OMF-MAC: An Opportunistic Matched Filter-based MAC in Cognitive Radio Networks

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Abstract—Opportunistic spectrum access (OSA) in cognitive radio has been proposed to improve spectrum efficiency in which secondary users can access the unused portion of the primary users’ spectrum temporally or spatially. OSA by multiple secondary users is controlled by the medium access control (MAC) protocol. OSA MAC protocols have to avoid or prevent collisions during transmission. In addition, OSA MAC protocols have to deal with the interference between the primary and secondary systems. The key challenge is to provide service to the secondary system while not causing interruption to the primary system. In this paper, we propose an Opportunistic Matched Filter-based (OMF) MAC protocol to jointly consider spectrum access of the idle portion of the spectrum and coordination among secondary users. We first model the channel as an ON-OFF process and then apply renewal theory to deal with spectrum access and derive the expression for the expected accumulated interference interval in terms of the transmission time. Using this expression for interference interval, we model the coordination and competition among SUs as a discrete time Markov chain. We then perform throughput and delay analysis to derive the closed-form expressions for the aggregate secondary system throughput and average secondary packet delay. Finally, we present simulations results to verify our analysis.

Index Terms—Renewal Theory; Distributed Coordination Function; Opportunistic Spectrum Access; Dynamic Spectrum Access; Cognitive Radio

I. INTRODUCTION

Opportunistic spectrum access (OSA) is an effective mechanism to mitigate the scarcity of the radio spectrum [1–3]. The radio spectrum can be considered as a resource, that is diminishing with respect to significant increases in the number of ubiquitous wireless devices. However, some of the spectrums licensed to primary users (PUs) are under-utilized, for example, TV white spaces. OSA enables the secondary users (SUs) with cognitive capability to dynamically access the idle radio spectrum. Spectrum access, how and when a user can transmit on the channel, is controlled by a Medium Access Protocol (MAC). A MAC protocol performs coordination among users in conventional systems such as IEEE 802.11 Wireless LAN [4] and IEEE 802.16 Wireless MAN. A centralized MAC protocol is used for Wireless MAN, while a distributed MAC protocol, distributed coordination function (DCF) [4], is implemented in Wireless LAN. At present, due to the flexible and scalable nature of DCF, Wireless LAN (or Wi-Fi) is the most popular wireless technology. As a consequence, the operating spectrum for Wireless LAN, 2.4 GHz ISM band is becoming more and more congested. OSA is one of the viable solutions and IEEE 802.11af task group is now currently working on this topic [5].

For OSA, a MAC protocol must deal with the mutual interference between the primary and secondary systems, in addition to coordination among SUs. From the MAC layer perspective, when both primary and secondary signals are present on the spectrum, there will be interference. This can happen in either of two ways: (i) an SU transmits while the primary signal is present, or (ii) a primary signal appears while an SU is transmitting. A MAC protocol must prevent the first case. However, we cannot fully prevent the second case and the MAC protocol must reduce the interference time interval. From the observation of these two cases for interference, we can identify two key issues for MAC in OSA: spectrum sensing and interference mitigation [6]. Spectrum sensing identifies whether the channel is occupied or free. If we can correctly identify the spectrum state without any errors, the first case for interference will be eliminated. On the other hand, interference mitigation concerns with the second case of interference, and thus involves reducing the interference time interval, i.e., stopping the SU transmission as soon as the primary signal appearance is detected.

Another key issue is spectrum sensing. There are many mechanisms for spectrum sensing; energy detection, matched-filter detection, cyclo-stationary detection, etc. [6]. Energy detection is the simplest and most widely used mechanism for spectrum sensing. In energy detection, a simple threshold on the received signal strength is employed for decision hypothesis [7]. Spectrum sensing using matched-filter detection is exactly the same as the traditional matched-filter detection technique deployed in digital receivers. In matched-filter detection, a correlation of the received signal and a known reference signal is performed [8]. This process detects the presence of the known reference signal in the received signal. Matched-filter detection is the optimal detection for channels with additive stochastic noise. For both energy detection and matched-filter detection, there is no additional hardware requirement. Based on these basic mechanisms, there are many proposals for spectrum sensing. The authors in [9] developed an estimator to schedule sensing periods, whereas...
in [10] cooperation between SUs was explored to improve accuracy of spectrum sensing. The authors in [11] proposed an indirect monitoring method to reduce the cost of sensing overhead. These works focus on how to successfully detect the primary signal. The drawback of these works is that all SUs have to remain silent while performing spectrum sensing thus adding sensing overhead to other existing costs. Moreover, the SUs have to perform another sensing step for detecting secondary transmissions as governed by the IEEE 802.11 DCF [4]. Therefore, the spectrum sensing and spectrum access become the classic exploration versus exploitation problem, which the authors in [12] give a full discussion. The reason for such a problem is that the mentioned spectrum sensing methods cannot explicitly identify whether the received signal is the primary or secondary signal. We argue that from the MAC layer perspective, a simple sensing mechanism with the ability for the SUs to differentiate between the primary and secondary signals is necessary. Moreover, the spectrum sensing mechanism must be compatible with the MAC without incurring any extra time overhead.

Firstly, to tackle the above mentioned problems of interference mitigation and spectrum sensing, we propose an Opportunistic Matched Filter-based (OMF) MAC protocol. OMF-MAC uses a DCF-based contention process between SUs for each channel, i.e. CSMA/CA with binary exponential backoff [4, Sec.9.3]. Similar to Clear Channel Assessment (CCA) in DCF, OMF-MAC employs short sensing intervals which are compatible with the DCF mechanism. Using these sensing intervals in sequence, OMF-MAC can differentiate between primary and secondary signals. When a primary signal is detected on the channel by SUs, all SUs must remain silent until the channel becomes free again. Secondly, we model our proposal using a two-step approach to close the discontinuity between the two different modeling methods discussed in related works. For the first step, we apply the renewal theory [13] to model interference and other related time intervals as well as their probabilities. Using the closed-form expressions from the first step, we employ the Markov chain to model the secondary network behavior. Furthermore, we perform throughput and delay analysis.

A. Related Works

There are many existing works on the issue of MAC for OSA. However, only a few proposals jointly model the inter-behavior between the primary and secondary systems and intra-behavior of SUs. The model for inter-behavior between the primary and secondary systems is usually separated from intra-behavior, i.e., MAC for SUs. The authors in [14] proposed CREAM-MAC in which they model the channel as an ON-OFF process and directly apply the work of [15] that provides a good approximation for the aggregate throughput. Two access mechanisms, CR-ALOHA and CR-CSMA were proposed in [16] which employ a two-level MAC protocol. They perform throughput and delay analysis based on their proposed model. However, in both [14] and [16], mutual interference between the primary and secondary systems cannot be observed. On the other hand, the authors in [17] provide a closed-form expression of interference duration in terms of transmission duration with a simple queuing model for MAC. This approach is not sufficient to model the intra-behavior among SUs.

