Coordinated Resource Partitioning and Data Offloading in Wireless Heterogeneous Networks

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***Abstract*—In this paper, a game-theoretic framework is proposed for coordinating resource partitioning and data offloading in LTE-based HetNets. The goal of this framework is to determine the amount of radio resources a macrocell should offer to neighboring small cells (SCs) and the amount of traffic each SC should admit from the macrocell. A two-stage Stackelberg game is applied to optimize the strategies of both the macrocell (the leader) and SCs (the followers). The macrocell's strategy is shown to be a mixed-boolean nonlinear program, which is NP-hard. To solve this problem efficiently, a branch and bound based method is proposed to obtain the global optimal. We also show that this two-stage game has a unique Stackelberg equilibrium. Numerical results show that the proposed framework outperforms the traditional design by 50% in term of offloaded data. Additionally, reduction of 14% was observed in term of cost paid by MBS.**

***Index Terms*—4G LTE, heterogeneous hetworks, data offloading, resource partitioning, game theory.**

I. Introduction

In wireless heterogeneous networks (HetNets), data traffic of macrocells can be routed or offloaded to small cells (SCs) such as picocells, femtocells, or WiFi networks [1],[2] for better radio resource management. Nevertheless, mobile data offloading should not increase the load at the SCs, especially given the limited available resources and the small coverage area of the SCs. Consequently, it is necessary to develop efficient resource partitioning mechanisms in order to achieve optimal data offloading.

This *data offloading* problem in heterogeneous wireless networks has recently received great attention [1],[2]. The authors in [1]-[2] investigated the economics aspect of mobile data offloading. The work in [1] studies how much economic benefits can be generated due to delayed WiFi offloading, by modeling the interaction between a single provider and users based on a two-stage sequential game. Using different approach, in [2], the authors introduce an iterative double auction mechanism that ensures the market where mobile network operators maximize their offloading benefits and SBSs minimize their offloading costs.

Meanwhile, there have been recent works on *resource partitioning* in HetNets [3]-[4]. The works in [3] and [4] consider joint resource partitioning and user offloading by using almost blank subframes (ABS) and cell selection bias (CSB). In [3], the authors solve the coupled problems of resource partitioning and user association in LTE HetNets. The proposed algorithm is developed based on the decomposition-based approach and subgradient descent-based dual update. The work in [4] derives the downlink rate distribution over the entire network and proposes an optimal strategy for joint resource partitioning and user offloading using Stochastic Geometry analysis. In this paper, we will jointly consider resource partitioning and data offloading using an incentive based approach.

To address the coordinated resource partitioning and data offloading problem in HetNets, we introduce a two-stage leader-followers game in which the macro base station (MBS) plays the leader role and the small cell base stations (SBSs) act as followers. In the first stage, the MBS proposes the fraction of resource (ABSs) that it is willing to sacrifice to each SBS for offloading its traffic. In the second stage, each SBS determines how much traffic to be admitted from the MBS corresponding to the total amount resource offered by the MBS in the first stage.

The main contributions of this paper are:

- A novel incentive based approach is proposed to address the coordinated resource partitioning (i.e., each SBS has variable number of orthogonal ABS subframe) and mobile data offloading (i.e., amount of offloaded data depends upon small cells admission capability). In this scheme, we develop a game-theoretic model to design an economic scheme that incentivizes each individual SBS to admit offloaded traffic for MBS in a noncooperative fashion and to determine its own traffic demand for optimizing its total utility.
- In this Stackelberg game, we show that the Stage-I problem of the MBS is a mixed-boolean nonlinear programming problem, which is NP-hard. Therefore, to obtain the optimal solution for Stage-I problem, we develop an algorithm based on the branch and bound (BnB) method [7], which can solve the MBS's problem efficiently. We show that there exists a unique Stackelberg equilibrium for the proposed game.
- Numerical results show that the proposed scheme outperform the fixed enhanced inter-cell interference coordination (eICIC) scheme by 50% in term of offloaded data. Additionally, reduction of 14% was observed in term of incentive cost paid by MBS.

