Optimal Pricing for Duopoly in Cognitive Radio Networks: Cooperate or Not Cooperate?

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***Abstract*—Pricing is an effective approach for spectrum access control in cognitive radio (CR) networks. In this paper, we study the pricing effect on the equilibrium behaviors of selfish secondary users' (SUs') data packets which are served by a CR base station (BS). From the SUs' point of view, a spectrum access decision on whether to join the queue of the BS or not is characterized through an individual optimal strategy that is joining the queue with a joining probability. This strategy also requires each SU to know the average queueing delay, which is a non-trivial problem. Toward this end, we provide queueing delay analysis by using the M/G/1 queue with breakdown. From the BS's point of view, we consider a duopoly market based on the two paradigms: the opportunistic dynamic spectrum access (O-DSA) and the mixed O-DSA & dedicated dynamic spectrum access (D-DSA). In the first paradigm, two co-located opportunistic-spectrum BSs utilize freely spectrum-holes to serve SUs. Then, we show the advantages of the cooperative scenario due to the unique solution that can be obtained in a distributed manner by using the dual decomposition algorithms. For the second paradigm, there are one opportunistic-spectrum BS and one dedicated-spectrum BS. We study a price competition between two BSs as a Stackelberg game. The cooperative behavior between two BSs is modeled as a bargaining game. In both paradigms, bargain revenues of the cooperation are always higher than those due to competition in both cases. Extensive numerical analysis is used to validate our derivation.**

***Index Terms*—cognitive radio, duopoly, Stackelberg game, bargaining game, M/G/1 queue.**

I. Introduction

THE radio spectrum is one of the most scarce and valuable resources for wireless communications. However, some surveys that report on actual measurements show that most of the allocated spectrum is largely under-utilized [1]. Similar views on the under-utilization of the allocated spectrum were reported by the Spectrum-Policy Task Force appointed by Federal Communications Commissions (FCC) [2]. Cognitive radio (CR) has been proposed as a way to improve spectrum efficiency by exploiting the unused spectrum in dynamically changing environments [3]. In a cognitive radio network (CRN), there are two types of users, namely, primary user (PUs) and secondary user (SUs). In CRN, the transmission channel is licensed to the PUs while the SUs opportunistically access the channel resources when it is not occupied by any PU.

Among the various dynamic spectrum access (DSA), the opportunistic DSA (O-DSA) and dedicated DSA (D-DSA) have been widely used in the literature [4]. D-DSA allows the dedicated-spectrum base station ($\mathrm{BS^d}$) operate without interruption from PUs (i.e., no PUs operation). O-DSA, on the other hand, forces the opportunistic-spectrum base station ($\mathrm{BS^o}$) to provide secondary services without harming the operations of PUs on the leased spectrum. Here, the interruption of the operations of the $\mathrm{BS^o}$ is modeled as the break down of M/G/1 queueing system. In this paper, we study pricing-based spectrum access to control a queueing system in CRN. We consider an arrival process of SU customers (e.g., calls, packets or sessions), arriving at the $\mathrm{BS^o}$ and $\mathrm{BS^d}$. The base stations (BSs) control the service provision of SU customers through pricing-based methods with two market models: the O-DSA model and a mixed O-DSA & D-DSA model.

In the first market model, O-DSA, by considering SUs that share a PU's single channel, we examine the effect of the $\mathrm{BS^o}$'s pricing on the equilibrium behaviors of noncooperative SU customers. Due to the higher priority of PUs, when PUs occupy the channel, the $\mathrm{BS^o}$ stops serving SU customers, i.e. the $\mathrm{BS^o}$ has a breakdown. Therefore, the $\mathrm{BS^o}$ oscillates between two states of ON/OFF as illustrated in Fig.1. Each SU customer can make a decision about whether to join the queue or to leave the queue, e.g., by discarding the packet. The waiting time in the queue incurs a cost. Certainly, there are situations in which the demand of a service is relatively inflexible, then, in such cases, SU customers can have a rule as follows: a SU customer will join the queue if the benefit to him/her of being served exceeds the cost of the average waiting time he/she experiences. Then, there are three questions to answer: first, given an admission price charged by the $\mathrm{BS^o}$, what is the individual optimal strategy of SU customers?; second, what is the pricing strategy of the $\mathrm{BS^o}$ to maximize its revenue in the monopoly O-DSA market (i.e., a market dominated by only one BS)?; and third, what are the pricing strategies in the duopoly O-DSA market (i.e., a market dominated by two BSs)?

Considering the first question from the SU customers' viewpoint, we first introduce an individual optimal strategy,

Manuscript received July 29, 2013; revised December 8, 2013; accepted February 2, 2014. The associate editor coordinating the review of this paper and approving it for publication was M. C. Vuran.

This research was partially supported by the Ministry of Science, ICT & Future Planning (MSIP), Korea, under the ITRC support program supervised by the NIPA (NIPA-2013-(H0301-13-2001)) and by Next-Generation Information Computing Development Program through the NRF funded by the MISP (2010-0020728). Dr. CS Hong is the corresponding author.

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Digital Object Identifier 10.1109/TWC.2013.131363

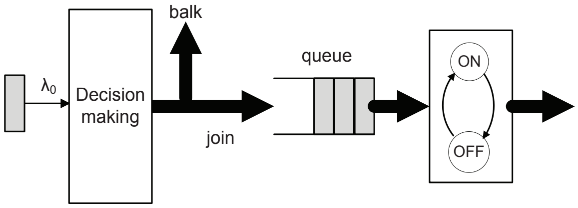


Fig. 1. Illustration of queuing system with breakdowns.

in which each SU customer as a player in a non-cooperative game makes its spectrum access decision based on its utility function that captures the queueing delay. We next show that there exists a unique symmetric Nash equilibrium of this game. In order to evaluate SUs' average queueing delay; we use M/G/1 queue with the server breakdowns model. Taking into account the BS's strategy in the second question, we use a revenue-optimal pricing policy to maximize the $\mathrm{BS^o}$'s revenue by solving a convex optimization problem. In order to answer the third question, we assume that two $\mathrm{BS^o}s$ may compete against each other. Game theory can also be used here, we can derive the Nash equilibrium solution in the O-DSA market. On the other hand, two $\mathrm{BS^o}s$ can cooperate in order to enhance network utilization. Then, the bargaining game is firstly used to answer how two $\mathrm{BS^o}s$ should cooperate. Furthermore, the Nash bargaining equilibrium of the price can be obtained in a distributed manner by using the dual decomposition algorithm.

In the second market model, mixed O-DSA and D-DSA, one $\mathrm{BS^d}$ and one $\mathrm{BS^o}$ interact with each other by varying their admission price. The Stackelberg equilibrium of the price in the mixed O-DSA and D-DSA is derived for competitive behavior. On the other hand, when the $\mathrm{BS^d}$ and $\mathrm{BS^o}$ are cooperative, we need to solve the bargaining problem. Unfortunately, the general bargaining problem is not a convex problem. However, by setting appropriate bargaining parameters, we prove that the bargaining problem with appropriate parameters is a convex problem and the Nash bargaining equilibrium of the price can be obtained in a distributed manner by using the dual decomposition technique.

The remainder of this paper is organized as follows. In Section II, we discuss related works. The system model is introduced in Section III. In Section IV, the expected queueing delay and the individual optimal behaviors of SU customers are derived. The non-cooperative and cooperative duopoly of O-DSA are discussed in the Section V. The duopoly market of the mixed O-DSA and D-DSA model is analyzed in Section VI. Finally, conclusions are given in Section VII.

II. Related works

In this work, we focus on the pricing strategy and its impact on the equilibrium strategy of SU customers and BSs in a queueing system, which can be traced from the original work of [5], [6], [7], [8]. Recent works such as [9] and [10] are categorized into pricing approaches in spectrum access control in CRN. There are several existing works that consider either the observable or unobservable queueing model. The observable queue models in [9] and [11] either require a centralized control server or a feedback mechanism with time overhead. In [9], the authors found the socially optimal strategy, from the viewpoint of each customer, in a CRN in which the server suffers from service interruption. However, the shortcoming is that each SU must observe the queue length to make a decision, whether to join the queue or not. The current queue length can be received by a broadcast packet from the BS. But the queue length is normally dynamic and changes rapidly. We, however, use an unobservable queue case [10], [12], [13] which models appropriately the non-cooperative and distributed nature of CRN where SUs have no information about each other.

In these queueing models, multiple service interruptions have also been examined in terms of server vacations or breakdowns models [9] and [10]. The work in [10] used the continuous model; however, the services time was restricted to the exponential distribution for ease of analysis. In [9], the authors used the discrete-time model where all distributions of arrivals and services were simply limited to be Binomial distributions. In [12], the authors have modeled the channel ON-OFF process by using renewal theory. To obtain the expected queueing delay, the authors must perform a Laplace transform, which requires the full information of the probability density function (pdf) of service time of PUs and SUs, respectively. To the best of our knowledge, we are the first to use M/G/1 queue subject to breakdowns where the PUs and SUs service time distributions can be of a general distribution. We model the channel ON-OFF process as the breakdown process of the $\mathrm{BS^o}s$. Therefore, we only require the first and second moment of service time of PUs and SUs.

Among the various DSA approaches, the O-DSA models have been widely considered in [9], [12], [13]. However, we firstly investigate the duopoly bargaining problem in the pricing-based approaches in CRN where two $\mathrm{BS^o}s$ are cooperative. On the other hand, the D-DSA models have not been discussed broadly except in [10] and [14]. Elias et al. have used a simple M/M/1 queue model in order to focus on the price of anarchy and the dynamic behavior of network users by using population games and replicator dynamics in [14]. In this paper, as far as we know, we are the first to address the cooperation between $\mathrm{BS^o}$ and $\mathrm{BS^d}$ in the mixed O-DSA & D-DSA model by using the Nash bargaining solution.

III. System Model

In this section, we first introduce the O-DSA model and server breakdown from PUs. Then, we explain about the D-DSA model.