The following are some other related works that employ the Markov chain and renewal theory for MAC protocols in OSA. In [15], the Markov Chain was modeled to derive the closed-form expression for normalized system throughput for the saturated network conditions. In [18], the authors extended the work of [15] and derived the closed-form expression for the average packet delay for the network for saturated network conditions. In [19], the licensed channel was modeled as an ON-OFF renewal process and it was used to find the optimal price for SUs. The Markov chain model was adopted and queuing analysis was performed in [20] and [21]. Applying the renewal theory, the authors in [22] proposed a protocol to proactively switch channels when the PU signal is estimated to appear. In [23], authors proposed a joint rate control and allocation under the packet collision constraint. The authors in [24] provides soft guarantees for minimum hop routing based on the interference components between PUs and SUs. The authors of [25] proposed hardware constrained MAC in which the optimal stopping problem is solved under sensing and transmission constraints. Recent works which apply renewal theory are [26] and [27]. In [26], the authors performed analysis of spectral holes in a multichannel cognitive radio system and derived probability distributions of their time intervals. In [27], the authors proposed a simple recovery mechanism for channel-hopping cognitive radio networks that uses a list of backup channels.

The rest of the paper is organized as follows. Firstly, the system model is described in Section II. Secondly, our proposed protocol OMF MAC is given in Section III. In Section IV, we derive the closed-form expression for the transition probabilities and interference to the primary network. In Section V, we model the secondary network with the discrete Markov chain to perform throughput and delay analysis. We present our simulation results in Section VI and conclusion are given in Section VII.

II. SYSTEM MODEL

As shown in Fig. 1, we assume that there is only one PU transmitter in each licensed channel. The primary network has a higher priority and can occupy the channel at any time while SUs can only access the channel if it is idle. As the PU is the original occupant of the channel, it does not sense the channel before its transmission. We assume that there are \( n \) SU devices in the secondary network. The secondary network does not know when the primary network will start or stop transmitting beforehand. Therefore, the SU devices must sense if the PU is present or absent on its channel. The \( n \) SU devices must perform coordination governed by the DCF in the IEEE 802.11 standard.

We assume that there is no cooperation between the primary system and secondary network. Therefore, an SU has no information about the exact communication mechanism of the primary system. In addition, we assume that an SU cannot
know the exact slot interval used by the primary system and synchronization cannot be performed. From the point of view of SUs, the channel is switching between the ON state and OFF state as depicted in Fig. 2. During the ON state, the PU is present; whereas, during the OFF state, the PU is absent and SUs can access the channel. We model the duration of the ON state and OFF state by two random variables, namely $T_{ON}$ and $T_{OFF}$. We assume that $T_{ON}$ and $T_{OFF}$ are statistically independent. Let $f_1(t)$ and $f_0(t)$ be the probability density functions (p.d.f’s) of $T_{ON}$ and $T_{OFF}$, respectively. This kind of ON-OFF behavior of the channel is an alternating renewal process [13]. From Fig. 2 the renewal interval is $T_p = T_{ON} + T_{OFF}$, and the distribution of $T_p$ is denoted by $f_p(t) = f_1(t) * f_0(t)$ where the operator $*$ represents the convolution operation. Notice here that our channel model is flexible, i.e., non-slotted, whereas the channel model for CR-ALOHA in [16] is structured and rigid, i.e., slotted. Since the OSA environment is highly dynamic, our model would be more compatible and adaptable.

III. OPPORTUNISTIC MATCHED FILTER-BASED MAC PROTOCOL

In this section, we discuss our proposed OMF-MAC protocol and the rationale behind the DCF-based design. We will now discuss some disadvantages and advantages of DCF mechanism. In an OSA environment, DCF cannot operate effectively due to design limitations: (i) DCF cannot distinguish between primary and secondary transmissions. (ii) DCF is designed to operate on a single channel. (iii) DCF mechanism is sub-optimal due to its ad-hoc nature. Despite its limitations, DCF is suitable for the OSA environment: (a) DCF is simple to implement and robust. (b) DCF is currently the most popular and widely used wireless technology. (c) DCF is compatible with the ad-hoc nature of resource availability of OSA environment. Our design goal is to keep OMF-MAC as simple as possible with least amount of additional overhead. The details of OMF-MAC are discussed in the following subsections.

A. Overview of DCF

The DCF can simply be explained as a listen-before-talk mechanism which employs Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) with the binary exponential backoff algorithm. DCF has two transmission modes, namely, Basic Service Set (BSS) mode and Request-To-Send/Clear-To-Send (RTS/CTS) mode, both of which employ the positive acknowledgement (ACK) or feedback. The BSS mode involves a two-way handshake process where the source sends the DATA packet and the destination replies with ACK if the data transmission is successful. The RTS/CTS mode improves on the BSS mode by the following sequence of packet exchanges; the source sends RTS packet; the destination replies with CTS packet; then the source send DATA packet and the destination replies with ACK if the data transmission is successful. We will call this (RTS-CTS-DATA-ACK) as a four way handshake process.

The basic time unit for DCF is slot-time which is defined by modulation technique at the PHY layer. For each slot-time, the listening phase of DCF is carried out by the CCA. Each slot-time contains a sensing interval [4] Sec.9.3.7. Each sensing interval employs Energy Detection (ED) at PHY layer with adequate reliability for identifying the FREE/BUSY channel (detection probability > 0.9) [4] Sec.18.3.10.6. For efficiency, DCF uses slot-time for backoff and integrate CCA into it.

The procedure for conventional DCF is as follows: When there is a packet to transmit, a node will choose a random discrete backoff counter value within the Contention Window (CW). Whenever the node senses a free channel, it decreases its backoff counter by one. Otherwise, it will freeze the backoff counter. When the counter reaches zero, and if the channel remains free for at least DCF Interframe Space (DIFS) interval, the node will transmit its packet. If the transmission is successful, the value of the CW is assigned the minimum value. If there was a collision, the transmitting node will not receive CTS or ACK packet, i.e. CTStimeout or ACKtimeout. Then, the transmitting node will double its CW value and choose a random discrete backoff counter value within the CW. If there are successive collisions, this backoff procedure repeats itself and the value of CW is doubled until it reaches its maximum value. For a complete detailed presentation of conventional DCF, please refer to the IEEE 802.11 standard [4] Sec.9.3.

B. OMF-MAC: Spectrum Sensing

Since OMF-MAC is based on DCF, it inherits the standard CCA for carrier sensing mechanism. However, as discussed
in previous subsection, the actual detection of the Signal of Interest (SoI) at the PHY layer is carried out using ED mechanism. ED mechanism draws BUSY or IDLE hypothesis by comparing the sum or average of the samples of received signal strength with a predefined threshold. Furthermore, ED is a non-coherent detection method and cannot distinguish between SoI and other signals operating on the same wireless channel [6]. In order for SUs to differentiate between primary
and secondary signals, a coherent detection is necessary. SUs need to decode the SoI for coherence, i.e. MAC layer detection. Matched filter detection can identify the presence of known reference signal, i.e. secondary signal, inside the received signal. To improve the reliability, we propose to perform complete decoding of MAC header in addition to conventional matched filter detection. Without loss of generality, we will refer to this complete procedure as the Matched-Filter (MF) detection. MF detection can be implemented without any extra hardware for the transceivers since every standard half-duplex Wi-Fi transceivers contain a matched-filter in the modem (modulate/demodulate) module. As MF detection has to decode the frame information (preamble, PHY header, MAC header) of the packet, it must wait until the start of another packet to successfully detect the SoI. SUs have two opportunities for using MF detection in the BSS transmission mode and four opportunities in the RTS/CTS transmission mode.

