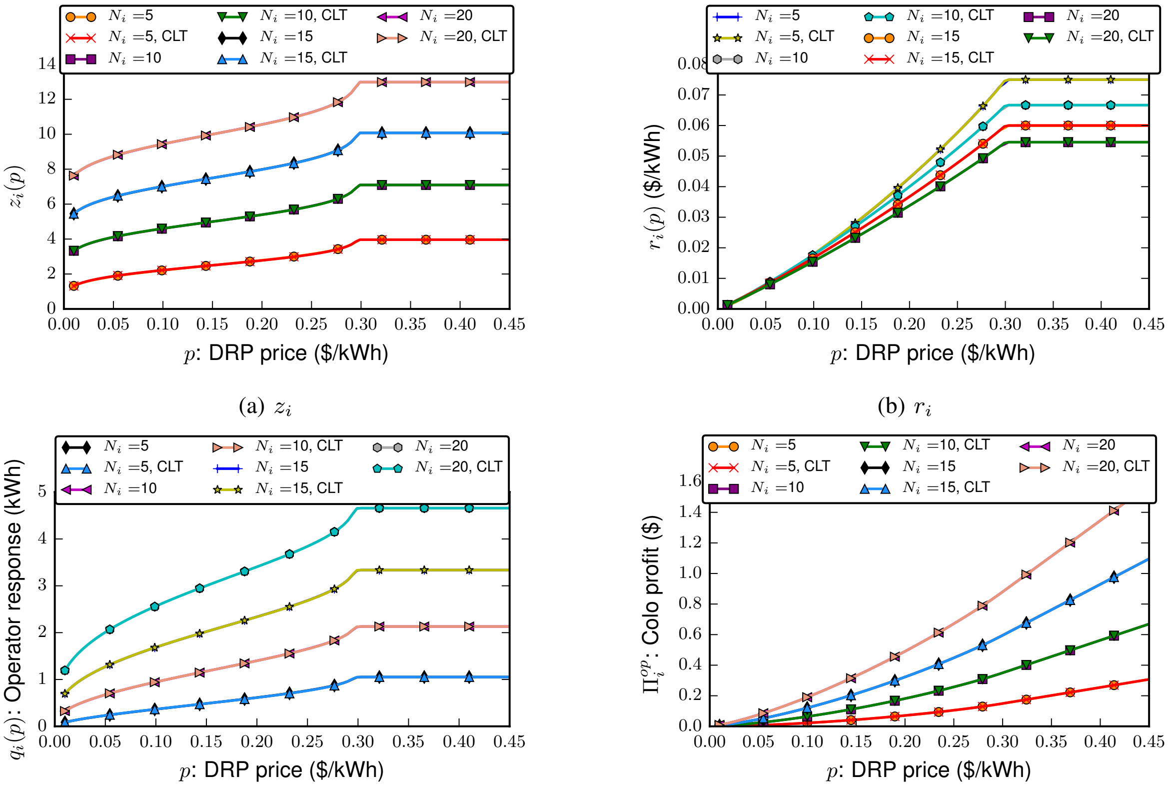
Applying (22) to (21) and then substituting (21) to (16), we obtain (37). ∎

B. Case Study

We consider a specific colocation i with $\alpha_i = 2$. The backup generator is assumed to run on diesel fuel that has unit cost 0.3 \$/kWh [6]. Henceforth, we assume that each tenant of this colocation has $\bar{\omega}_n$ uniformly distributed in $[0, 1]$ \$/kWh. The uniform distribution is often used to model distribution of user valuations/costs for computing, communication, and networking services [31]. Therefore, W_i is an Irwin-Hall distribution, which is also known as a uniform sum (of N_i) distribution [32].

We evaluate the performance of the operator and its tenants by varying 100 values of the DRP compensation price from 0 to 0.45 \$/kWh and increasing N_i from 5 to 20 with step size 5. Especially, we also compare the exact analysis (i.e., W_i distribution is known, c.f. Theorem 1) with the CLT-based approximation (c.f. Corollary 1) when the DRP compensation price is less than β.