A. O-DSA Model

We start by defining the model for a system with a single PU's channel. The PUs' channel oscillates between two states of ON and OFF. Suppose that when the PUs' channel is ON, PUs would release the channel at an exponential rate β. With perfect sensing, the probability of the CR base station will be able to serve a SU customer for an additional time z without breaking down is $e^{-\beta z}$. Once the PU occupies the channel, the service time of PU is assumed to be a random variable X, with the pdf $f_X(x)$. Assume that the SU customer is serving by the CR base station when a breakdown occurs, can

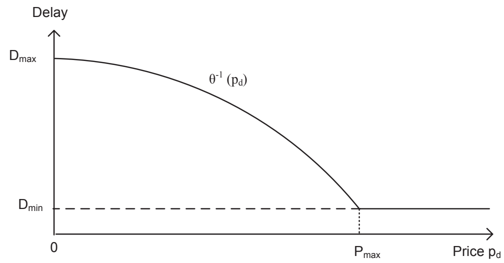


Fig. 2. An example of delay function $\theta^{-1}(p_d)$.

resume the unfinished transmission instead of retransmitting the whole connection [15]. The service time of SU customers is a random variable Y, with the pdf $f_Y(y)$. The total number of SU customers arrive at the network according to a Poisson process with arrival rate Λ. With these above aspects, the spectrum usage model in this paper is based on the M/G/1 break down model [16].

B. D-DSA Model

We assume the BS$^{\text{d}}$ can lease a part of the dedicated spectrum. This spectrum chunk is divided into multiple bands, each of which has the same bandwidth as the single band of the BS$^{\text{o}}$. Since there is no PU traffic on these dedicated bands, the SU services are not interrupted in this case. We consider that the BS$^{\text{d}}$ always has sufficient numbers of dedicated bands. The SU customers' service times are exponential with parameter θ. Then, the expected queueing delay of SU customers is equal to $1/\theta$. In both previous works [10] and [14], the authors assume that the parameter $1/\theta$ is constant although the SU customers pay different prices for admission. However, in this paper, we assume that the expected queueing delay in D-DSA $1/\theta$ is a concave function of the admission price p_d. The higher admission price, p_d, the more leased dedicated bandwidth can be used for serving SU customers; consequently, the less expected queueing delay SU customer is. For example, the expected queueing delay $1/\theta$ can be expressed as a concave function as follows

$$\theta^{-1}(p_d) = \zeta \log(e^{\frac{D_{max}}{\zeta}} - p_d), p_d \in [0, P_{max}], \quad (1)$$

where $\zeta > 0$ is the predefined delay sensitivity level. Fig. 2 illustrates the expected queueing delay function $\theta^{-1}(p_d)$, where D_{max} is the maximum delay that the SU customer can tolerate and D_{min} is the minimal expected queueing delay that the BS$^{\text{d}}$ can provide to SU customers due to bandwidth limitation. Here, P_{max} is the maximum admission price charged by the BS$^{\text{d}}$ and can be obtained as $P_{max} = e^{\frac{D_{max}}{\zeta}} - e^{\frac{D_{min}}{\zeta}}$. That is, when the SU customers pay more than P_{max} for admission, the BS$^{\text{d}}$ cannot supply better services with lower expected queueing delay than D_{min}. This assumption is reasonable due to the fact that the SU customer paying more should obtain better service (e.g. service with a lower-than-expected queueing delay).

IV. INDIVIDUALLY OPTIMAL STRATEGY

In this section, we discuss the optimal strategy of each SU customer. We first explain about the SU's individual utility. Then, we analyze the expected queueing delay and derive the individually optimal strategy.

A. SU's Individual Utility

When an SU customer wants to be served at the BS$^{\text{o}}$, the SU decides whether to let the SU customer to join the BS$^{\text{o}}$'s queue or leave it. A first-in-first-out (FIFO) rule can be implemented in the queue of the BS$^{\text{o}}$. There exists a waiting cost of C units per time unit, which is continuously accumulated from the time that the SU customer arrives at the system until the time the SU customer leaves after being served. In practical systems, the cost C represents the penalty for the delay or traffic congestion. The admission fee, p, is charged by the BS$^{\text{o}}$ as the subscriber fee (i.e., SUs are price-takers). Every SU customer receives a reward or a service value of R units for finishing with a service. For example, given the admission price p_d of the BS$^{\text{d}}$, the reward R equals $p_d + C\theta^{-1}(p_d)$, that is, the cost that SU customers pay to obtain service from the BS$^{\text{d}}$ when SU customer choose balk from the BS$^{\text{o}}$. We assume that the SU customers' decisions are made only at their arrival time. Similar to [9], the net benefit of an SU customer that stays in the system for $\overline{T}$ time units and successfully finishes the service is

$$U = R - C\overline{T} - p. \quad (2)$$

Obviously, the net benefit could be negative when the delay $\overline{T}$ is sufficiently large. We assume that the SU customer will choose to join the queue if the net benefit is not negative. If the SU customer chooses not to join the queue, the corresponding net benefit will be zero. In order to perform the SU customer's individual optimal strategy, each SU customer must estimate the mean queueing delay, which will be analyzed in the following subsection.

B. Queueing Delay Analysis

We use the M/G/1 queueing model with breakdowns to analyze the average queueing delay $\overline{T}$ (waiting time + serving time). By using the traffic parameters (i.e., SU customers' arrival rate λ, PUs occupy the channel at an exponential rate β, the pdf of the service time of PU $f_X(x)$ and the pdf of the service times of SU customers $f_Y(y)$), which are assumed to be estimated by existing methods [17], the average queueing delay $\overline{T}(\lambda)$ induced by arbitrary SU customers' arrival rate λ at the BS$^{\text{o}}$ is analyzed as follows.

Due to multiple breakdowns at the BS$^{\text{o}}$, the original service time of the SU customer is increased as illustrated in Fig. 3. We call this increased service time as the effective service time which is denoted by a random variable Y_e. Then, the M/G/1 queueing system with server breakdowns can be represented as the M/G/1 queue with its average service time $E[Y_e]$. Moreover, this queue is stable when the condition $\lambda < 1/E[Y_e]$ is satisfied.

We start the analysis by denoting $\overline{W}(\lambda)$ as the average waiting time in the queue induced by arrival rate λ. Then, we obtain

$$\overline{T}(\lambda) = \overline{W}(\lambda) + E[Y_e]. \quad (3)$$

Due to the Pollaczek-Khinchin formula [18], the average waiting time is calculated as follows

$$\overline{W}(\lambda) = \frac{\lambda E[Y_e^2]}{2(1-\lambda E[Y_e])}. \tag{4}$$

Then, the problem requires the derivation of $E[Y_e]$ and $E[Y_e^2]$.

1) $E[Y_e]$ Derivation: Let $N(y)$ denote the number of times that the BS breaks down while it is serving the SU customer given that the service time of the SU customer requires y units; then we assume $X_1, X_2, \ldots, X_{N(y)}$ are, respectively, the amounts of time of the different PUs who are occupying the channel. Then, we have

$$Y_e = \sum_{i=1}^{N(y)} X_i + y, \tag{5}$$

where the number $N(y)$ of PUs occurs in $(0,y)$ is a Poisson random variable with mean βy. Thus, the random variable $S = \sum_{i=1}^{N(y)} X_i$ is a compound Poisson random variable with Poisson mean βy. We have

$$E[S] = \beta y E[X], \tag{6}$$

$$\mathrm{Var}[S] = \beta y E[X^2]. \tag{7}$$

Therefore, the conditional expectation of Y_e given $Y = y$ is

$$E[Y_e|Y=y] = E\left[\sum_{1}^{N(y)} X_i|Y=y\right] + y = E[S] + y = \beta y E[X] + y. \tag{8}$$

Therefore, the unconditional expectation of Y_e is obtained as follows

$$E[Y_e] = E[Y(1+\beta E[X])] = E[Y](1+\beta E[X]). \tag{9}$$

2)$E[Y_e^2]$ Derivation: Similarly, the conditional variance of Y_e given $Y = y$ is

$$\mathrm{Var}[Y_e|Y=y] = \mathrm{Var}\left[\sum_{1}^{N(y)} X_i|Y=y\right] = \mathrm{Var}(S) = \beta y E[X^2]. \tag{10}$$

Using the conditional variance, we have

$$\begin{aligned}\mathrm{Var}[Y_e] &= E[\mathrm{Var}[Y_e|Y]] + \mathrm{Var}[E[Y_e|Y]] \\ &= \beta E[Y]E[X^2] + (1+\beta E[X])^2 \mathrm{Var}[Y].\end{aligned} \tag{11}$$

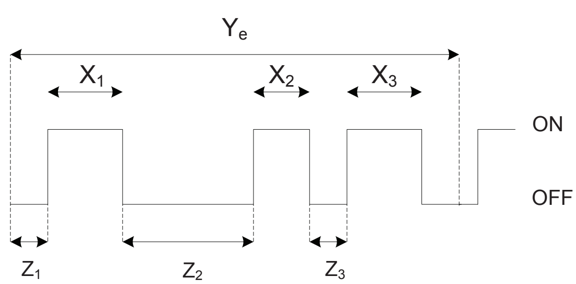


Fig. 3. A sample ON-OFF process with a realization of an effective service time Y_e where the PUs' channel at state OFF will continue being OFF for an additional time Z_1, Z_2 and Z_3 before changing to state ON. X_1, X_2 and X_3 are corresponding ON periods, respectively.

Then, using (9), we obtain

$$E[Y_e^2] = \mathrm{Var}[Y_e] + (E[Y_e])^2 = \beta E[Y]E[X^2] + (1+\beta E[X])^2 E[Y^2]. \tag{12}$$

3) The expected queueing delay $\overline{T}(\lambda)$: characteristics and examples with analysis and simulation comparisons. Using (3) and (4), we obtain the final results as follows

$$\overline{T}(\lambda) = \frac{\lambda E[Y_e^2]}{2(1-\lambda E[Y_e])} + E[Y_e], \tag{13}$$

where $E[Y_e]$ and $E[Y_e^2]$ are defined by (9) and (12), respectively. Note that the stable condition of the queue is $\lambda < 1/E[Y_e] = \frac{1}{E[Y](1+\beta E[X])}$.

In order to characterize the function $\overline{T}(\lambda)$, let us consider its first and second derivatives in the interval $(0, 1/E[Y_e])$. Then, we easily prove that $\overline{T}'(\lambda) > 0$ and $\overline{T}''(\lambda) > 0$. Hence, $\overline{T}(\lambda)$ is a convex and strictly increasing continuous function in $(0, 1/E[Y_e])$.

We give a comparison between analysis and simulation through three following cases.