Thus, we propose a sensing mechanism that encompasses both PHY and MAC layer detection. Firstly, every SUs will perform CCA on each time-slot similar to DCF. When a SU detects a BUSY channel with CCA, it will invoke MF detection to distinguish the type of the signal. However, SUs only have the signal information on the secondary signal, i.e. preamble, PHY header, MAC header. Therefore, if the SUs can decode the signal using MF detection, the SoI is secondary. Otherwise, if the SUs cannot decode the signal within predefined interval, namely SENSEtimeout, the SUs will conclude that PU is present and vacate the channel. It is important to note that the time interval SENSEtimeout must not be longer than the maximum allowable interfering time duration of the primary system. However, this protective approach may lead to SUs identifying the secondary signal as primary signal due to limited time window. In terms of sensing time interval, as MF detection invokes the modem module of the transceiver, it takes longer time to decode the SoI. One of the reason SUs only use MF detection when CCA already identifies the channel as BUSY is that MF is more expensive than ED. In case of detecting FREE channels, CCA with ED is sufficient and this does not require long decoding time period of MF detection.

The sensing operation of OMF-MAC is depicted in Figs. 3-4. Basically, OMF-MAC follows the same procedure as the conventional DCF except for unsuccessful packet transmission. In OSA, there are two possibilities for unsuccessful packet transmission. The first one is collision among SUs and the second one is the interruption by the primary signal. As shown in Figs. 3-4, SUs must detect the header of packets transmitted using MF detection within SENSEtimeout interval. Every detected SoI is considered as a primary signal in any BUSY time-slot unless it can be decoded by MF detection. Therefore, OMF-MAC can distinguish between the primary and secondary signals and take appropriate actions.

C. OMF-MAC: Backoff Procedures

In general, the binary exponential backoff procedure for OMF-MAC is the same as DCF except for the presence of PU. Figs. 3-4 depict the operations of OMF-MAC when the primary signal is present and absent, respectively. As discussed in previous subsection, spectrum sensing will identify the two different cases of unsuccessful packet transmissions. In the case of collision between SUs, similar to DCF, the CW of the SUs involved will be doubled and new values for backoff counters will be randomly chosen. The detailed steps taken when a secondary transmission is interrupted by the primary signal are shown in Fig. 3. Fig. 3 displays source and destination SUs detecting the PU signal. At that time instance, the source and destination SUs will wait for the end of PU transmission. During the waiting period, as depicted in Fig. 3, as the sequence of slot-times, the source and destination SUs performed CCA. When the source SU detect FREE channel after the end of primary transmission, it will perform handshaking again with destination SU for re-transmission.

D. OMF-MAC: Network Allocation Vector

The Network Allocation Vector (NAV) is also known as virtual carrier sensing. The main purpose of NAV is to save energy for idle SUs by entering asleep or doze mode while the PU or other SUs are transmitting. We have the same two scenarios as depicted in Figs. 3-4 for the NAV procedure. Usually, NAV counter values are set by overhearing RTS/CTS handshakes. When the PU is absent, SUs can follow the conventional NAV procedure as depicted in Fig. 4 by using the transmission time duration information contained in RTS/CTS packets. However, SUs cannot decode the primary signal and there is no synchronization between primary and secondary systems. Therefore, when PU is present, idle SUs cannot precisely set the value for NAV counters. We propose to set the NAV counter value to the last assigned values of NAV by SUs when there was no primary signal. As shown in Fig. 3, the idle SUs will wake up periodically and go back into sleep mode when they detect the presence of primary signal. As depicted in Fig. 3, at the end of the final NAV period, SUs will wake up and discover the absence of primary signal. At that time instance, there may be other SUs already awake and transmitting data packets. At this point, the SUs will remain awake until the end of current secondary packet transmission. They will then revert back to the conventional NAV mechanism.

IV. Transition Probabilities, Interference Duration for PU and Waiting Time for SU

In this section, we discuss the state transition probabilities, interference duration to the PU and waiting time for the SUs. We apply renewal theory to obtain two basic matrices: the transition probabilities and accumulated time intervals related to the secondary system. We can view this as a two-stage renewal process, where, in the first stage, the channel is switching between ON and OFF for the secondary network. When the channel becomes free, the secondary system goes into the second stage in which the SUs either transmit or wait. Note that both stages are alternating renewal processes. For simplicity of analysis, we will assume that secondary transmission time duration is constant, i.e., the constant secondary packet size.
Let \( t \) be the time taken for a transmission interval. Let \( X \) and \( Y \) denote the OFF and ON state time intervals, respectively, as depicted in Fig. 5. Let \( X \) and \( Y \) be the recurrence times (residual times) of \( X \) and \( Y \), respectively. If \( f_0(x) \) and \( f_1(y) \) are the p.d.f's of \( X \) and \( Y \), respectively, the density functions (c.d.f) and means of \( X \) and \( Y \) are given by:

\[
f_X(\tilde{x}) = 1 - F_0(x)/\mu_0, \quad f_Y(y) = 1 - F_1(y)/\mu_1, \tag{1}\]

where \( F_0(x) \), \( F_1(y) \) and \( \mu_0, \mu_1 \) are the cumulative distribution functions (c.d.f) and means of \( X \) and \( Y \), respectively.

**A. State Transition Probabilities**

We consider an ON-OFF alternating renewal process which is in equilibrium. Therefore, we only need to account for the last renewal. First, let us consider the case where there is no renewal since the last renewal, i.e., if the channel is OFF (ON) at time \( t_s \), it will be OFF (ON) at time \( t_s + t \). In this case, the equilibrium state transition probability \( \pi_{00}(t) \) is given by:

\[
\pi_{00}(t) = \int_t^\infty \frac{1 - F_0(x)}{\mu_0} \, dx + \int_0^t h_{01}(x) \{1 - F_0(t-x)\} \, dx, \tag{2}\]

where \( h_{01}(x) \) is the renewal density for arrivals of the ON state given that the state of the channel at \( t_s \) is OFF. Denote by \( f^*(s) \) as the Laplace transform of \( f(t) \). When we take the Laplace transform, (2) becomes:

\[
\pi_{00}^*(s) = \left\{ \frac{\mu_0 s - 1 + f_0^*(s)}{\mu_0 s^2} \right\} + h_{01}^*(s) \left\{ \frac{1 - f_0^*(s)}{s} \right\}. \tag{3}\]