Colocation operator: Fig. 1 evaluates the operator performance, including the solutions and objectives of the problem ($\mathbf{P_{opr}}$). We see that the solutions z_i^*, r_i^*, q_i^* and the profit $\Pi_i^{op}(z_i^*, r_i^*|p)$ are non-decreasing with respect to the increasing DRP compensation price and N_i in Figs. 1a, 1b, 1c, and 1d, respectively. Since q^* depends on z_i^* and r_i^*, when $p < \beta$, due



(a) z_i (b) r_i

(c) q_i (d) Π_i

Fig. 1: Colocation operator parameters with varying N_i

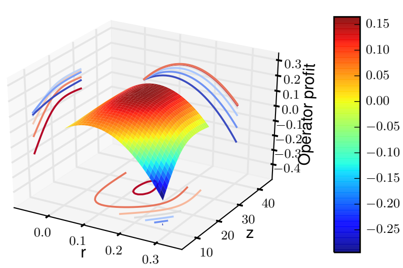


Fig. 2: Colocation profit: $p < \beta$.

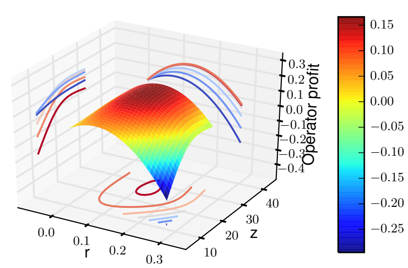


Fig. 3: Colocation profit: $p < \beta$, CLT.

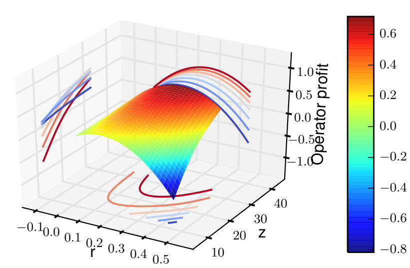


Fig. 4: Colocation profit: $p > \beta$.

to the specific distribution of W_i that affects both z_i^* and r_i^*, all curves of these three parameters are peculiarly non-convex. When $p > \beta$, obviously we have z_i^*, r_i^*, and q^* are constants, while $\Pi_i^{op}(z_i^*, r_i^*|p)$ increases with p.

We also examine how the operator profit changes when varying both z_i and r_i. When $p = 0.27 < \beta$, the operator profit with the exact analysis and with the CLT-based approximation are shown in Figs. 2 and 3, respectively. When $p = 0.45 > \beta$, the operator profit function with exact analysis is shown in Fig. 4. We remark on two observations: (a) The CLT approach has a good approximation with the exact analysis, and (b) the unique optimal operator profit in the graphs is obtained correctly with our analytical results. Specifically, the analytical results of the exact analysis and CLT approach are $z_i^* = 25.9, r_i^* = 0.128, \Pi_i^{op}(z_i^*, r_i^*|p) = 0.166$, and $z_i^* = 25.7, r_i^* = 0.116, \Pi_i^{op}(z_i^*, r_i^*|p) = 0.165$ when $p = 0.27$, and that of the case $p = 0.45$ is $z_i^* = 31.8, r_i^* = 0.15, \Pi_i^{op}(z_i^*, r_i^*|p) = 0.58$, which are matched with the optimal values of the graphs.

Tenants: In Fig. 5, tenant performance is examined in terms of expected tenant supply, tenant surplus, and tenant cost in Figs. 5a, 5b, and 5c, respectively, where r is the resultant r_i^* from Theorem 1. We clearly see that all metrics increase with respect to increasing DRP compensation price and N_i. Furthermore, each tenant supply curve has a diminishing return effect due to its sensitive parameter $\alpha \geq 2$, which induces the positive increasing rate of each tenant surplus curve. The environmental impact is also reflected in Fig. 5d: When compensation price is small enough (less than 0.15), the tenant supply can fulfill the operator's energy procurement q_i so that the operator needs not turn on the backup generator. On the other hand, when compensation price increases, the proportion of backup energy increases since the tenant supply is not