1) The first case is that all X and Y have the exponential distributions with $f_X(x) = \mu_X e^{-\mu_X x}$ and $f_Y(y) = \mu_Y e^{-\mu_Y y}$, respectively. This combination is called the *Exp case*, and we obtain

$$E[Y_e] = \frac{1}{\mu_Y}\left(1 + \frac{\beta}{\mu_X}\right), \tag{14}$$

$$E[Y_e^2] = \frac{2}{\mu_Y^2} + \frac{2\beta^2}{\mu_X^2 \mu_Y^2} + \frac{2\beta}{\mu_X^2 \mu_Y} + \frac{4\beta}{\mu_X \mu_Y^2}. \tag{15}$$

2) The second case is that all X and Y have the Erlang distribution with $f_X(x) = \mu_X^2 x e^{-\mu_X x}$ and $f_Y(y) = \mu_Y^2 y e^{-\mu_Y y}$, respectively. This combination is called the *Erl case*, and we have

$$E[Y_e] = \frac{1}{\mu_Y}\left(1 + \frac{\beta}{\mu_X}\right), \tag{16}$$

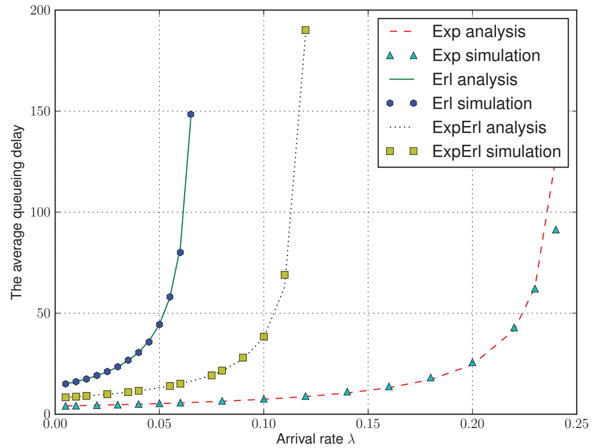
$$E[Y_e^2] = \frac{6}{\mu_Y^2} + \frac{12\beta}{\mu_X^2 \mu_Y} + \frac{12\beta}{\mu_X \mu_Y^2} + \frac{24\beta^2}{\mu_X^2 \mu_Y^2}. \tag{17}$$

3) The third case is called the *ExpErl case*: X has the exponential distribution with $f_X(x) = \mu_X e^{-\mu_X x}$ and Y has the Erlang distribution with $f_Y(y) = \mu_Y^2 y e^{-\mu_Y y}$. We obtain

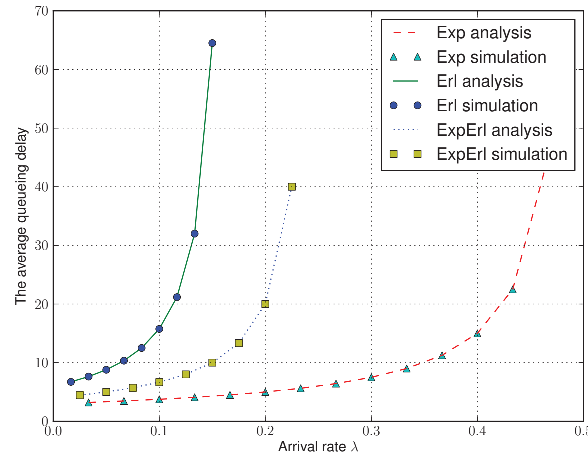
$$E[Y_e] = \frac{2}{\mu_Y}\left(1 + \frac{\beta}{\mu_X}\right), \tag{18}$$

$$E[Y_e^2] = \frac{6}{\mu_Y^2} + \frac{4\beta}{\mu_X^2 \mu_Y} + \frac{12\beta}{\mu_X \mu_Y^2} + \frac{6\beta^2}{\mu_X^2 \mu_Y^2}. \tag{19}$$

In order to demonstrate our queueing analysis, we simulate a single-server queue subject-to-server break down. We fix $\mu_x = 0.5$, $\mu_Y = 1$ in all of the Exp, Erl and ExpErl cases. The comparison between analysis and simulation is presented in two scenarios: Fig. 4(a) illustrates a PUs heavy traffic model in urban areas with $\beta = 1.5$, while Fig. 4(b) represents for a PUs light traffic model in rural areas with $\beta = 0.5$. As can be seen from these two figures, the queueing delays of the PU's heavy traffic model are higher than those of the PU's light traffic model. Despite the variation of numerical settings,



(a) PUs' heavy traffic model $\beta = 1.5$, $\mu_X = 0.5$ and μ_Y=1.



(b) PUs' light traffic model $\beta = 0.5$, $\mu_X = 0.5$ and μ_Y=1.

Fig. 4. Average queueing delay performance comparison

Fig. 4 shows that our analysis correctly coincides with the simulation results.

C. Individual Equilibrium Strategy

In this subsection, we investigate the SU customers' strategies based on the queueing delay estimation, the Nash equilibrium, and the equilibrium convergence.

We consider a stream of potential arriving SUs who are self-optimizing, which means that each SU customer concerned only with his or her own benefit. Specifically, upon arrival, each potential SU customer has to make an individual decision about whether to join the queueing system or balk with the goal of obtaining a non-negative expected net benefit. In the context of game theory, the potential SU customers behave like players in a noncooperative game, and the decisions about joining or balking are their strategies.

We start by analyzing SU customers' behavior in the equilibrium when the potential SU customer arrival rate is Λ (i.e, the arrival rate of SU customers who intend to access the BS^{O}).

A definition of an individual optimal strategy is provided as follows. We consider the SU customers' strategies described by a probability q which is the probability an SU customer decides to join the queue (thus, with probability $1-q$ the SU customer decides to leave the queue).

Since SU customers are assumed to be selfish, they will individually and selfishly choose q: each SU customer wants to obtain a non-negative expected net benefit. The net benefit for an SU customer who joins the queue and finishes his or her service with effective arrival rate λ (i.e., the arrival rate of SU customers who have already decided to join the queue) is: $U = R - C\overline{T}(\lambda) - p$. The SU customer who balks receives zero net benefit. For a given effective arrival rate λ, the individually optimizing SU customer who joins with probability q receives an expected net benefit as follows

$$q(R - C\overline{T}(\lambda) - p) + (1-q)0 = q(R - C\overline{T}(\lambda) - p). \tag{20}$$

To avoid a trivial solution, we make the following assumption: $p + C\overline{T}(0) < R$. Motivated by the concept of symmetric Nash equilibrium, we define an individually optimal or equilibrium joining probability q_e (and associated equilibrium arrival rate $\lambda_e = q_e\Lambda$), by the property that no individual SU customer trying to obtain a non-negative expected net benefit has any incentive to deviate unilaterally from joining probability $q_e(\lambda_e)$. Then, given an admission price p, we have two cases:

1) $p + C\overline{T}(\Lambda) \le R$. Thus, all SU customers will join with probability $q_e = 1$, and hence their expected utility is $R - C\overline{T}(\Lambda) - p \ge 0$.
2) $p + C\overline{T}(0) < R < p + C\overline{T}(\Lambda)$. Since the average queueing delay $\overline{T}(\lambda)$ is a continuous and monotonically increasing function with variable effective arrival rate λ, given p, there exists a unique equilibrium arrival rate $\lambda_e(p)$ such that $R = p + C\overline{T}(\lambda_e)$ as follows

$$\lambda_e(p) = \frac{2(R - p - CE[Y_e])}{2RE[Y_e] - 2pE[Y_e] + CE[Y_e^2] - 2CE[Y_e]^2}. \tag{21}$$

For a given effective arrival rate $\lambda_e(p)$, the expected net benefit is

$$q(R - C\overline{T}(\lambda_e) - p) = 0, \tag{22}$$

and it does not depend on the joining probability q. Thus, SU customers are indifferent among all joining probability q such that $0 \le q \le 1$, so that they have no incentive to deviate from the joining probability

$$q_e = \frac{\lambda_e(p)}{\Lambda} = \frac{\frac{2(R-p-CE[Y_e])}{2RE[Y_e]-2pE[Y_e]+CE[Y_e^2]-2CE[Y_e]^2}}{\Lambda}. \tag{23}$$

We supplement the individual equilibrium strategy analysis with the numerical results by different cases. The relationship between SU individual arrival rate $\lambda_e(p)$ and admission price p is described in Fig. 5. The more the price increases, the less the SU customers enter the system. Therefore, we can conclude that the pricing mechanisms can be used by the BS^{O} to regulate the SU customer arrival rate to obtain a specific objective.

Equilibrium Convergence: We consider a discrete-time model with time periods indexed $t = 1, 2, \ldots$. At each period t, the SU customers' joining probability is q^t during a period t,

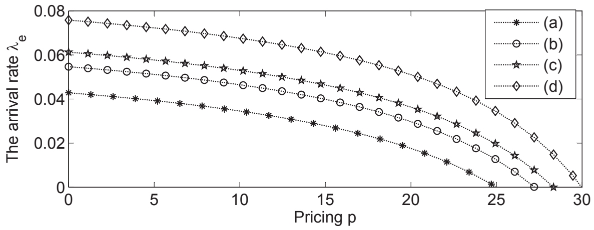


Fig. 5. SU customer individual arrival rate λ_e vs. pricing p with $R=40, C=1$ and the Erl channel in four case: (a) $\beta=2$, $\mu_Y=1.2$ and $\mu_X=0.5$; (b) $\beta=2$, $\mu_Y=1.2$ and $\mu_X=0.6$; (c) $\beta=1.5$, $\mu_Y=1.2$ and $\mu_X=0.5$; (d) $\beta=1.5$, $\mu_Y=1.2$ and $\mu_X=0.6$.

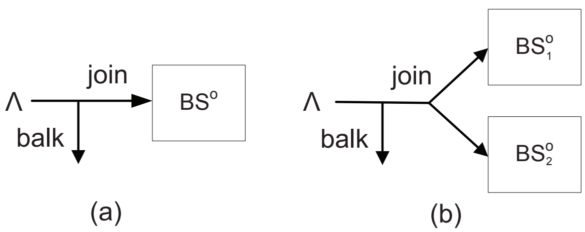


Fig. 6. SU customer market in the O-DSA model: (a) Monopoly and (b) Duopoly.

which is assumed to last sufficiently for the system to reach the stable state. From the same initial joining probability q^0, the dynamics of SU customers' joining probability can be updated via a gradient algorithm as follows

$$q^{t+1} = \left[q^t - \alpha(t)F'(q^t)\right]_0^1 = \left[q^t - \alpha(t)\overline{T}(q^t\Lambda)\right]_0^1, \tag{24}$$

where $[x]_0^1$ denotes the projection of x on $[0,1]$ and the function $F(q)$ is defined as

$$F(q) = \int_o^q \left[\overline{T}(x\Lambda) - \frac{R-p}{C}\right] dx \tag{25}$$

Since $\overline{T}(q\Lambda)$ is a convex function with respect to q, $F(q)$ is a convex function. When $F'(q_e)=0$, $F(q)$ has the minimum point at q_e. Thus, with appropriate step sizes $\alpha(t)$, the iteration in (24) converges to the equilibrium joining probability q_e for any starting point $q^0 \in [0,1]$ [27].