Moreover, \( h_{01}^*(s) \) is expressed in [13] p.85 as:

\[
h_{01}^*(s) = \frac{f_1^*(s) \{1 - f_0^*(s)\}}{\mu_0 s^2 \{1 - f_0^*(s)f_1^*(s)\}}. \tag{4}\]

By substituting (4) into (3), we obtain

\[
\pi_{00}^*(s) = \frac{1}{s} \left\{ \frac{1 - f_0^*(s)}{\mu_0 s^2} \right\} \left\{ 1 - f_0^*(s) \right\} \left\{ 1 - f_1^*(s) \right\}. \tag{5}\]

However, we know from the limit theorems that, as \( t \to \infty \),

\[
\lim_{t \to \infty} \pi_{00}(t) = \mu_0/(\mu_0 + \mu_1). \tag{6}\]

Therefore, we can rewrite the equilibrium state transition probability \( \pi_{00} \) as:

\[
\pi_{00}(t) = \mu_0/(\mu_0 + \mu_1). \tag{7}\]

Taking the Laplace transform of (7) and substituting the result into (5), we have:

\[
\omega^*(s) = \frac{\mu_0 \mu_1}{(\mu_0 + \mu_1)s} - \frac{1 - f_0^*(s)}{s^2} \left\{ 1 - f_0^*(s)f_1^*(s) \right\}. \tag{8}\]

In general, for the equilibrium alternating renewal process, given that the initial state of the channel at \( t_s \), the probabilities of the state of the channel after time duration, \( t \), are given in the matrix \( \Pi \) as:

\[
\Pi = \begin{bmatrix} \pi_{00}(t) & \pi_{01}(t) \\ \pi_{10}(t) & \pi_{11}(t) \end{bmatrix} = \frac{\mu_0}{\mu_0 + \mu_1} \begin{bmatrix} \omega(t) & \frac{\mu_1}{\mu_0 + \mu_1} \omega(t) \\ \frac{\mu_0}{\mu_0 + \mu_1} \omega(t) & \frac{\mu_0}{\mu_0 + \mu_1} \omega(t) + \frac{\mu_1}{\mu_0 + \mu_1} \omega(t) \end{bmatrix}. \tag{9}\]

From (6) and (7), we can see that the transition probabilities depend on the type of the distributions of the ON and OFF states. Moreover, \( \omega(t) \) is distribution specific and we can calculate it by taking the inverse Laplace transform of (8).

**B. Analysis of Accumulated Time Intervals**

In [9] and [17], the authors derived the sojourn (or holding) times for interference intervals, i.e., taking Laplace’s transforms of distribution functions of time intervals to derive the closed-form expression. We derive the same approach as in [17], which is the standard technique given in [13]. For our purpose, we apply the following generic quantities, namely, \( T_{SU}(t), T_I(t), T_W(t) \), and \( T_H(t) \), which reflect different aspects of the secondary network. The interference time intervals for two different cases with three scenarios are depicted in

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**Fig. 5.** The density functions of residual times for ON and OFF periods.

**Fig. 6.** Interference intervals for two different cases, \( T_I(t) \) for the modified renewal process and \( \tilde{T}_I(t) \) for the ordinary renewal process, with three scenarios.
Let $T_1(t)$ and $\tilde{T}_1(t)$ be the expected accumulated interference intervals of the respected cases within time interval $t$. The first case is the general case of interference which is a modified renewal process and we want to find a closed-form expression. However, as depicted in Fig. 3 for the third scenario, $T_1(t)$ is a function of $\tilde{T}_1(t)$, which is a case of the ordinary renewal process. Thus, for an ordinary renewal process, $T_1(t)$ can be written as:

$$T_1(t) = \int_{t<\tau<x} (t-x) f_{XY}(x,y) dx dy + \int_{x+y<t} (y + \tilde{T}_1(t-x-y)) f_{XY}(x,y) dx dy.$$  \hfill (10)

Since $X$ and $Y$ are independent, $f_{XY}(x,y) = f_0(x)f_0(y)$ and let the random variable $W = X + Y$, and $f_W(w) = f_0(t) * f_1(t)$. Thus, we can rewrite (10) as:

$$\tilde{T}_1(t) = \int_0^t (t-x) f_0(x) dx + \int_0^t \tilde{T}_1(t-x-y) f_0(x)f_1(y) dx dy \nonumber$$
$$- \int_{x+y<t} (t-x-y) f_0(x) f_1(y) dx dy, \nonumber$$
$$= \int_0^t (t-x) f_0(x) dx + \int_0^t \tilde{T}_1(t-w) f_w(w) dw \nonumber$$
$$- \int_0^t (t-w) f_w(w) dw, \nonumber$$
$$= t * f_0(t) + \tilde{T}_1(t) * f_w(t) - t * f_w(t), \nonumber$$
$$= t * f_0(t) + \tilde{T}_1(t) * f_0(t) * f_1(t) + t * f_0(t) * f_1(t). \nonumber$$  \hfill (11)

Taking the Laplace transform of (11), we obtain:

$$\tilde{T}_1(s) = \frac{f_0(s)}{s^2} + \frac{\tilde{T}_1(s) f_0(s) f_1(s) - f_0(s) f_1(s)}{s^2} \nonumber$$
$$= \frac{f_0(s)}{s^2} \left\{ 1 - f_1(s) \right\} \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nono
transform pairs are $f_0^*(s) = \lambda_0/(s + \lambda_0)$, and $f_1^*(s) = \lambda_1/(s + \lambda_1)$. Thus,
\[
\omega(t) = \frac{1}{\lambda_0 + \lambda_1} e^{-(\lambda_0 + \lambda_1)t},
\]
\[
\pi_{01}(t) = \frac{\lambda_0}{\lambda_0 + \lambda_1} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),
\]
\(T_I(t) = \frac{\lambda_0}{\lambda_0 + \lambda_1} t - \frac{\lambda_0}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right), \quad (20)
\]
\(T_W(t) = \frac{\lambda_0}{\lambda_0 + \lambda_1} t + \frac{\lambda_1}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right),
\]
\(T_H(t) = \frac{\lambda_1}{\lambda_0 + \lambda_1} t - \frac{\lambda_1}{(\lambda_0 + \lambda_1)^2} \left(1 - e^{-(\lambda_0 + \lambda_1)t}\right).
\]

Note that due to the memory-less nature of the exponential distributions, $T_I(t) = \bar{T}_I(t)$, $T_W(t) = \bar{T}_W(t)$ and $T_H(t) = \bar{T}_H(t)$.