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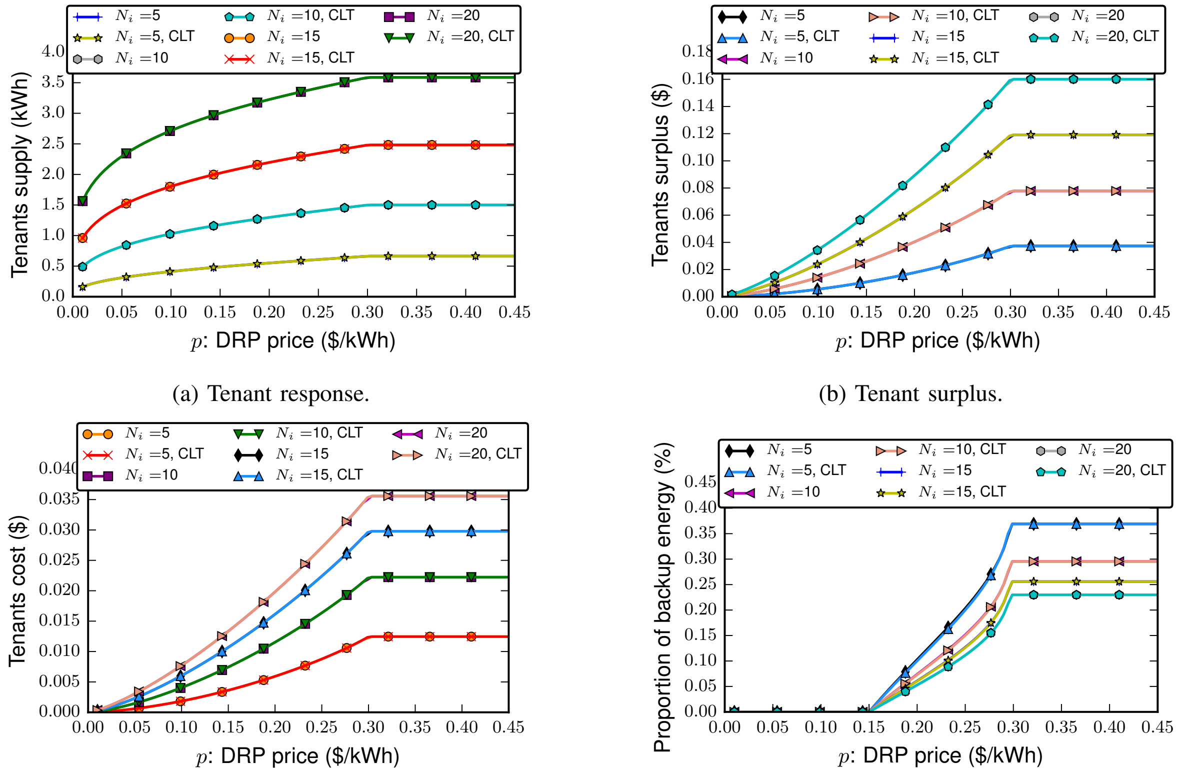

(a) Tenant response.

(b) Tenant surplus.

(c) Tenant cost.

(d) Proportion of backup energy.

Fig. 5: Tenant performance.

sufficient for the energy procurement.

CLT-based approximation versus exact analysis: Finally, by examining all of the above numerical case study figures, we confirm that the performance of the CLT-based approximation is the same as that of exact analysis, which helps the operator alleviate the burden to learn the W_i distribution.

V. DRP Decision in the First Stage

In this section, we first formulate the DRP cost minimization problem and its solution. We then compare the performance of the DRP cost against the social cost using an illustrative case study.

A. DRP Optimal Compensation Price

According to Theorem 1, the operator response curve can be expressed explicitly as follows:

$$q_i^*(p) = \begin{cases} q_{i,max}^* := F_{W_i}^{-1}(1)\left(\frac{\beta}{\alpha_i^2}\right)^{\frac{1}{\alpha_i - 1}}, & \text{if } p \geq \beta; \\ \left(\frac{\beta}{\alpha_i^2 \mathbb{E}[W_i]} \int\limits_0^{p/\beta} F_{W_i}^{-1}(x)\, dx\right)^{\frac{1}{\alpha_i - 1}} F_{W_i}^{-1}\left(\frac{p}{\beta}\right), & \text{if } p \leq \beta. \end{cases} \tag{23}$$

We denote the maximum voluntary procurement from all colocations by

$$Q_{max} := \sum\nolimits_{i \in I} q_{i,max}^*. \tag{24}$$

We see that the form of $q_i^*(p)$ is cumbersome, which can complicate the DRP to solve its problem $(\mathbf{P_{drp}})$. However, we propose an efficient algorithm, named OptPrice, to search for a solution of $(\mathbf{P_{drp}})$ in Algorithm 1. Before delving into an explanation of OptPrice, we provide the following lemma, which supports our algorithm design.