V. Monopoly and Duopoly in O-DSA market model

This section answers the question: what are pricing strategies in the duopoly scenario in the O-DSA model? In order to understand the behavior of $\mathrm{BS^O}s$ in the duopoly market, we introduce the individual optimal pricing strategy of a single $\mathrm{BS^O}$ who aims to maximize its own revenue in a monopoly market. In particular, the SU customer will make its decision to join or balk based on the prices charged by the $\mathrm{BS^O}$ as illustrated by Fig. 6(a). Then, we discuss the duopoly model by two scenarios into two parts: i) two $\mathrm{BS^O}s$ are competitive; ii) two $\mathrm{BS^O}s$ are cooperative through bargaining in the O-DSA model. In the O-DSA duopoly market (cf. Fig. 6(b)), there are two O-DSA base stations denoted by $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$, and SU customers make a decision to join either $\mathrm{BS}_1^{\mathrm{O}}$ or $\mathrm{BS}_2^{\mathrm{O}}$ (or neither).

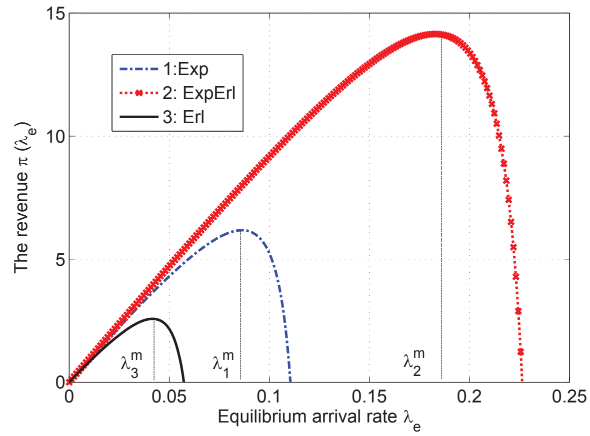


Fig. 7. The revenue vs. equilibrium arrival rate λ_e in three examples with the parameters: $R=100$, $C=1$, $\beta=2$, $\mu_Y=1.2$ and $\mu_X=0.5$; (1) the channel follows the ExpErl case; (2) the channel follows the Exp case; (3) the channel follows the Erl case.

A. Monopoly Market: BS^O's Revenue Maximizing

We assume that there is one $\mathrm{BS^O}$. We consider the system from the point of view of the $\mathrm{BS^O}$ whose goal is to set an admission price to maximize its revenue. Specifically, when charging a price p, the revenue of the $\mathrm{BS^O}$ can be defined as $\pi(p)=\lambda_e(p)p$, and the revenue maximizing problem is expressed as

$$\begin{aligned} \max_{p\geq 0} \quad & \pi(p)=\lambda_e p \\ \text{s.t.} \quad & p = R - C\overline{T}(\lambda_e). \end{aligned} \tag{26}$$

In order to transform the problem (26) into a convex form, we change the variable p to λ_e and obtain an equivalent problem as follows

$$\begin{aligned} \max_{\lambda_e} \quad & \pi(\lambda_e)=\lambda_e\left[R - C\overline{T}(\lambda_e)\right] \\ \text{s.t.} \quad & 0 \leq \lambda_e \leq \min\{\Lambda, 1/E[Y_e]\}. \end{aligned} \tag{27}$$

Since $\overline{T}(\lambda_e)$ is a convex and increasingly continuous function, $\pi(\lambda_e)$ is a strictly concave function in the interval $(0, 1/E[Y_e])$. Thus, we obtain the unique optimal solution λ_e^m by setting the first derivative of $\pi(\lambda_e)$ to zero. Then, we have

$$\lambda_e^m = \min\{\frac{1}{E[Y_e]} - \frac{\sqrt{CE[Y_e^2]\Omega}}{E[Y_e]\Omega}, \Lambda\}, \tag{28}$$

where $\Omega = CE[Y_e^2] + 2RE[Y_e] - 2CE[Y_e]^2$. The optimal price p^m of (26) is given as follows

$$p^m = R - C\overline{T}(\lambda_e^m). \tag{29}$$

In conclusion, by setting the admission price p^m and SUs employ the individual optimal strategy, the $\mathrm{BS^O}$ can regulate the arrival rate of SU customers at rate λ_e^m such that it achieves the maximum revenue $\pi^m = \lambda_e^m p^m$.

Numerical results: In order to examine the shape of the revenue function $\pi(\lambda_e)$, we provide three examples. The shapes of the revenue function $\pi(\lambda_e)$ are shown in Fig. 7. All revenue functions are concave and obtain the maximum at $(\lambda_1^m, \lambda_2^m, \lambda_3^m) = (0.086, 0.183, 0.042)$, respectively.

B. Duopoly Market: Non-cooperative Model

We consider a duopoly market where BS_1^O and BS_2^O compete with each other by setting the admission price to maximize their revenues as shown in Fig. 6(b). We assume that the arriving SU customers are individual optimizers. Then, given a particular admission price p_i $(i=1,2)$ of the BS_i^O $(i=1,2)$, the SU customer's equilibrium arrival rate λ_i at the BS_i^O satisfies the equilibrium conditions $\lambda_i(R - C\overline{T}_i(\lambda_i) - p) = 0$. As a player, for a given admission price p_1 of the BS_1^O, the BS_2^O will determine the best reply admission price p_2. Motivated by the concept Nash equilibrium, we define equilibrium admission prices (p_1^{nc}, p_2^{nc}), due to the property that no BS_i^O trying to maximize its own revenue has any incentive to deviate unilaterally from the value of its admission price. In this noncooperative game, we assume that the BS^Os (i.e., players) know the other's utility function so that they can determine the Nash Equilbirum by using the following procedure.

Both BS_1^O and BS_2^O fix their admission prices simultaneously. Given the admission price p_2, then the best response of BS_1^O that maximizes the revenue at BS_1^O is obtained as follows.

$$\begin{aligned} \max_{p_1 \geq 0, \lambda_1} \quad & \pi_1(\lambda_1) = \lambda_1 p_1 \\ \text{s.t.} \quad & R = p_1 + C\overline{T}_1(\lambda_1), \\ & \lambda_1 + \lambda_2 \leq \Lambda, \\ & 0 \leq \lambda_1 \leq \frac{1}{E[Y_1](1+\beta_1 E[X_1])}, \end{aligned} \tag{30}$$

where Λ is the total arrival rate of all SU customers.

Similarly, given the admission price p_1, the best response of BS_2^O that maximizes the revenue at BS_2^O is given as follows

$$\begin{aligned} \max_{p_2 \geq 0, \lambda_2} \quad & \pi_2(\lambda_2) = \lambda_2 p_2 \\ \text{s.t.} \quad & R = p_2 + C\overline{T}_2(\lambda_2), \\ & \lambda_1 + \lambda_2 \leq \Lambda, \\ & 0 \leq \lambda_2 \leq \frac{1}{E[Y_2](1+\beta_2 E[X_2])}. \end{aligned} \tag{31}$$

We divide two cases in terms of variable Λ by the critical point $\lambda_1^m + \lambda_2^m$, where λ_1^m and λ_2^m is the optimal solution of the revenue maximizing of the monopoly in (27).

(1) Case 1: $\lambda_1^m + \lambda_2^m \leq \Lambda$. Both $\pi_1(\lambda_1)$ and $\pi_2(\lambda_2)$ are concave functions with the maximum value $\pi_1(\lambda_1^m)$ and $\pi_2(\lambda_2^m)$ since $\overline{T}_1(\lambda_1)$ and $\overline{T}_2(\lambda_2)$ are convex and strictly increasing continuous functions. Then, the optimal solutions of (30) and (31) are $(\lambda_1^m, \lambda_2^m)$ which are also the Nash equilibrium arrival rate $(\lambda_1^{nc}, \lambda_2^{nc})$. Thus, the Nash equilibrium admission prices are $p_1^{nc} = R - C\overline{T}_1(\lambda_1^m)$ and $p_2^{nc} = R - C\overline{T}_2(\lambda_2^m)$.

(2) Case 2: $\Lambda < \lambda_1^m + \lambda_2^m$. We have the theorem as follows.

Theorem 1: The optimal equilibrium solutions $(\lambda_1^{nc}, \lambda_2^{nc})$ of (30) and (31) must satisfy $\lambda_1^{nc} + \lambda_2^{nc} = \Lambda$.