Case-2: For the special case where both the OFF and ON time intervals have uniform distributions, i.e., $X \sim f_0(x) = 1/b_0$ and $Y \sim f_1(x) = 1/b_1$, the Laplace transform pairs are $f_0^*(s) = 1/sb_0$, and $f_1^*(s) = 1/sb_1$. Therefore, we have
\[
\omega(t) = t + \frac{C_5}{2} e^{-t/C_1} + \frac{C_4}{2} e^{t/C_1} - C_5,
\]
\[
\pi_{01}(t) = \frac{b_1}{C_2} \left(1 - \frac{b_0}{b_1} \left(t + \frac{C_5}{2} e^{-t/C_1} + \frac{C_4}{2} e^{t/C_1} - C_5\right)\right),
\]
\(T_I(t) = \frac{t(2C_4 - t) - \sqrt{b_0}}{b_0} \left(C_3 e^{-t/C_1} - C_4 e^{t/C_1}\right) - 4b_1,
\]
\(T_W(t) = \frac{t(t - C_6) - \sqrt{b_1}}{b_1} \left(C_3 e^{-t/C_1} - C_4 e^{t/C_1}\right) + 4b_0,
\]
\(T_H(t) = \frac{t(2C_2 - t) + \sqrt{b_0}}{b_1} \left(C_3 e^{-t/C_1} - C_4 e^{t/C_1}\right) - 4b_0,
\]
where $C_i$’s are constants values, $i \in [1, 2]$, and are given by:
\[
C_1 = \sqrt{b_0} \sqrt{b_1}, \quad C_2 = b_0 + b_1,
\]
\[
C_3 = \frac{1}{\sqrt{b_0} + \sqrt{b_1}}^2, \quad C_4 = \frac{\sqrt{b_0} - \sqrt{b_1}}{2},
\]
\[
C_5 = \frac{2b_0 + b_1 + 2b_2}{b_0 + b_1}, \quad C_6 = 2b_0 + b_1.
\]

Case-3: For the special case where both the OFF and ON time intervals have Erlang distributions with $k = 2$, i.e., $X \sim f_0(x) = \lambda_0^2 x e^{-\lambda_0 x}$ and $Y \sim f_1(y) = \lambda_1^2 y e^{-\lambda_1 y}$, the Laplace transform pairs are $f_0^*(s) = \lambda_0^2/(s + \lambda_0)^2$, and $f_1^*(s) = \lambda_1^2/(s + \lambda_1)^2$. Therefore, we have
\[
\omega(t) = \frac{D_1 e^{-D_1 t} + D_2 e^{-D_2 t} - D_3 e^{-D_3 t}}{\lambda_0 \lambda_1 - 2\lambda_0 \lambda_1},
\]
\[
\pi_{01}(t) = \frac{\lambda_0}{\lambda_1 - 2} \left(\frac{D_1 e^{-D_1 t} + D_2 e^{-D_2 t} - D_3 e^{-D_3 t}}{2\lambda_0 \lambda_1}\right),
\]
\(T_I(t) = \frac{\lambda_0}{\lambda_1 - 2} \frac{D_1 e^{-D_1 t} + D_2 e^{-D_2 t} - D_3 e^{-D_3 t}}{4\lambda_0 \lambda_1 D_3} - \frac{D_4 e^{-D_4 t}}{2\lambda_0 \lambda_1},
\]
\(T_W(t) = \frac{\lambda_1}{\lambda_1 - 2} \frac{D_1 e^{-D_1 t} + D_2 e^{-D_2 t} - D_3 e^{-D_3 t}}{4\lambda_0} + \frac{D_4 e^{-D_4 t} + D_5 e^{-D_5 t}}{8\lambda_1 D_3},
\]
\(T_H(t) = \frac{\lambda_1}{\lambda_1 - 2} \frac{D_1 e^{-D_1 t} + D_2 e^{-D_2 t} - D_3 e^{-D_3 t}}{4\lambda_0} + \frac{D_4 e^{-D_4 t} + D_5 e^{-D_5 t}}{8\lambda_1},
\]
where $D_i$’s are constants values, $i \in [1, 6]$, and they are given as follows:
\[
D_1 = \lambda_0 + \lambda_1,
\]
\[
D_2 = \lambda_0 - \lambda_1,
\]
\[
D_3 = \sqrt{\lambda_0^2 - 6\lambda_0 \lambda_1 + \lambda_1^2},
\]
\[
D_4 = D_3^2/\lambda_0 + \lambda_1,
\]
\[
D_5 = (D_3 + \lambda_0 + \lambda_1)/2,
\]
\[
D_6 = (D_3 - \lambda_0 - \lambda_1)/2,
\]
\[
D_7 = D_3(\lambda_0 + \lambda_1) + D_2^2,
\]
\[
D_8 = D_3(\lambda_0 + \lambda_1) - D_2^2.
\]

D. Discussion

We would like to discuss similarities and differences of our renewal models compared to others in [9], [17], [20] and [27]. Firstly, our renewal analysis model and those of [9] and [17] consider a single channel whereas the works in [20] and [27] consider multiple channels. Our motivation in using a single channel model is rooted in our MAC protocol design. Single channel MAC protocols are simpler and more robust than multi-channel MAC protocols. Multi-channel MAC is more complex because network nodes (or devices) need to coordinate their channel access by using a control channel or by channel hopping. Secondly, our analysis approach is unique in that we combine the renewal analysis and the Markov chain analysis to portray the OMF-MAC operating in an OSA environment. On the other hand, in [20], the authors focus solely on theoretical analysis of spectral hole duration and in [27], the authors focus on the recovery process. Although, these works are exemplary, there remains an underlying problem of the MAC protocol design that achieves efficient access coordination SU's in multiple channels. Thirdly, concerning the closed-form expressions, in [17], the authors performed the analysis only for a specific case where both arrival and departure distributions are exponential. We performed generic analysis with three example distribution function combinations in Section IV.C. Note that for Case-1, the closed-formed expression we derived is the same as the result in [17]. Our resulting closed-from expression are different from [9], in which the authors derived their expressions by using differential substitution in Laplace transform steps. On the other hand, we derive our expression in terms of the basic term $\omega^*(s)$, described in [9], in the Laplace transform steps.

V. PERFORMANCE ANALYSIS

We will continue the analysis of our proposed OMF-MAC in the second stage. In this second stage, we will employ the discrete Markov chain method used in [15] as our basic framework due to its simplicity. To represent and model the intrinsic properties of the PU activities, one more degree of freedom is necessary. Therefore, we add an extra dimension to the existing two-dimensional Markov chain in [15] to represent the dynamics of PU activities. As depicted in Fig. 7, the three dimensional Markov chain can now represent the PU activities associated with an OSA environment. We assume that: (i) the secondary network consists of $n$ contending SU devices, (ii) each SU device always has a packet available for transmission, and (iii), the conditional collision probability
between SU devices, \( p \), of a transmitted SU packet is constant and independent of the re-transmissions which this packet has suffered in the past.