Lemma 1. *For a given $D > 0$, the optimal demand response capacity of the problem $(\mathbf{P_{drp}})$ satisfies $d^* \leq \min\{D, Q_{max}\}$, and d^* is procured using a unique optimal compensation price $p^* \leq \beta$.*

Proof: First, we show that $d^* \leq D$. We consider two cases:

1) $D \geq Q_{max}$: We obviously have $d^* \leq Q_{max} \leq D$.
2) $D < Q_{max}$: We use contradiction. Suppose $D < d^* \leq Q_{max}$ and d^* is procured by a unique p', then the objective of $(\mathbf{P_{drp}})$ can be written as

$$\Pi^{lse}(p', d^*) = \sum\nolimits_{i=1}^{I} p' \, q_i^*(p') + \lambda\Big(\sum\nolimits_{i=1}^{I} q_i^*(p') - D\Big)^2. \tag{25}$$

However, because $q_i^*(p)$ is a strictly increasing function due to $\frac{d}{dp} q_i^*(p) > 0$, $\forall i \in I$, and $p \leq \beta$, and accordingly $p\, q_i^*(p)$ is also strictly increasing, we see that $\Pi^{lse}(p', d^*)$ decreases if we decrease p'. Therefore, p' and $d^* (> D)$ are not optimal, which shows a contradiction.

Second, for any optimal $0 < d^* \leq \min\{D, Q_{max}\}$, there exists a unique p^* satisfying $\sum_{i=1}^{I} q_i^*(p^*) = d^*$ because $q_i^*(p)$ is a strictly increasing function, and $q_i^*(0) = 0$, $\forall i \in I$ and $p \leq \beta$. Furthermore, because $d^* \leq \min\{D, Q_{max}\}$, we have $p^* \leq \beta$. ■

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Algorithm 1 OptPrice: Optimal Compensation Price

Input: D
Output: p^*, d^*
1: **if** $D < Q_{max}$ **then**
2: Find $\bar{p}$ such that $\sum_{i=1}^{I} q_i^*(\bar{p}) = D$;
3: Solve the following problem

$$(\mathbf{P'_{drp}}): \quad \text{min.} \quad \sum_{i=1}^{I} p\, q_i^*(p) + \lambda\big(d - D\big)^2$$
$$\text{s.t.} \quad \sum_{i=1}^{I} q_i^*(p) = d, \qquad (26)$$
$$0 \le p \le \bar{p}. \qquad (27)$$

4: Return the solution p^* and d^*.
5: **else**
6: Solve the problem $(\mathbf{P'_{drp}})$ with the constraint (27) replaced by $0 \le p \le \beta$.
7: **end if**

Algorithm Discussion: Based on Lemma 1, by comparing D with Q_{max}, OptPrice solves $(\mathbf{P_{drp}})$ by considering two cases

1) $D < Q_{max}$: OptPrice reduces the search interval from $p \ge 0$ to $0 \le p \le \bar{p}$ (line 2) such that any feasible p in this interval with its corresponding d satisfies $\sum_{i=1}^{I} q_i^*(p) = d \le D$. Hence, solving $(\mathbf{P_{drp}})$ is equivalent to solving $(\mathbf{P'_{drp}})$ (lines 1-4).
2) $D \ge Q_{max}$: Similarly, solving $(\mathbf{P_{drp}})$ is equivalent to solving $(\mathbf{P'_{drp}})$ but the search interval is changed to $0 \le p \le \beta$ (line 6) according to Lemma 1.

We see that $(\mathbf{P'_{drp}})$ has no convexity structure. However, with the restricted search space and by absorbing the constraint (26) into the objective such that the problem has a single variable p, we can use a simple numerical algorithm (e.g. bisection) to find a local solution, or a global optimization method (e.g., branch-and-bound) to find its global solution. Interestingly, in the following case study with various settings, we show that $(\mathbf{P_{drp}})$ is a curve with a valley such that a unique optimal price can be found.

Based on the above algorithm discussion, we state the result of this algorithm.

Proposition 2. *For a given $D > 0$,* OptPrice *always returns a feasible solution to the DRP problem $(\mathbf{P_{drp}})$.*

Proof: In OptPrice, since $(\mathbf{P'_{drp}})$ optimizes its continuous objective over compact sets, and $(\mathbf{P'_{drp}})$ is equivalent to $(\mathbf{P_{drp}})$ as discussed, the result follows according to [33]. ■

B. Case Study

1) Settings: We consider a DRP that covers a service area of 8 colocations. In the first 4 colocations, the number of their tenants are 5, 10, 15 and 20, respectively, and the tenant weight $\bar{\omega}_n$ are uniformly distributed on $[0, 1]$ ($/kWh). In the remaining 4 colocations, their tenant number are set to 20, 40, 80 and 100, respectively, and the tenant weight $\bar{\omega}_n$ is exponentially distributed with mean value 1. We note that, in this setting, even though some colocations have the same tenant weight distribution, the number of tenants is set differently to make the statistical attributes W_i of the colocation distinguishable, i.e., the Irwin-Hall and Erlang distributions with different shape parameters [32], [34]. Furthermore, the α_i of each colocation is varied from 2 to 6 in order to reflect the heterogeneity of the colocation sensitivity. With these settings, Q_{max} is calculated to be 68 kWh.