Proof: We assume that there exists an optimal equilibrium solution (λ_1, λ_2) such that $\lambda_1 + \lambda_2 < \Lambda < \lambda_1^m + \lambda_2^m$ and $(\pi_1(\lambda_1); \pi_2(\lambda_2))$ is the maximum value. We have either $\lambda_1 < \lambda_1^m$ or $\lambda_2 < \lambda_2^m$ (or both). Suppose we have $\lambda_1 < \lambda_1^m$. Due to strict concavity of $\pi_1(\cdot)$, $\pi_1(\cdot)$ is increasing in (λ_1, λ_1^m). Denote $\lambda_1' = \min\{\lambda_1^m, \Lambda - \lambda_2\}$, then $\pi_1(\lambda_1') > \pi_1(\lambda_1)$ and $\lambda_1' + \lambda_2 \leq \Lambda$. Therefore, by unilaterally changing from λ_1 to λ_1', we have a better solution (λ_1', λ_2) such that $\pi_1(\lambda_1') > \pi_1(\lambda_1)$. Therefore, (λ_1, λ_2) cannot be the optimal solution, which contradicts with our assumption. On the other hand, if $\lambda_1 + \lambda_2 = \Lambda$, we cannot improve $\pi_1(\lambda_1)$ or $\pi_2(\lambda_2)$ by replacing λ_1 by $\lambda_1' = \min\{\lambda_1^m, \Lambda - \lambda_2\}$. Therefore, we have proved theorem 1. ■

Using Theorem 1, problem (31) can be rewritten as follows

$$\begin{aligned} \max_{p_2 \geq 0, \lambda_2} \quad & \pi_2(\lambda_2) = \lambda_2 p_2 \\ \text{s.t.} \quad & R = p_2 + C\overline{T}_2(\lambda_2), \\ & \lambda_1 + \lambda_2 = \Lambda, \\ & 0 \leq \lambda_2 \leq \frac{1}{E[Y_2](1+\beta_2 E[X_2])}. \end{aligned} \tag{32}$$

Using the first equality constraint of (30) and (32), we can rewrite the problem (32) as follows

$$\begin{aligned} \max_{p_2 \geq 0, \lambda_2} \quad & \lambda_2(p_1 + C\overline{T}_1(\Lambda - \lambda_2) - C\overline{T}_2(\lambda_2)) \\ \text{s.t.} \quad & p_1 + C\overline{T}_1(\lambda_1) = p_2 + C\overline{T}_2(\lambda_2), \\ & 0 \leq \lambda_2 \leq \frac{1}{E[Y_2](1+\beta_2 E[X_2])}. \end{aligned} \tag{33}$$

The above optimization is solved by differentiating the objective function with respect to λ_2 to determine the (necessary) first-order condition for the value of λ_2 to be optimal value such as

$$p_2 = \lambda_2(C\overline{T}_1'(\Lambda - \lambda_2) + C\overline{T}_2'(\lambda_2)). \tag{34}$$

Similarly, using the symmetric relation, the first-order condition for the value of λ_1 to be optimal given the admission price p_2 is obtained as follows

$$p_1 = \lambda_1(C\overline{T}_1'(\lambda_1) + C\overline{T}_2'(\Lambda - \lambda_1)). \tag{35}$$

Combining (34) and (35), we have

$$p_1 - p_2 = (2\lambda_1 - \Lambda)(C\overline{T}_1'(\lambda_1) + C\overline{T}_2'(\Lambda - \lambda_1)). \tag{36}$$

From the first equality constraint of (30) and (32), we obtain

$$p_1 - p_2 = C\overline{T}_2(\Lambda - \lambda_1) - C\overline{T}_1(\lambda_1). \tag{37}$$

Then, we finally obtain

$$\overline{T}_2(\Lambda - \lambda_1) - \overline{T}_1(\lambda_1) = (2\lambda_1 - \Lambda)(\overline{T}_1'(\lambda_1) + \overline{T}_2'(\Lambda - \lambda_1)). \tag{38}$$

Finding the solution of (38) is equivalent to finding the root of $G(\lambda_1) = 0$, where $G(\lambda_1) = \overline{T}_2(\Lambda - \lambda_1) - \overline{T}_1(\lambda_1) - (2\lambda_1 - \Lambda)(\overline{T}_1'(\lambda_1) - \overline{T}_2'(\Lambda - \lambda_1))$. In order to find a root of $G(\lambda_1) = 0$, we can resort to root-finding algorithms. One possible numerical method is the bisection method with logarithmic complexity [19]. Then, the value λ_2^{nc} is easily obtained by using the equation $\lambda_1^{nc} + \lambda_2^{nc} = \Lambda$. Using the first equality constraints of (30) and (31), we obtain the Nash equilibrium admission prices $(p_1^{nc}, p_2^{nc}) = \left(R - C\overline{T}_1(\lambda_1^{nc}), R - C\overline{T}_2(\lambda_2^{nc})\right)$.

Note that given the admission price p_1, from (33), the best response of BS_2^O can be expressed in terms of the arrival rate variable λ_2 as

$$\begin{aligned} \max_{\lambda_2} \quad & \lambda_2\left(p_1 + C\overline{T}_1(\Lambda - \lambda_2) - C\overline{T}_2(\lambda_2)\right) \\ \text{s.t.} \quad & 0 \leq \lambda_1 \leq \frac{1}{E[Y_1](1+\beta_1 E[X_1])}. \end{aligned} \tag{39}$$

TABLE I
EQUILIBRIUM ARRIVAL RATE IN SIX SCENARIOS: (I) $\beta=2$, $\mu_Y=1.2$ AND $\mu_X=0.5$; (II) $\beta=2$, $\mu_Y=1.2$ AND $\mu_X=0.6$; (III) $\beta=1.5$, $\mu_Y=1.2$ AND $\mu_X=0.5$; (IV) $\beta=1.5$, $\mu_Y=1.2$ AND $\mu_X=0.6$; (V) $\beta=3$, $\mu_Y=1.2$ AND $\mu_X=0.6$; (VI) $\beta=3$, $\mu_Y=1.2$ AND $\mu_X=0.5$; $C=1$. THE FIRST CHANNEL FOLLOWS THE EXPERL CASE AND THE SECOND CHANNEL FOLLOWS THE EXP CASE.

Scenario	Total arrival rate Λ	Equilibrium arrival rate $(\lambda_2^{nc},\lambda_1^{nc})$	The product revenue $\pi_1(\lambda_1)\pi_2(\lambda_2)$
(i)	0.120	(0.053,0.067); (0.042,0.078);	27.42; 24.06
(ii)	0.138	(0.064,0.074); (0.043,0.095)	38.4; 30.7
(iii)	0.150	(0.070,0.080); (0.046,0.104)	45.7; 35.8
(iv)	0.171	(0.081,0.090); (0.047,0.124)	61.7; 45.0
(v)	0.100	no existence	no existence
(vi)	0.083	no existence	no existence

Solving the above problem, we obtain the best response arrival rate $\lambda_2^B(p_1)$ of the $\mathrm{BS}_2^{\mathrm{O}}$. From the first constraint of (33), the best response admission price p_2 corresponding to a given price p_1 is obtained as follows

$$p_2^B(p_1)=p_1+C\overline{T}_1\left(\Lambda-\lambda_2^B(p_1)\right)-C\overline{T}_2\left(\lambda_2^B(p_1)\right). \quad (40)$$

Similarly, given a price p_2, the best response price of $\mathrm{BS}_1^{\mathrm{O}}$ is obtained as follows

$$p_1^B(p_2)=p_2+C\overline{T}_2\left(\Lambda-\lambda_1^B(p_2)\right)-C\overline{T}_1\left(\lambda_1^B(p_2)\right). \quad (41)$$

The Nash equilibrium points can be found by identifying the intersection points of the reaction curve of both BS^{O}s. We can draw the best response price of the $\mathrm{BS}_1^{\mathrm{O}}$ as a function of the price p_2. Similarly, we can draw the best response price of the $\mathrm{BS}_2^{\mathrm{O}}$. When a solution of (38) does exist, the two reaction curves have an intersection point that is an equilibrium point. However, the solution of (38) may be neither unique nor even exists. Therefore, it may lead to a multiple Nash equilibrium points scenario or a non-convergent oscillation scenario. By comparing the product revenue between multiple Nash equlibrium points, the BS^{O}s would choose the most efficient price equilibrium.

We clarify the equilibrium analysis in the non-cooperative game through numerical results by the following six scenarios. As can be seen in Table I, the equilibrium does not exist for the two scenarios (v) and (vi) because the roots of (38) are non-real or non-positive. There are two equilibria for four scenarios (i), (ii), (iii) and (iv) with different product revenue values $\pi_1(\lambda_2)\pi_2(\lambda_2)$. In the next subsection, we will discuss the duopoly in the cooperative game, which has a unique solution and can be solved in a distributed manner. By comparing the product revenue value $\pi_1(\lambda_2)\pi_2(\lambda_2)$, we will show the advantages of the cooperative model.

C. Duopoly Market: Cooperative Model

In this subsection, we assume that $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$ are not competitive but cooperative through bargaining. Bargaining theory is categorized in cooperative game theory [20], [21], [22]. Here, we will find a Nash bargaining solution of the cooperative game between $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$.

1) Nash Bargaining Solution: A bargaining game is defined as a situation in which two (or more) players can mutually benefit from reaching a certain agreement but have conflicting interests in their the agreement. Therefore, we can model this case as the bargaining game between $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$ who share the SUs' customer market. We again assume the utility functions of $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$ are $\pi_1(\lambda_1)=\lambda_1 p_1=\lambda_1\left[R-C\overline{T}_1(\lambda_1)\right]$ and $\pi_2(\lambda_2)=\lambda_2 p_2=\lambda_2\left[R-C\overline{T}_2(\lambda_2)\right]$. Then, mathematically, the bargaining problem can be formulated as follows

$$\begin{aligned}\max_{\lambda_1,\lambda_2}\quad & [\pi_1(\lambda_1)-d_1]^{w_1}\,[\pi_2(\lambda_2)-d_2]^{w_2}, \qquad (42)\\ \text{s.t.}\quad & 0\le\lambda_1\le\frac{1}{E[Y_1](1+\beta_1E[X_1])},\\ & 0\le\lambda_2\le\frac{1}{E[Y_2](1+\beta_2E[X_2])},\\ & 0\le\lambda_1+\lambda_2\le\Lambda,\end{aligned}$$

where the pair (d_1,d_2) is the disagreement point that is the outcome if two $\mathrm{BS}_i^{\mathrm{O}}$'s fail to reach an agreement [22], (w_1, w_2) are constant and denote the bargaining power of $\mathrm{BS}_1^{\mathrm{O}}$ and $\mathrm{BS}_2^{\mathrm{O}}$, respectively.