A. Markov Chain Model and Analysis

For efficiency, DCF employs a discrete-time exponential backoff scheme. Before any packet transmission, the value of each SU device’s backoff timer is uniformly chosen in the range \([0, W_i − 1]\), where \( W_i \) is the current contention window (CW) size and \( i \) is the backoff stage. We have \( W_i = 2^i \cdot W_0, i ∈ [0, m] \), where \( W_0 \) is the minimum CW size and \( m \) is the maximum backoff stage such that \( CW_{max} = 2^m \cdot W_0 \). At the first transmission attempt of an SU packet, \( CW_{min} = W_0 \). After each successive unsuccessful transmission due to collision among SU devices, \( W_i \) is doubled up to the value \( CW_{max} \).

Let \( s(θ) \) be the backoff stage, \( b(θ) \) be the backoff timer and \( h(θ) \) be the state of the channel for a given SU device at time \( θ \). Fig. 7 depicts the discrete-time Markov chain used to model a tri-dimensional process \( \{s(θ), b(θ), h(θ)\} \), similar to the bi-dimensional process mentioned in [15]. The transition probabilities are given by:

\[
\begin{align*}
P\{i, k, 0| i, k + 1, 0\} & = 1 − β, \quad k ∈ (0, W_i − 2), i ∈ (0, m), \\
P\{0, k, 0| i, 0, 0\} & = \frac{1−β−p}{W_0}, \quad k ∈ (0, W_i − 1), i ∈ (0, m), \\
P\{i, k, 0| i − 1, 0, 0\} & = \frac{p}{W_i}, \quad k ∈ (0, W_i − 1), i ∈ (1, m), \\
P\{m, k, 0|m, 0, 0\} & = \frac{p}{W_m}, \quad k ∈ (0, W_i − 1), \\
P\{i, k, 1| i, k, 0\} & = β, \\
P\{i, k, 0| i, k, 1\} & = α, \\
P\{i, k, 1| i, k, 1\} & = 1 − α,
\end{align*}
\]

(25)

Let \( b_{i,k,h} = \lim_{θ→∞} P\{s(θ) = i, b(θ) = k, h(θ) = z\} \) be the stationary distribution of the Markov chain where \( i ∈ (0, m), k ∈ (0, W_i − 1) \) and \( z ∈ \{0, 1\} \). We will now solve the balanced
equations of this Markov chain. First, discrete channel state transition probabilities $\alpha$ and $\beta$ can be calculated using [9] as: $\alpha = \pi_{10}(\sigma)$ and $\beta = \pi_{01}(\sigma)$ which are assumed to be constant. Note that,

\[
b_{i,k,1} = \beta \cdot b_{i,k,0} + (1 - \alpha) \cdot b_{i,k,1},
\]

\[
= \beta / \alpha \cdot b_{i,k,0}, \quad k \in (0, W_i - 1), \quad i \in (0, m).
\]

Furthermore,

\[
p \cdot b_{i-1,0,0} = (1 - \beta) \cdot b_{i,0,0},
\]

\[
b_{i,0,0} = \frac{p^i}{(1 - \beta)^i} \cdot b_{0,0,0}, \quad 0 < i < m,
\]

\[
p \cdot b_{m-1,0,0} = (1 - \beta - p) \cdot b_{m,0,0},
\]

\[
b_{m,0,0} = \frac{p^m}{(1 - \beta - p)^m} \cdot b_{0,0,0}.
\]

Due to the regularities of the Markov chain, for each $k \in (0, W_i - 1)$, we have

\[
b_{i,k,0} = \frac{W_i - k}{W_i} b_{i,k,0}, \quad k \in (0, W_i - 1), i \in (1, m).
\]

Therefore, from (26), (27), and (28), all the values of $b_{i,k,h}$ can be expressed as a function of the value $b_{0,0,0}$ and the conditional collision probability among SUs $p$. Thus, $b_{0,0,0}$ is calculated by applying the normalization condition of the Markov chain that simplifies as follows:

\[
1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k,0} + \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k,1},
\]

\[
= \left( 1 + \frac{\beta}{\alpha} \right) \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k,0},
\]

\[
\frac{\alpha}{\alpha + \beta} = \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,0,0} = \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} \frac{W_i - k}{W_i}.
\]

For simplicity, let the constants $A = \alpha / (\alpha + \beta)$ and $B = 1 - \beta$. Then (30) becomes:

\[
A = \frac{b_{0,0,0}}{2} \left[ W \left( \sum_{i=0}^{m-1} (2p)^i B^i + (2p)^m (B - p) B^{m-1} \right) + \frac{B}{B - p} \right]
\]

from which

\[
b_{0,0,0} = \frac{2A(B - p)(B - 2p)B^{m-1}}{(W + 1)(B - 2p)B^{m-1} + Wp[B^{m-1} - (2p)^m]}.
\]

An SU transmission occurs when the backoff time counter reaches zero, regardless of the backoff stage. Then, the probability $\tau$ that an SU device transmits a packet in a randomly chosen slot time can be obtained as:

\[
\tau = \sum_{i=0}^{m} b_{i,0,0} = \frac{1 - \beta}{1 - \beta - p} b_{0,0,0}
\]

\[
= \frac{2A(B - 2p)B^m}{(W + 1)(B - 2p)B^{m-1} + Wp[B^{m-1} - (2p)^m]}.
\]

On the other hand, the conditional collision probability $p$ of a transmitted secondary packet can be calculated as follows:

\[
p = 1 - (1 - \tau)^{n-1}.
\]

(33) and (34) forms a nonlinear system with two unknowns $\tau$ and $p$ which can be solved by numerical methods. Note that for conventional DCF without the primary system on a channel, $\alpha = 1$, $\beta = 0$ and (32) equals to its respective counterpart in [13].

B. Secondary Packet Transmission Probability and Transmission Time Interval

Let $P_{tr}$ be the probability that there is at least one successful SU device in the contention process for transmission at any randomly slot-time. Since $n$ SU devices contend on the channel, and each transmits with probability $\tau$,

\[
P_{tr} = 1 - (1 - \tau)^n.
\]

The probability $P_s$ that there is no collision among SU transmissions is given by the probability that exactly one SU device transmits conditioned on the fact that at least one SU device transmits, i.e.,

\[
P_s = \frac{n \tau(1 - \tau)^{n-1}}{P_{tr}} = \frac{n \tau(1 - \tau)^{n-1}}{1 - (1 - \tau)^n}.
\]

C. Behavior of SUs’ Transmissions

For the secondary system, as in [15], the transmission time intervals are as follows:

\[
T_c = DIFS + RTS + CTS + H + E[L]
\]

\[
+ ACK + 3SIFS + 4\delta,
\]

\[
T_e = DIFS + RTS + \delta,
\]

where $T_c$ is the time interval in which a secondary packet is transmitted successfully, and $T_e$ is the time interval in which the channel is sensed busy due to a collision among SUs. $H$ is the packet header and $E[L]$ is the expected data payload size. $\delta$ is the propagation delay. We can consider $T_a$ as the time duration for continuous uni-directional transmission instead of several bi-directional transmissions since there is no interruption from other SU devices.
D. Throughput and Delay Analysis

We start the analysis by the effective time duration required for a successful transmission of a secondary packet. The time $T_s$ is required for the actual successful transmission of the secondary packet. Each SU must contend for the channel to transmit its packet and time duration of $1 - P_{tr}$ is spent during contention. Moreover, SUs suffer collision among themselves for a time duration of $T_c = \frac{(1 - P_s)}{P_s}$. Therefore, the effective time duration of a secondary packet can be defined as:

$$T_{ef} = T_s + \sigma \cdot \frac{(1 - P_{tr})}{P_s} + T_c < \frac{(1 - P_s)}{P_s},$$

where $\sigma$ is the length of the time slot for the discrete backoff counter and its value depends on the PHY layer. Using this effective transmission time, we calculate the interference duration as $T_I(T_{ef})$. Thus, $T_{ef} + T_I(T_{ef})$ is spent for every successful secondary packet transmission.