2) Benchmark: In this stage, we compare the R2R equilibrium against the optimal social cost benchmark, which is defined as follows:

$$(\mathbf{P_{soc}}): \quad \min_{p,d \ge 0} \quad \Pi^{soc}(p, d)$$
$$\text{s.t.} \quad \sum_{i=1}^{I} q_i(p) = d, \qquad (28)$$

where $\Pi^{soc}(p, d)$ is defined to be

$$\sum_{i=1}^{I} \sum_{n=1}^{N_i} \mathbb{E}\left[C_n(S_n(r_i)) + \beta\Big[q_i - \sum_{n=1}^{N_i} S_n(r_i)\Big]^+ + \lambda\big(d - D\big)^2\right], \qquad (29)$$

with r_i and q_i specified in Theorem 1 as functions of p, and $\sum_{n=1}^{N_i} C_n(S_n(r_i)) = \sum_{n=1}^{N_i} \omega_n\, S_n(r_i)^{\alpha_i} = W_i s_i(r_i)^{\alpha_i}$ based on (3) and (4).

The social cost (SOC) is defined as the aggregate cost of tenants, operator, and DRP. The operator reward to tenants and DRP payment to operators are transfered internally in the R2R system, so they have no impact on the social cost and are excluded. Obviously, in practice, the DRP cannot always have full information to solve the SOC problem $(\mathbf{P_{soc}})$, especially the tenant cost $C_n(S_n(r_i))$ [6], [18].

3) Results: The performance metrics we evaluate are the DRP and social costs with the impact of D and λ. The rationale behind these impacts are: (a) Since the R2R mechanism considers a single demand response period, by varying D, we emulate a sequence of independent demand response periods to explore how the DRP pricing policy behaves with different values of D. (b) On the other hand, since λ controls how strictly the target D is obtained via the energy procurement of its colocations, by varying λ, we examine the deviation of DRP with either its individual cost or social cost. Henceforth, the units of D and λ are kWh and $/kWh^2, respectively.

DRP and social costs with varying prices: In Fig. 6, we compare the DRP with the social cost evaluated at 100 price values ranging from 0 to 0.45 (i.e., 1.5β).
Impact of D: In Fig. 6a, we fix $\lambda = 0.001$ and alter three values of D: 10, 40, and 70, which represent low, medium, and high demand response targets in our setting, respectively, (compared with $Q_{max} = 68$). Fig. 6a reveals the trajectory of the minimum DRP and social cost by evaluating over a range of increasing prices. The minimum value is found at the valley of each curve, and the curves clearly exhibit the convexity of the cost versus price, which numerically provides a unique optimum. The graphs show that the minimum DRP cost is higher than that of social cost for all cases. Furthermore, the difference between the minimum value of DRP and social costs increases with respect to increasing D, which will be

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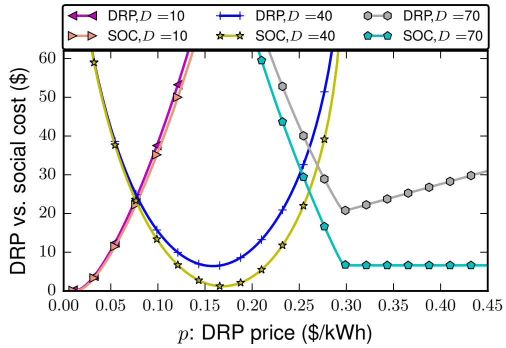
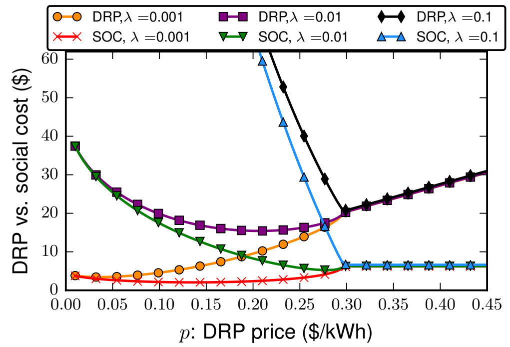


(a) Fixed λ, different values of D.