2) The Dual Decomposition Algorithm: The Nash bargaining solution of the cooperative game can be solved in a distributed manner by using the dual decomposition algorithm as follows. In order to decompose problem (42), we rewrite problem (42) as follows

$$\begin{aligned}\max_{\lambda_1,\lambda_2}\quad & w_1\log(\pi_1(\lambda_1)-d_1)+w_2\log(\pi_2(\lambda_2)-d_2), \qquad (43)\\ \text{s.t.}\quad & 0\le\lambda_1\le\frac{1}{E[Y_1](1+\beta_1E[X_1])},\\ & 0\le\lambda_2\le\frac{1}{E[Y_2](1+\beta_2E[X_2])},\\ & 0\le\lambda_1+\lambda_2\le\Lambda.\end{aligned}$$

Both $\pi_1(\lambda_1)$ and $\pi_2(\lambda_2)$ are strictly concave functions since $T_1(\lambda_1)$ and $T_2(\lambda_2)$ are convex and strictly increasing continuous functions. Then, problem (43) is convex. We can solve problem (43) in the distributed manner by using the dual decomposition algorithms [23], [24], [25] and [26]. We first form the Lagrangian function as follows

$$\begin{aligned}& L(\lambda_1,\lambda_2,\nu)\\ &=w_1\log(\pi_1(\lambda_1)-d_1)+w_2\log(\pi_2(\lambda_2)-d_2)\\ &-\nu(\lambda_1+\lambda_2-\Lambda),\\ &=[w_1\log(\pi_1(\lambda_1)-d_1)-\nu\lambda_1]\\ &+[w_2\log(\pi_2(\lambda_2)-d_2)-\nu\lambda_2]+\nu\Lambda, \qquad (44)\end{aligned}$$

where $\nu\ge 0$ is the Lagrange multiplier associated with the inequality constraint $\lambda_1+\lambda_2\le\Lambda$. We want to maximize the $L(\cdot)$ function from which we can decompose it into two different problems, presented as follows

$$\begin{aligned}\max_{\lambda_i}\quad & w_i\log(\pi_i(\lambda_i)-d_i)-\nu\lambda_i \qquad (45)\\ \text{s.t.}\quad & 0\le\lambda_i\le\frac{1}{E[Y_i](1+\beta_iE[X_i])}, i=1,2,\end{aligned}$$

which has unique solution $\lambda_i^*(\nu)$ for given ν due to the strict concavity of $\log(\pi_i(\lambda_i)-d_i)$.

The dual function is given as

$$g(\nu) = [\log(\pi_1(\lambda_1^*) - d_1) - \nu\lambda_1^*] + [\log(\pi_2(\lambda_2^* - d_2)) - \nu\lambda_2^*] + \nu\Lambda. \quad (46)$$

The master dual problem is

$$\min_{\nu \geq 0} \; g(\nu). \quad (47)$$

Using the gradient method, the Lagrange multiplier ν is updated as follows

$$\nu(t+1) = [\nu(t) - \alpha(\Lambda - \lambda_1^*(t) - \lambda_2^*(t))]^+, \quad (48)$$

where t is the iteration index, $\alpha > 0$ is a sufficiently small positive step-size and $[\cdot]^+$ denotes the projection onto the nonnegative orthant. The dual variable $\nu(t)$ will converge to the dual optimal ν^* as $t \to \infty$ since the duality gap for the problem (43) is zero and the solution to (45) is unique; the primal variable $\lambda_i^*(t)$ obtained by solving (45) will also converge to the primal optimal value λ_i^{co}. Finally, we have the dual algorithm to determine the optimal arrival rate $(\lambda_1^{co}, \lambda_2^{co})$ of problem (43) in Algorithm 1. Since problem (43) is convex and the Slater's condition is satisfied, the optimal duality gap is zero [27]. Thus, the solution $(\lambda_1^*(t),\, \lambda_2^*(t))$ will converge to the optimal solution $(\lambda_1^{co}, \lambda_2^{co})$. Then, the equilibrium prices are $(p_1^{co}, p_2^{co}) = \big(R - C\overline{T}_1(\lambda_1^{co}), R - C\overline{T}_2(\lambda_2^{co})\big)$.

Algorithm 1 Dual Algorithms to find the Nash bargaining solution

- Parameters: each BS_i^{o} $(i = 1, 2)$ can estimate the parameters of the utility function $\pi_i(\cdot)$ and the SU customer arrival rate Λ based on existing estimation methods [17];
- Initialize $t = 0$ and $\nu(0)$ equals to a certain nonnegative value;

1) Each BS_i^{o} $(i = 1, 2)$ locally solves its problem by computing (45) and then broadcasts the solution $\lambda_i^*(t)$;
2) Each BS_i^{o} $(i = 1, 2)$ updates the Lagrange multiplier $\nu(t+1)$ with the gradient iterate (48);
3) Set $t + 1 \to t$ and go back to step 1 (until satisfying the termination criterion);

3) Numerical Results: In order to compare the numerical results with the non-cooperative model, we use six scenarios (i), (ii), (iii), (iv), (v) and (vi) as shown in Table I in the duopoly market in the non-cooperative model section. Fig. 8(a) presents the convergence of the SU customers' equilibrium arrival rate $\lambda_1^*(t)$ which is updated according to the dual algorithms. With all six scenarios mentioned in the previous section, we obtain six equilibrium arrival rates $(\lambda_1^{co}, \lambda_2^{co})$ as (0.056,0.064), (0.065,0.073), (0.071,0.079), (0.082, 0.089), (0.046,0.054), (0.039,0.044), respectively. With the appropriate step size α, the dual algorithm converges quickly to the optimal value as shown in Fig. 8(a).

In order to show the advantage of the cooperative model, we compare the bargaining product revenue $\pi_1(\lambda_1)\pi_2(\lambda_2)$ between the two models: the non-cooperative and cooperative model in four scenarios. Since the equilibrium does not exist in the two scenarios of (v) and (vi) in the non-cooperative model.

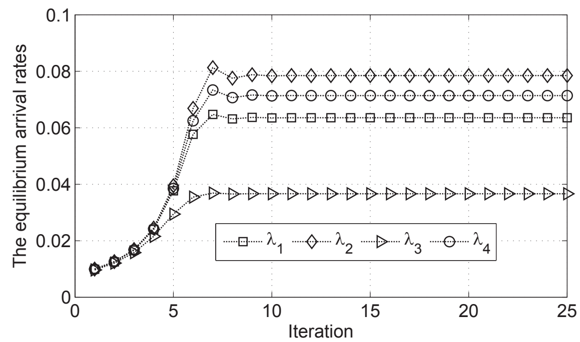


Fig. 9. The convergence of the equilibrium arrival rate with four BS_i^{o} with $d_i = 0$, $w_i = 1$ $(i = 1,2,3,4)$, $\Lambda = 0.2$, $R = 100$ and $C = 1$: (1) BS_1^{o} with the ExpErl channel, $\beta = 2$, $\mu_Y = 1.2$ and $\mu_X = 0.5$; (2) BS_2^{o} with the Exp channel, $\beta = 2$, $\mu_Y = 1.2$ and $\mu_X = 0.5$; (3) BS_3^{o} with the Erl channel, $\beta = 2$, $\mu_Y = 1.2$, $\mu_X = 0.5$; (4) BS_4^{o} with the Exp channel, $\beta = 1.5$, $\mu_Y = 1.5$ and $\mu_X = 0.2$.

Fig. 8(b) shows that the product revenue of the cooperative model is always higher than the product revenue of the non-cooperative model in four of the scenarios.

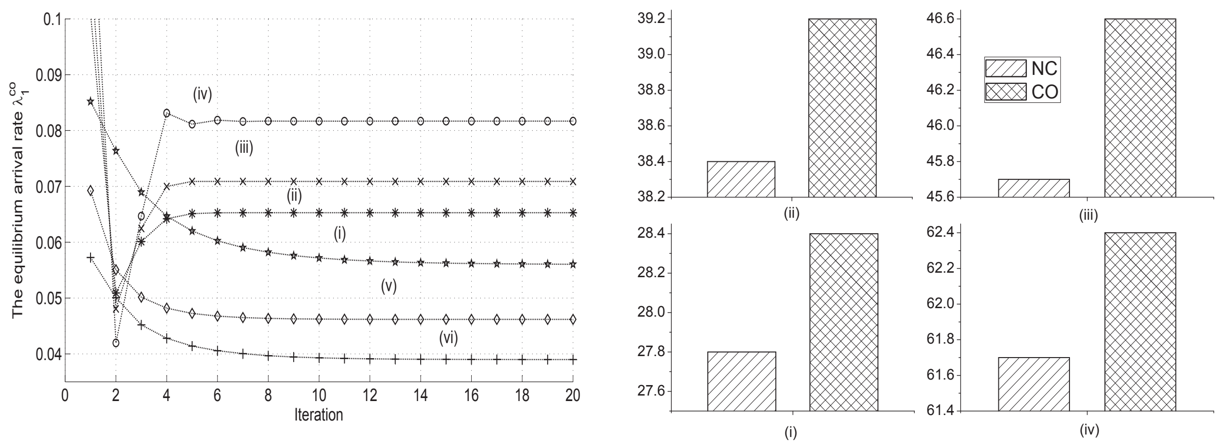
4) Multiple Base Stations Scenario: The price setting problem can also be analyzed when several BS^{o}s, each of which operate in a different PU band, are available. The SU customer's decision in this case is joining to one of the BS^{o}s based on the estimated delay and the admission prices. Suppose that there are N $(N > 2)$ BS_i^{o} $(i = 1, \ldots, N)$ in the SU markets, then the duopoly market can be extended to consist of multiple BS_i^{o}. The bargaining game between N BS_i^{o} can be formulated as

$$\begin{aligned} \max_{\lambda_i} \quad & \prod_{i=1}^{N} [\pi_i(\lambda_i) - d_i]^{w_i} \\ \text{s.t.} \quad & 0 \leq \lambda_i \leq \frac{1}{E[Y_i](1 + \beta_1 E[X_i])}, i = 1, \ldots, N, \\ & 0 \leq \textstyle\sum_{i=1}^{N} \lambda_i \leq \Lambda. \end{aligned} \quad (49)$$

By using the dual decomposition algorithm, we can obtain the equilibrium arrival rate of the above problem. Therefore, by using the bargaining game theory, we can easily extend the duopoly scenario to multiple BS^{o}s. Furthermore, the advantages of bargaining game is that it can be solved in a distributed manner, which helps the policy maker design a good model to optimize resource allocation. We demonstrate the multiple BS^{o}s by an example with four channels. Fig. 9 shows the quick convergence of the equilibrium arrival rate obtained by the dual decomposition algorithms and demonstrates that the cooperative model can be applied for not only the duopoly model but also for multiple BSs scenarios.

VI. Duopoly in mixed O-DSA and D-DSA market model

We consider a cognitive radio system in which there is one D-DSA base station denoted by BS^{d} and one BS^{o}. The BS^{d} can rent a licensed dedicated band for a certain cost. Given the total arrival rate Λ, SU customers choose to join the queue of the BS^{o} with an admission price p_o or join the queue of the BS^{d} with an admission price p_d as illustrated in Fig. 10.



(a) The convergence of the equilibrium arrival rate λ_1^{co} in six scenarios. (b) Comparison the bargain product revenue $\pi_1(\lambda_1)\pi_2(\lambda_2)$ between non-cooperative (NC) and cooperative (CO) model.