Let $S$ be the normalized aggregate system throughput of the secondary system. As defined in [13], $S$ is the fraction of time in which the channel is used successfully to transmit secondary packets. We can write $S$ as follows:

$$S = \frac{P_{tr} P_s E[L]}{P_{tr} P_s (T_s + T_I(T_{ef})) + (1 - P_{tr}) \sigma + P_{tr} (1 - P_s) T_c} = \frac{P_{tr} E[L]}{T_{ef} + T_I(T_{ef})}.$$

The average amount of payload information successfully transmitted in a slot-time not occupied by PU signal is $E[L]$, and a successful SU contention process occurs with probability $P_s P_{tr}$. Note that SUs can only transmit when the channel is idle which is $P_0$ proportion of the time.

Let $E[N_s]$ refer to the average number of slot-times required for a successful secondary packet transmission. From the Markov chain, we obtain $E[N_s] = 1/b_{0,0,0}$. Let $E[D]$ be the average delay for a successfully transmitted secondary packet. As in [13], the packet delay is defined as the time interval from the instance a packet at the head of its MAC queue of an SU device until it is successfully received at its intended destination. The average secondary packet delay is given by:

$$E[D] = \frac{P_{tr} P_s}{P_0} E[N_s] \cdot (T_s + T_I(T_{ef})) = \frac{P_{tr} P_s}{P_0} b_{0,0,0} (T_s + T_I(T_{ef})).$$

VI. Simulation Results

We use Matlab to build an event-driven simulator to perform Monte Carlo’s simulations. The simulation results are taken from an average of 20 runs. We run the simulation for five minutes, i.e., $\theta = 300 \times 10^3$ milli-seconds. Unless stated otherwise, the following results have been obtained using the parameters provided in Table. 1. The difference between the simulation results and numerical results are well within the range of a 95% confidence interval and discrepancies are lower than 0.02.

<table>
<thead>
<tr>
<th>Secondary packet payload</th>
<th>8184 bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAC header</td>
<td>272 bits</td>
</tr>
<tr>
<td>PHY header</td>
<td>192 bits</td>
</tr>
<tr>
<td>ACK</td>
<td>112 bits + PHY header</td>
</tr>
<tr>
<td>RTS</td>
<td>160 bits + PHY header</td>
</tr>
<tr>
<td>CTS</td>
<td>112 bits + PHY header</td>
</tr>
<tr>
<td>Channel Bit Rate</td>
<td>1 Mbit/s</td>
</tr>
<tr>
<td>Propagation Delay</td>
<td>1 $\mu$s</td>
</tr>
<tr>
<td>Slot Time</td>
<td>20 $\mu$s</td>
</tr>
<tr>
<td>SIFS</td>
<td>10 $\mu$s</td>
</tr>
<tr>
<td>DIFS</td>
<td>50 $\mu$s</td>
</tr>
<tr>
<td>ACK Timeout</td>
<td>50 $\mu$s</td>
</tr>
<tr>
<td>CTS Timeout</td>
<td>50 $\mu$s</td>
</tr>
</tbody>
</table>

A. Verification of Analysis Model

We performed a series of experiments to verify our analysis model. For the first experiment, Fig. 8, we fixed the number of secondary users in the system to a constant, i.e., $n = 50$ and vary the size of the secondary packet. As can be observed from Fig. 8 the simulation results agree with the numerical results. We run the simulation for special cases with the following parameters:

Case-1: both OFF and ON time intervals have exponential distributions, i.e., $X \sim f_0(x) = \lambda_0 e^{-\lambda_0 x}$ and $Y \sim f_1(y) = \lambda_1 e^{-\lambda_1 y}$, with $\mu_0 = 700$ ms and $\mu_1 = 300$ ms.

Case-2: both OFF and ON time intervals have uniform distributions, i.e., $X \sim f_0(x) = 1/b_0$ and $Y \sim f_1(x) = 1/b_1$, with $\mu_0 = 600$ ms and $\mu_1 = 400$ ms, and

Case-3: for the special case where both OFF and ON time intervals have Erlang distributions with $k = 2$, i.e., $X \sim f_0(x) = \lambda_0^2 x e^{-\lambda_0 x}$ and $Y \sim f_1(y) = \lambda_1^2 y e^{-\lambda_1 y}$, with $\mu_0 = 500$ ms and $\mu_1 = 500$ ms.

Fig. 8(a) depicts the transition probability of the channel versus the size of the secondary packet. In other words, the transition probability of the channel is the probability that interference to the PU can occur. As the size of the secondary packet increases, the transmission time increases, which in turn raises the probability of PU appearance during the secondary packet transmission.

Fig. 8(b) shows the expected accumulated interference versus the size of the secondary packet. Note that this value is calculated with effective transmission time where the cumulative interference is spread over every individual successful transmission.

Fig. 8(c) displays the aggregate throughput of the secondary network versus the size of the secondary packet. The aggregate secondary network throughput increases with respect to the size of the secondary packet. For a successfully transmitted packet, the overhead (RTS, CTS, ACK, header, etc.) is constant. Therefore, if we increase the payload, we can have better throughput performance. Moreover, we can see that the rate of increase in throughput slows as the secondary packet size becomes larger, i.e., exponential c.d.f.

Fig. 8(d) depicts the average secondary packet delay versus the size of the secondary packet. From Fig. 8(d), we note that the average delay time increases linearly with the size of
the packet. Note that the expression of average delay already includes the waiting time for the secondary users while the channel is busy. From Fig. 8(c) and Fig. 8(d), we can see better overall secondary system throughput for larger secondary packets than smaller ones. However, larger secondary packets also increase the average delay of each individual SU. In other words, the larger the packet, the better the overall secondary system performance, the longer the individual packet delay.

For the second experiment, Fig. 9, we fixed the secondary packet size to 1023 bytes and vary the number of SUs in the system. We use the same parameters as in the first experiment with three special cases. Simulations results from Fig. 9 show some variances from our derived numerical expressions which is well within the acceptable range.