(b) Fixed D, different values of λ.

Fig. 6: Comparison of DRP and social costs with varying price.

explained later when we evaluate the optimal prices and costs by varying D.

Impact of λ*:* In Fig. 6b, we fix $D = 10$ and use three values of λ: 0.001, 0.01 and 0.1, which represent low, medium, and high deviation penalties, respectively. Similar to Fig. 6a, Fig. 6b shows the trajectories of DRP and social cost with respect to price where the optimal value is achieved at the valley of each curve; furthermore, we also observe that higher λ induces higher difference between the minimum value of DRP and social cost. These common trends will be explained in the following part, where we evaluate the performance of DRP and social costs with the optimal prices.

DRP and social optimal prices and costs: We compare the individual with the social objective of the DRP in terms of the optimal prices and costs in Figs. 7 and 8, respectively. *Impact of* D*:* By fixing $\lambda = 0.001$ and varying 100 values of D from 10 to 70, we see that both optimal prices in Fig. 7a and optimal costs in Figs. 8a increase, which is obvious. Furthermore, the gap between the optimal DRP's individual and social prices, as well as the gap between the DRP's individual and social cost, increases. The rationale behind this fact is explained in Fig.8a: While the social price is increased to keep the *deviation cost* $\lambda (d - D)^2$ small, the DRP tends to give lower prices to balance the deviation cost with its own cost $\sum_{i=1}^{I} p\, q_i^*(p)$.

Impact of λ*:* By fixing $D = 10$ and varying 100 values of λ from 0.001 to 0.1, we observe in Fig. 7b that (a) the DRP optimal price is lower than the social price and (b) both quickly increase and approach a constant value, which produces similar curves of optimal DRP and social costs in Fig. 8b. While (a) is obvious since the DRP always gives as low of a price as possible for its own procurement cost sake, (b) can be explained in that a sufficiently high λ makes the deviation cost dominant, which forces both optimal DRP and social prices close to $\bar{p} = 0.013$ (with $D = 10$), which minimizes the deviation cost.

VI. R2R: Implemenation in the Stackelberg Equilibrium

In previous sections, we have shown that the Stackelberg equilibrium of R2R can be obtained by backward induction. In this section, we present the R2R implementation that centers around this equilibrium, which is shown as follows.

R2R **Implemenation:**

Step 1: Each colocation, which is a demand response participant, sends its parameters $N_i, \alpha_i, \mu_i, \sigma_i$, and (W_i^l, W_i^u)[8] to the DRP. Based on these parameters and colocation response curves, the DRP solves $(\mathbf{P_{drp}})$ to find (p^*, d^*) and broadcasts p^* to all I colocations.

Step 2: After receiving p^*, each operator $i \in I$ sets its reward $r_i^*(p^*)$ and response $q_i^*(p^*)$ according to Corollary 1 and broadcasts r_i^* to all N_i tenants.

Step 3: After receiving r_i^*, each tenant $n \in N_i$ decides its supply $S_n(r_i^*)$ according to (6) and also reports $S_n(r_i^*)$ to the operator i.

Step 4: After receiving $S_n(r_i^*)$ from all tenants and comparing it with q_i^*, the operator i will trigger the backup generator if a response deficit occurs.

Step 5: Finally, the demand response is exercised: (a) Tenants reduce their energy (e.g., switch off servers) by an amount $S_n(r_i^*)$, and their rewards are proceeded. (b) Operators receive compensation from DRP.

Obviously, the R2R implementation takes only one round. Furthermore, this implementation requires coordination between the operator and DRP so that the DRP can precisely obtain the colocation response curves $q_i^*(p)$ (Step 1).

VII. Conclusions

To recap, this work showed a first attempt to design an incentive mechanism for colocation economic demand response with the role of DRP, which is ignored in previous studies. We first proposed R2R, a mechanism that uses reward/price to incentivize colocations to reduce energy consumption for economic demand response. The R2R is based on two-stage sequential decisions where the DRP first decides its compensation price for the colocation, and the colocation later decides its reward for each tenant. Due to the strategic interaction between the DRP and colocations, we formulated

[8]These parameters can be estimated by the operator via interacting with its tenant for a sufficient time.