Fig. 8. Numerical parameters: $d_1 = d_2 = 0$ and $w_1 = w_2 = 1$ in six scenarios (i), (ii), (iii), (iv), (v) and (vi).

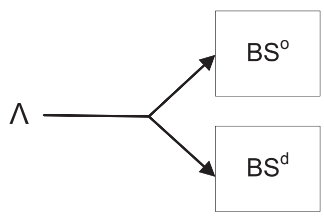


Fig. 10. Duopoly market in mixed O-DSA and D-DSA model.

We organize this section into three subsections. The first one analyzes the non-cooperative model between the BS$^{\text{o}}$ and the BS$^{\text{d}}$ by using the Stackelberg competition in the duopoly model. In the second subsection, we investigate the cooperative behavior between the BS$^{\text{o}}$ and BS$^{\text{d}}$, and solve the bargaining problem by the dual composition algorithm. The numerical results are shown in the third subsection.

Given the prices p_o and p_d, SU customers will individually determine a strategy q_o of the probability that SU customers decide to join the BS$^{\text{o}}$ queue (thus, with probability $q_d = 1 - q_o$ SU customers acquire the BS$^{\text{d}}$). The expected cost when acquiring the BS$^{\text{d}}$ is given by

$$C\theta^{-1}(p_d) + p_d, \tag{50}$$

where $\theta^{-1}(p_d)$ and p_d are the expected queueing delay and the admission fee of the BS$^{\text{d}}$. Thus, given the equilibrium SU customer arrival rate $\lambda_o = q_o\Lambda$ at the BS$^{\text{o}}$, the total cost of an SU customer who chooses the BS$^{\text{o}}$ is given by

$$p_o + C\overline{T}(\lambda_o). \tag{51}$$

At the equilibrium point, the cost of the BS$^{\text{o}}$ is equal to the cost of the BS$^{\text{d}}$. Therefore, λ_o can be obtained by solving the equilibrium equation

$$C\theta^{-1}(p_d) + p_d = p_o + C\overline{T}(\lambda_o). \tag{52}$$

To avoid a trivial solution in equality constraint (52), we assume that there exists a set of prices $[p_d^l, p_d^u] \in [0, P_{max}]$ such that $p_d + C\theta^{-1}(p_d) > C\overline{T}(0), \forall p_d \in [p_d^l, p_d^u]$. The revenue obtained by the BS$^{\text{d}}$ is defined as follows

$$\pi^d \triangleq \lambda_d p_d, \tag{53}$$

where $\lambda_d = q_d\Lambda$ is the equilibrium SU customer arrival rate at the BS$^{\text{d}}$. Similarly, the revenue obtained by the BS$^{\text{o}}$ is given by

$$\pi^o \triangleq \lambda_o p_o. \tag{54}$$

A. Non-cooperative Model

We now investigate the non-cooperative model in which the BS$^{\text{d}}$ and BS$^{\text{o}}$ selfishly maximize their own revenues. In order to compete with each other, the BS$^{\text{o}}$ sets the price p_o to maximize its own revenue given the price p_d of the BS$^{\text{d}}$, and vice versa. Specifically, we model the strategic interaction between the BS$^{\text{d}}$ and BS$^{\text{o}}$ as a Stackelberg competition in the duopoly market [28], [29]. Here, the expected queueing delay for SU customer accessing the BS$^{\text{o}}$ depends on the quality of the PU's channel (i.e., pdf $f_Y(y)$ and β). However, the BS$^{\text{d}}$ owns the license and possibly decides to decrease the expected queueing delay $1/\theta$ by acquiring more bandwidth to serve SU customers. Hence, the BS$^{\text{d}}$ can be a dominant provider by keeping the price and expected queueing delay of SU customers sufficiently small. Thus, we assume that the BS$^{\text{d}}$ is the game leader and the BS$^{\text{o}}$ is the game follower. In the Stackelberg game, the BS$^{\text{d}}$ has the so-called first-move advantage, which means that the BS$^{\text{d}}$ adapts its decisions to maximize its revenue by anticipating the BS$^{\text{o}}$'s response. Then, we use backward induction to derive the Stackelberg equilibrium of the prices, which are denoted by (p_d^S, p_o^S), in a duopoly as follows.

1) Follower BS$^{\text{o}}$'s Revenue Maximization: First, given the BS$^{\text{d}}$'s admission price p_d, the BS$^{\text{o}}$ aims to determine the optimal SU customer arrival rate λ_o^m at the BS$^{\text{o}}$ and optimal price p_o^m by solving the following problem:

$$\begin{aligned} \max_{\lambda_o, p_o} \quad & \lambda_o p_o \\ \text{s.t.} \quad & p_o = p_d + C\theta^{-1}(p_d) - C\overline{T}(\lambda_o), \\ & 0 \leq \lambda_o \leq \min\{\Lambda, 1/E[Y_e]\}, \\ & 0 \leq p_o \leq P_{max}, \end{aligned} \tag{55}$$

where P_{max} is the maximum price SU customer may afford. By replacing p_o in the first constraint and setting the first derivative of objective function to zero, we obtain the optimal arrival rate λ_o^m as follows

$$\lambda_o^m(p_d) = \min\{\frac{1}{E[Y_e]} - \frac{\sqrt{CE[Y_e^2]\Omega'}}{E[Y_e]\Omega'}, \Lambda\} \tag{56}$$

where $\Omega' = CE[Y_e^2] + 2\left(C\theta^{-1}(p_d) + p_d\right)E[Y_e] - 2CE[Y_e]^2$. Then, we obtain the optimal price p_o^m of (55) as follows

$$p_o^m(p_d) = p_d + C\theta^{-1}(p_d) - C\overline{T}(\lambda_o^m). \tag{57}$$

2) Leader $\mathrm{BS^d}$'s Revenue Maximization: Knowing the $\mathrm{BS^o}$'s best-response λ_o^m and p_o^m, the $\mathrm{BS^d}$ determines its admission price p_d by solving the following problem

$$\begin{aligned} \max_{p_d} \quad & \pi^d(p_d) = p_d[\Lambda - \lambda_o^m(p_d)] \\ \text{s.t.} \quad & 0 \leq p_d \leq P_{max}. \end{aligned} \tag{58}$$

3) Stackelberg Equilibrium Summary: The maximization (58) can be solved by finding the root of the first derivation $\pi'^d(p_d) = 0$. As before, we can use a standard root-finding algorithm such as the bisection method with logarithmic complexity [19]. Thus, the $\mathrm{BS^o}$'s Stackelberg equilibrium of admission price is given as

$$p_o^S = p_o^m(p_d^S) = p_d^S + C\theta^{-1}(p_d^S) - C\overline{T}\left(\lambda_o^m(p_d^S)\right). \tag{59}$$

B. Cooperative Model

In this subsection, we investigate the cooperative behavior between the $\mathrm{BS^o}$ and $\mathrm{BS^d}$. We assume that there is a revenue sharing contract which encourages coordination between the $\mathrm{BS^d}$ and $\mathrm{BS^o}$. Then, we will see how the cooperation makes the revenue better off as compared to the case in which they selfishly maximize their own profits from the social point of view. We consider the cooperative problem as the following bargaining problem

$$\begin{aligned} \max_{\lambda_o, p_o, \lambda_d, p_d} \quad & (\pi^d - d_d)^{w_d}(\pi^o - d_o)^{w_o} \\ \text{s.t.} \quad & p_o + C\overline{T}(\lambda_o) = p_d + C\theta^{-1}(p_d), \\ & \lambda_o + \lambda_d = \Lambda, \\ & 0 \leq \lambda_o \leq \min\{\Lambda, 1/E[Y_e]\}, \\ & 0 \leq \lambda_d \leq \Lambda, \\ & 0 \leq p_o, p_d \leq P_{max}. \end{aligned} \tag{60}$$

The first constraint of (60) can be expressed as $h(\lambda_o, p_o, \lambda_d, p_d) = 0$ where $h(\lambda_o, p_o, \lambda_d, p_d) = p_o + C\overline{T}(\lambda_o) - p_d - C\theta^{-1}(p_d)$. Since $h(\lambda_o, p_o, p_d)$ is not affine, then problem (60) is not a convex optimization problem.

In order to transform the original bargaining problem (60) into a convex problem, we go through two steps as follows:

Step 1: We set $d_d = d_o = 0$ and take the logarithm of the objective function in order to obtain the following objective function

$$\max \quad w_d \log(\lambda_d p_d) + w_o \log(\lambda_o p_o). \tag{61}$$

Step 2: We relax the equality $h(\lambda_o, p_o, \lambda_d, p_d) = 0$ in (60) by a convex inequality $h(\lambda_o, p_o, \lambda_d, p_d) \leq 0$ since $h(\lambda_o, p_o, \lambda_d, p_d)$ is a jointly convex function. Thus, we have the following convex problem:

$$\begin{aligned} \max_{\lambda_o, p_o, \lambda_d, p_d} \quad & w_d \log(\lambda_d p_d) + w_o \log(\lambda_o p_o) \\ \text{s.t.} \quad & h(\lambda_o, p_o, \lambda_d, p_d) \leq 0, \\ & \lambda_o + \lambda_d = \Lambda, \\ & 0 \leq \lambda_o \leq \min\{\Lambda, 1/E[Y_e]\}, \\ & 0 \leq \lambda_d \leq \Lambda, \\ & 0 \leq p_o, p_d \leq P_{max}. \end{aligned} \tag{62}$$

Lemma 2: Problem (60) and the convex problem (62) are equivalent.