Fig. 9(a) depicts the transition probability of the channel versus the number of SUs. The numerical results show that the probability decreases to a minimum point and rises again. However, the simulations results cannot verify this because we make an increment to the number of SUs by five. Fig. 9(b) shows the expected accumulated interference versus the number of SUs. Similar to Fig. 9(a), the numerical results in Fig. 9(b) also depict the minimum points in the accumulated interference duration. Despite the variances, we can still use the results from Fig. 9(a) and Fig. 9(b) for throughput and delay analysis because they are well within the accepted error range.

Fig. 9(c) displays the aggregate throughput of the secondary network versus the number of SUs. As the number of SUs increases, there is more competition among SUs, and therefore, the aggregate system throughput of the secondary network decreases. Note that Fig. 9(c) displays the throughput of the overall secondary network, and thus, the decrease in the overall network performance is gradual.

Fig. 9(d) depicts the average secondary packet delay versus the number of SUs. From Fig. 9(d), the higher the number of SUs in the network, the more delay each individual SU suffers. Note that the average delay corresponds to the performance of each SU. Thus, the delay increases linearly with respect to the number of SUs. We want to mention that the results in Fig. 9(d) have slower convergence rate than those in Fig. 9(c) due to the fact that sampling of each SU device takes more time. From Fig. 9(c) and Fig. 9(d), we can observe that the characteristics of performance matrices remain similar to that of conventional DCF. However, from the supply and demand perspective, supply of spectrum, i.e., transmission opportunities, solely depends on the primary system and out of the control of the secondary system. The SUs can only choose the supplier, i.e., channel, with varying prices, i.e., delay in this case.

From Fig. 9, we can observe that the primary and secondary systems behave similarly to supply and demand market economics. For our previous two experiments, we fixed the average OFF and ON time periods of the primary system, i.e., fixed supply. Furthermore, we vary the secondary packet size and the number of SUs, i.e., the value of merchandise and competition among SUs. For our third experiment, we observe

![Fig. 8. (a) Transition probability of channel versus size of secondary packet, (b) expected accumulated interference versus size of secondary packet, (c) aggregate throughput of secondary network versus size of secondary packet, and (d) average secondary packet delay versus size of secondary packet, $W_0 = 32$, $m = 5$, $n = 50$.](image-url)
the effect of increasing supply, i.e., transmission opportunities.

For our third experiment, we only perform for the case where both OFF and ON periods are exponentially distributed. We run the experiment with two network sizes, \(n = 50\) and \(n = 100\). We vary the spectrum opportunities, \(P_0 \in [0, 1]\). When \(P_0 = 0\), there is no opportunity for the secondary network, and the primary system is fully utilizing the channel. On the other hand, when \(P_0 = 1\), no primary incumbent is present, and only the secondary network operates on the channel which is the case for conventional DCF. Fig. 10 depicts the average secondary packet delay versus the availability of the secondary network. Intuitively, the smaller the network, the shorter the duration of the secondary packet delay. From Fig. 10 it is very important to know that the secondary packet delay decreases exponentially with respect to the increase in channel availability. From Fig. 10 we can deduce that, given a fixed secondary network load, the secondary network will perform better when there are more spectrum opportunities, i.e., more supplies. We can further argue that SU devices are motivated to search for higher spectrum opportunities. From the point of view of SU devices, spectrum utilization is less important than its own profit: greater throughput and shorter packet delay.

B. Performance Comparison

We will now compare performance of OMF-MAC with CR-ALOHA and CR-CSMA protocols [16]. As introduced earlier in Section I CR-ALOHA and CR-CSMA protocols are both single channel MAC protocols similar to OMF-MAC. CR-CSMA even uses the similar backoff procedure as OMF-MAC. However, there are some fundamental differences especially in the spectrum sensing mechanisms. Both CR-ALOHA and CR-CSMA protocols employ a periodic spectrum sensing mechanism in which detection of primary and secondary signals is carried out separately. As depicted in Figs. 11-12, this increases the overhead significantly. In contrast, OMF-MAC uses a novel approach (ED and MF) where CCA is adapted to perform both primary and secondary signals detection. The proposed sensing mechanism design does not incur extra overhead compared to the conventional CCA because MF is only performed when BUSY channel is detected.

Fig. 11 displays the aggregate throughput versus the number of SUs in the network for different cases of channel availability: (a) \(P_0 = 0.7\), (b) \(P_0 = 0.6\) and (c) \(P_0 = 0.5\). OMF-MAC outperforms both the CR-ALOHA and CR-CSMA protocols. As can be observed from Fig. 11 all the throughput curves
of three protocols increases with respect to the increasing number of SUs up to their respective maximum values. After reaching their maximum values, the aggregate throughput of all three protocols decreases with respect to increasing number of SUs. This phenomenon can be explained in two portions as follows: For the first portion, the number of SUs in the network is small and thus the channel is not fully utilized. As the number of SUs increases, the aggregate throughput value also increases. In the second portion, as the number of SUs continues to increase, the collision between SUs also increases. This results in more wasted time and thus the aggregate throughput value decreases with respect to the increasing number of SUs. In comparing the increasing portion, OMF-MAC reaches its maximum value first, depicting better utilization of the available channel. In comparing the decreasing portion, the decreasing slope of OMF-MAC is more gradual compared to CR-ALOHA and CR-CSMA protocols depicting less increase in collisions than other protocols. For the network size $55 < n < 70$, aggregate throughput of OMF-MAC and CR-CSMA protocols are approximately the same.

Fig. 11 depicts the average delay versus the number of SUs in the network for different cases of primary user occupancy: (a) $P_0 = 0.7$, (b) $P_0 = 0.6$ and (c) $P_0 = 0.5$. The delay slope of OMF-MAC is the least among the three protocols for all three cases of primary user occupancy. Intuitively and as shown in Fig. 12 as the number of SUs increases, the delay increases for all three protocols. The average delay CR-ALOHA protocol increases exponentially with respect to the number of SUs whereas the average delay CR-CSMA and OMF-MAC increase linearly. As can be seen from Fig. 12 CR-CSMA protocol performs better than OMF-MAC for small network sizes, $n < 20$. However, as the number of SUs increases, OMF-MAC outperforms CR-CSMA protocol in terms of average delay for a successful packet transmission.

VI. Conclusion

In this paper, we have proposed the DCF-based OMF-MAC in which we design a spectrum sensing mechanism that can distinguish between the primary and secondary signals. Moreover, we model the system using a novel approach that bridges the gap between the renewal process model of interference and the Markov chain model of secondary users in the network. We derive the closed-form expression for the expected accumulated interference in terms of the secondary packet transmission time and give examples for some specific distributions. Using these, we derive the aggregate system throughput and average secondary packet delay. Our simulation results show that our model can closely estimate the
real network scenario. Our performance comparison displays that OMF-MAC performs better than existing protocols. We have proposed to apply our derived expressions to evaluate the real secondary network for configuration. Without any additional hardware, SUs employing our proposed OMF-MAC can distinguish the type of signal responsible for unsuccessful packets. Using this feature, OM-MAC clearly reduces secondary packet delay and improve aggregate throughput of the secondary network.

REFERENCES


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