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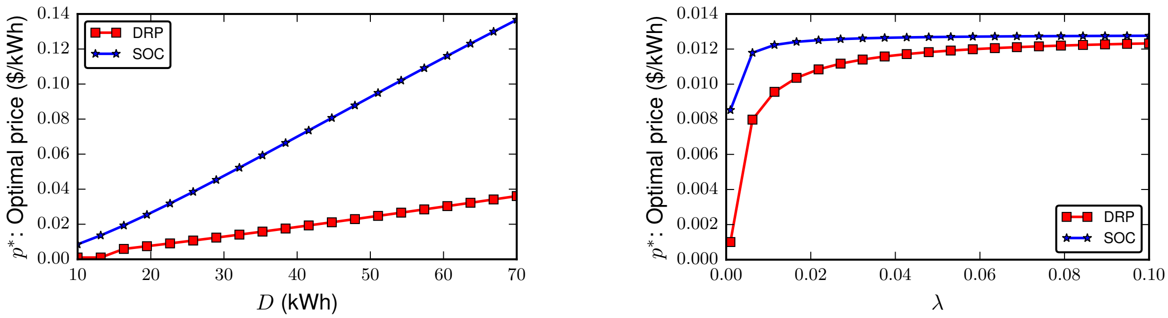
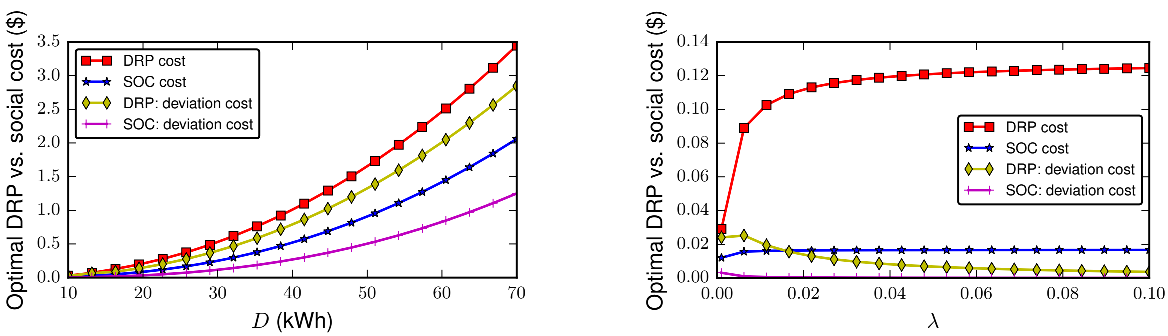


(a) Varying target D.

(b) Varying deviation weight λ.

Fig. 7: Comparison of optimal DRP compensation price with social price.



(a) Varying target D.

(b) Varying deviation weight λ.

Fig. 8: Comparison of optimal DRP cost with social cost.

R2R as a two-stage Stackelberg game, where the DRP is the leader and colocations are the followers. We also showed the existence of the Stackelberg equilibrium such that, for any given compensation price, there exists a unique competitive equilibrium where the expected operator profit is minimized in the second stage. Based on this competitive equilibrium, the DRP will choose an optimal market-clearing price that mimimizes its cost to match the colocation response to the demand response capacity.

In order to complement the exact analysis of the second-stage competitive equilibrium that can be impractical due to the required full knowledge of the tenant cost distribution, we also proposed an approximate approach with limited required information that can provide a comparable performance to the exact analysis. We validated the approximation methods through extensive case studies, which show that the approximation method can give the same performance as the exact analysis. On the other hand, even though finding the first-stage market-clearing price is generally non-convex, we designed an algorithm that can reduce the search space and thus the searching time. We provided various case studies to demonstrate the optimal compensation price finding and its corresponding compared DRP individual and social cost performance.

Appendix A
Proof of Theorem 1

We use the sequential minimization approach as in [27]. We fix r_i and find z_i^* first.