Proof: Since $h(\lambda_o, p_o, \lambda_d, p_d)$ is monotonically increasing in p_o, according to [27], we can guarantee that at any optimal solution $(\lambda_o^*, p_o^*, \lambda_d^*, p_d^*)$ of the convex problem (62), we have $h(\lambda_o^*, p_o^*, \lambda_d^*, p_d^*) = 0$. It can be proved by using contradiction as follows. Suppose there is an optimal solution $(\lambda_o^*, p_o^*, \lambda_d^*, p_d^*)$ of (62) such that $h(\lambda_o^*, p_o^*, \lambda_d^*, p_d^*) < 0$. Since $h(\lambda_o, p_o, \lambda_d, p_d)$ and the objective function of (61) are monotonically increasing in p_o, we can increase p_o while staying in the boundary. Thus, by increasing p_o we increase the objective and increase the function $h(\cdot)$. It contradicts the supposition that $\mathrm{h}(\lambda_o^*, p_o^*, \lambda_d^*, p_d^*)$ is the maximum value. ∎

Using Lemma 2, we can solve the (nonconvex) problem (60) by solving the convex problem (62). We then form the Lagrangian function as

$$\begin{aligned} & L(\lambda_o, \lambda_d, p_o, \lambda_d, p_d, \nu, \beta) \\ & = w_d \log(\lambda_d p_d) + w_o \log(\lambda_o p_o) \\ & - \nu(\lambda_o + \lambda_d - \Lambda) - \eta[p_o + C\overline{T}(\lambda_o) - C\theta^{-1}(p_d) - p_d], \end{aligned} \tag{63}$$

where ν, $\eta \geq 0$ are the Lagrange multipliers associated with an equality and inequality constraints. We take a dual decomposition approach, and (62) is decomposed into the two subproblems:

$$\begin{aligned} \max_{\lambda_d, p_d} \quad & w_d \log(\lambda_d p_d) - \nu\lambda_d + \eta p_d + \eta C\theta^{-1}(p_d) \\ \text{s.t.} \quad & 0 \leq \lambda_d \leq \Lambda, \\ & 0 \leq p_d \leq P_{max}, \end{aligned} \tag{64}$$

and

$$\begin{aligned} \max_{\lambda_o, p_o} \quad & w_o \log(\lambda_o p_o) - \nu\lambda_o - \eta C\overline{T}(\lambda_o) - \eta p_o \\ \text{s.t.} \quad & 0 \leq \lambda_o \leq \min\{\Lambda, 1/E[Y_e]\}, \\ & 0 \leq p_o \leq P_{max}. \end{aligned} \tag{65}$$

The optimal solutions (λ_d^*), (p_d^*), (λ_o^*) and (p_o^*) of (64) and (65) for a given set of Lagrange multipliers ν and β define the dual function as follows

$$g(\nu,\eta) = \nu(\Lambda - \lambda_o^* - \lambda_d^*) + \eta[C\theta^{-1}(p_d^*) + p_d^* - p_o^* - C\overline{T}(\lambda_o^*)] + w_d \log(\lambda_d^* p_d^*) + w_o \log(\lambda_o^* p_o^*). \quad (66)$$

Then, the master dual problem is given as

$$\min_{\nu,\eta \geq 0} g(\nu,\eta) \quad (67)$$

Since (λ_d^*), (p_d^*), (λ_o^*) and (p_o^*) in (64) and (65) are unique due to the strict concavity of $C\theta^{-1}(p_d)$ and $-\overline{T}(\lambda_o)$, by using the gradient method, we can solve the dual problem by the following updating Lagrangian multipliers:

$$\nu(t+1) = \nu(t) - \alpha_1(\Lambda - \lambda_o^*(t) - \lambda_d^*(t)), \quad (68)$$

$$\eta(t+1) = [\eta(t) - \alpha_2(C\theta^{-1}(p_d) + p_d^*(t) - p_o^*(t) - C\overline{T}\,[\lambda_o^*(t)])]^+, \quad (69)$$

where t is the iteration index, $\alpha_1 > 0$ and $\alpha_2 > 0$ are sufficiently small positive step-sizes. Then, we have the cooperative dual algorithm that implements the Nash bargaining solution distributively between the BS^{o} and BS^{d} in Algorithm 2. As a consequence of the assumption $p_d^u + C\theta^{-1}(p_d^u) > C\overline{T}(0)$, with sufficient small ϑ, ε and the continuity of $\overline{T}(\cdot)$ and $\theta^{-1}(\cdot)$, the point $(\lambda_o, p_o, \lambda_d, p_d) = (\vartheta, 0, \Lambda - \vartheta, p_d^u - \varepsilon)$ satisfies the Slater's condition. Since problem (62) is convex, the optimal duality gap is zero [27]. Thus, the solution $(p_o^*(t)$, $p_d^*(t))$ and $(\lambda_o^*(t)$, $\lambda_d^*(t))$ will converge to the optimal solution.

Algorithm 2 Cooperative dual algorithms

- Input parameters: function $C\overline{T}(.)$, $\theta^{-1}(p_d)$, C and the SU customer arrival rate Λ;
- Initialize $t = 0$ and $\nu(0)$, $\eta(0)$ equal to some value;

1) BS^{d} locally solves its problem by computing (64); then sends the solution $\lambda_d^*(t)$ and $p_d^*(t)$ to BS^{o};
2) BS^{o} locally solves its problem by computing (65); then sends the solution $\lambda_o^*(t)$ and $p_o^*(t)$ to BS^{d};
3) Both BS^{o} and BS^{d} update the Lagrange multiplier $\nu(t+1)$ and $\beta(t+1)$ with the gradient iterate (68) and (69);
4) Go back to step 1 (until satisfying the termination criterion);

C. Numerical results

We supplement the equilibrium analysis through the numerical results by the following four scenarios. Table II shows the comparison between the noncooperative (NC) model and cooperative (CO) model. From Table II, the product revenue of cooperative model is always higher than the product revenue of the non-cooperative model in four scenarios. Furthermore, Fig. 11 shows that the revenue π^d of the BS^{d} in the cooperative model is always higher than that in the noncooperative model. At the equilibrium of cooperative model in Table II, the arrival rate λ_d increases in (vii) and (viii) cases, but it decreases in (ix) and (x) cases. This shows that the change of the arrival rate from the BS^{d} to the BS^{o} and vice versa does not imply an increase of the BSs' revenue. The main reason of the revenue increase is the rise of the equilibrium price in cooperation. In other words, the competition keeps the equilibrium price low which in turn leads to the low revenue of the BSs.

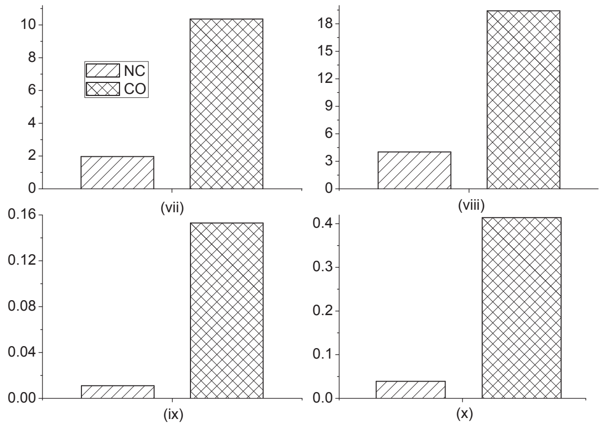


Fig. 11. The revenue π^d comparison between noncooperative model (NC) and cooperative model (CO).

D. Multiple Base Stations Scenario

The study of the general setting with multiple N BS^{d} and N BS^{o}, where each BS competes to all others, is very difficult. However, once additional assumptions are added, it may be possible to solve for the equilibrium point. We consider N to be the number of PU bands in the system. By a certain spectrum allocation mechanism, each PU band i is assigned to a $\mathrm{BS}_i^{\mathrm{d}}$. We assume that there is a $\mathrm{BS}_i^{\mathrm{o}}$ which only operates on the PU band i and competes with the $\mathrm{BS}_i^{\mathrm{d}}$. The spectrum allocation mechanism also assigns a group of SU customer type i who is operating in the spectrum PU band i. Each $\mathrm{BS}_i^{\mathrm{o}}$ independently sets the price p_o^i to compete with the $\mathrm{BS}_i^{\mathrm{d}}$. We suppose that N BS^{d}s belong to a Primary Operator (PO) and the PO sets the same admission price p_d for all N BS^{d}s. Then the cooperative or non-cooperative game between N $\mathrm{BS}_i^{\mathrm{d}}$ and $\mathrm{BS}_i^{\mathrm{o}}$ can be considered partial separately as in Subsections VI.B and VI.C.

VII. Conclusion

In this paper, we considered the decision-making process of SU customers and the optimal pricing of the BS. The impact of PU's emergence is modeled as a server with breakdowns. Explicit expressions for the equilibrium in SU customers' behaviors are obtained. In the O-DSA model, the BS^{o}'s pricing strategy for revenue maximization is formulated and is shown to be a convex optimization problem, which is solved directly. Then, the unique Nash bargaining solution for the cooperative duopoly scenarios are obtained by the decomposition algorithm. In the mixed O-DSA & D-DSA model, we formulate competitive and cooperative behaviors of the BS^{o} and the BS^{d} by Stackelberg and bargain game theory, respectively. By choosing appropriate bargaining parameters, we obtain the bargaining solution by the decomposition algorithm. The numerical results not only validate our analysis but also present the behaviors of BSs in the duopoly market. In both models, the cooperation between BSs helps them achieve higher product revenues. Furthermore, by using the decomposition, the Nash bargaining equilibrium of the admission price can be obtained in a distributed manner that does not reveal the BSs information.

TABLE II

COMPARISON OF NONCOOPERATIVE MODEL (NC) AND COOPERATIVE MODEL (CO) IN FOUR SCENARIOS WITH $w_d = w_o = 1$, $C = 2$, $\zeta = 10$ AND $P_{max} = 50$: (VII) EXP CHANNEL, $\beta = 2$, $\mu_Y = 1.2$ AND $\mu_X = 0.5$; $\Lambda = 0.15$; (VIII) EXP CHANNEL, $\beta = 1.5$, $\mu_Y = 1.2$, $\mu_X = 0.5$; $\Lambda = 0.2$; (IX) ERL CHANNEL, $\beta = 2$, $\mu_Y = 1.2$, $\mu_X = 0.5$; $\Lambda = 0.03$; (X) ERL CHANNEL, $\beta = 1.5$, $\mu_Y = 1.2$, $\mu_X = 0.5$; $\Lambda = 0.04$.

	(vii)	(viii)	(ix)	(x)
Equilibrium arrival rate (λ_d, λ_o) of NC	0.046, 0.104	0.053, 0.147	0.025, 0.005	0.028, 0.012
Equilibrium arrival rate (λ_d, λ_o) of CO	0.083, 0.067	0.109, 0.091	0.010, 0.020	0.024, 0.016
Equilibrium price (p_d, p_o) of NC	26.45, 15.66	28.23, 18.45	29.18, 2.79	25.28, 4.55
Equilibrium price (p_d, p_o) of CO	49.87, 37.25	49.90, 39.21	49.48, 14.92	49.62, 21.06
Product Revenue $\pi^d \pi^o$ of NC	1.97	4.02	0.011	0.039
Product Revenue $\pi^d \pi^o$ of CO	10.36	19.42	0.153	0.414

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