By calculating the first and second partial derivatives

$$\frac{\partial \Pi_i^{op}}{\partial z_i} = s_i(r_i)\big(p - \beta F_i(z)\big), \tag{30}$$

$$\frac{\partial^2 \Pi_i^{op}}{\partial z_i^2} = -\beta s_i(r_i) f_i(z) < 0, \tag{31}$$

we obtain a unique z_i^* as

$$z_i^* = \begin{cases} F_i^{-1}\left(\frac{p}{\beta}\right), & \text{if } p \le \beta; \\ F_i^{-1}(1) = W_i^u, & \text{if } p \ge \beta. \end{cases} \tag{32}$$

Hence, we have

$$\begin{aligned} &\Xi(z_i^*, r_i) \\ &= p s_i(r_i)\big(\mathbb{E}[W_i] - z_i^*\big) + \beta s_i(r_i)\mathbb{E}[z_i^* - W_i]^+ \\ &= p s_i(r_i)\big(\mathbb{E}[W_i] - z_i^*\big) + \beta s_i(r_i)\int_0^{z_i^*} (z_i^* - u)\, f_i(u)\, du \\ &= p s_i(r_i)\big(\mathbb{E}[W_i] - z_i^*\big) + \beta s_i(r_i)\int_0^{p/\beta} \big(z_i^* - F_i^{-1}(x)\big)\, dx \\ &= p s_i(r_i)\mathbb{E}[W_i] - \beta s_i(r_i)\int_0^{p/\beta} F_i^{-1}(x)\, dx, \end{aligned} \tag{33}$$

where the third equality of (33) follows by changing the variable $x = F_i(u)$ so that $u = F_i^{-1}(x)$.

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Therefore, we have

$$\Pi_i^{op}(z_i^*, r_i) = \Psi(r_i) + \Xi(z_i^*, r_i) \tag{34}$$
$$= -r_i s_i(r_i)\mathbb{E}[W_i] + \beta s_i(r_i)\int_0^{p/\beta} F_i^{-1}(x)\,dx.$$

As the next part of the sequential minimization, we find r_i^* using the first-order condition, i.e.,

$$\frac{\partial \Pi_i^{op}(z_i^*, r_i)}{\partial r_i} \tag{35}$$
$$= -\big(r_i s'(r_i) + s_i(r_i)\big)\mathbb{E}[W_i] + \beta s'(r_i)\int_0^{p/\beta} F_i^{-1}(x)\,dx$$
$$= s'(r_i)\Big[-r_i\mathbb{E}[W_i] + \beta\int_0^{p/\beta} F_i^{-1}(x)\,dx\Big] - s_i(r_i)\mathbb{E}[W_i]$$
$$= \frac{1}{\alpha_i - 1}\Big[-\mathbb{E}[W_i] + \frac{\beta}{r_i}\int_0^{p/\beta} F_i^{-1}(x)\,dx\Big] - \mathbb{E}[W_i] = 0.$$

In the last equality of (35), we use the fact that

$$\frac{r_i s_n'(r_i)}{s_n(r_i)} = \frac{r_i S_n'(r_i)}{S_n(r_i)} = \zeta_n = \frac{1}{\alpha_i - 1} \tag{36}$$

defined in (5).

Therefore, we have

$$r_i^* = \frac{\beta}{\alpha_i}\cdot\frac{\int_0^{p/\beta} F_i^{-1}(x)\,dx}{\mathbb{E}[W_i]}. \tag{37}$$

Consider $\int_0^{p/\beta} F_i^{-1}(x)\,dx$, by changing variable $x = F_i(u)$ such that $u = F_i^{-1}(x)$, we have

$$\int_0^{p/\beta} F_i^{-1}(x)\,dx = \int_0^{F_i^{-1}\left(\frac{p}{\beta}\right)} u\,dF_i(u) = \tag{38}$$
$$\begin{cases} \int_0^{W_i^u} u\,dF_i(u) = \mathbb{E}[W_i], & \text{if } p \geq \beta; \\ \frac{\mathbb{E}\left[W_i \mathbf{1}_{\left\{W_i \leq F_i^{-1}(p/\beta)\right\}}\right]}{\mathbb{P}\left[W_i \leq F_i^{-1}(p/\beta)\right]} = \mathbb{E}\left[W_i \mid W_i \leq F_X^{-1}(p/\beta)\right], & \text{if } p < \beta, \end{cases}$$

where $\mathbf{1}_{\{A\}}$ is an indicator function of an event A. From (37) and (38), we have

$$r_i^* = \begin{cases} \frac{p}{\alpha_i}\cdot\frac{\mathbb{E}\left[W_i \middle| W_i \leq F_i^{-1}\left(\frac{p}{\beta}\right)\right]}{\mathbb{E}[W_i]}, & \text{if } p/\beta \in [0,1); \\ \frac{\beta}{\alpha_i}, & \text{if } p/\beta \geq 1. \end{cases} \tag{39}$$

Combining (32) and (39), we complete the proof.

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