II. System Model

We consider a downlink two-tier HetNet with one MBS and a set $\mathcal{M} \triangleq \{1, 2, ..., M\}$ of SBSs serving their own small cell user equipments (SUEs). The MBS and all SBSs use the same frequency bands to transmit data. The MBS has a group of macro user equipments (MUEs) which are randomly distributed within the MBS and SBSs' coverage areas. The MUEs' location and traffic may change over time but for simplicity, they are considered fixed within the considered time period T (e.g., equal to a number of LTE subframes). Let $\mathcal{N} \triangleq \{1, 2, ..., N\}$ be the set of N MUEs whose traffic could be potentially offloaded to SBSs. We denote by $\boldsymbol{D} \triangleq (d_0, d_1, ..., d_N)$ as the MUEs' traffic demand vector where d_n is the demand of MUE $n \in \mathcal{N}$, and d_0 is the total MBS's traffic that cannot be offloaded to any SBS, i.e., those generated by MUEs not in the coverage area of any

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SBS. Each SBS has it own coverage range. We assume that the traffic of an MUE can be offloaded to a particular SBS only if the MUE is in the coverage area of the corresponding SBS. Let $\boldsymbol{\xi}$ denote the reciprocal location availability relation between MUEs and SBSs where $\xi_{m,n} = 1$ indicates that SBS m covers MUE n; otherwise, $\xi_{m,n} = 0$. Let $\mathcal{S}_m$ and $\mathcal{K}_m$ be the set of all MUEs within the coverage of SBS m and the set of all users associated with SBS m, respectively. Let $\mathcal{K}_0$ be the set of all MUEs associated with the MBS.

LTE-based Resource Partitioning. Let α_m be the fraction of ABS which are reserved by the MBS for the SBS m. In LTE-HetNets, a fixed eICIC pattern could be used, which means that there is a fixed number of ABSs in a certain number of subframes, e.g., the MBS can mute in 5, 10, or 15 ABS within a period of 40 LTE subframes [3]. In this work, orthogonality of ABSs among SBSs is assumed. Therefore, an SBS can transmit within its allocated ABSs without intra-tier interference.

We assume that the transmission rate between SBS m and its associated SUE i is denoted by $C_{m,i}$, which depends on corresponding SUE's SINR. Thus, if SBS m is allocated α_m fraction from ABSs then the fraction resource which MBS offers to SBS m is $\alpha_m = \sum_{i \in \mathcal{K}_m \cup \mathcal{S}_m} \alpha_{m,i}$ where $\alpha_{m,i}$ denotes the resource allocated to user i associated with SBC m. We assume that transmit powers of SBSs are fixed over the ABS time period and hence the rate $C_{m,i}$, $\forall m, i$, for data transmission and data offloading are constant during each resource partitioning period.

Data Offloading. Let l_m^{tot} denote the total traffic demand of all MUEs associated with SBS m (i.e., MUEs in set $\mathcal{S}_m$). We have $l_m^{tot} = \sum_{i \in \mathcal{S}_m} d_i$, where d_i is the traffic demand of MUE $i \in \mathcal{S}_m$. SBS m can offload an amount of traffic $\sum_{i \in \mathcal{S}_m} l_{m,i} \leq l_m^{tot}$. Let $x_{m,i}$ denote the SBS m's own traffic demand. The maximum amount of data that it can serve within the time period T is:

$$\sum_{i \in \mathcal{K}_m} \frac{x_{m,i}}{C_{m,i}} + \sum_{i \in \mathcal{S}_m} \frac{l_{m,i}}{C_{m,i}} \leq \alpha_m T. \tag{1}$$

The data rate between MBS and its associated the MUE i in the considered period T is given by $C_{0,i}$ and total resource for MUEs $\sum_{i \in \mathcal{K}_0 \cup \{\mathcal{S}_m\}} \alpha_{0,i} = (1 - \sum_m \alpha_m)$. We assume that each MUE is allocated an equal fraction resource $\alpha_{0,i} = (1 - \sum_m \alpha_m)/|\mathcal{K}|$, where $\mathcal{K} \triangleq \mathcal{K}_0 \cup \{\mathcal{S}_m\}, \forall m$. Hence, the effective amount of data that the MBS can serve must satisfy

$$\frac{d_{0,i}}{C_{0,i}} \leq (1 - \sum_{m \in \mathcal{M}} \alpha_m) T / |\mathcal{K}|, \forall i \in \mathcal{K}_0, \tag{2}$$

$$\frac{d_i - l_{m,i}}{C_{0,i}} \leq (1 - \sum_{m \in \mathcal{M}} \alpha_m) T / |\mathcal{K}|, \forall i \in \mathcal{S}_m, \forall m. \tag{3}$$

Without loss of generality, we normalize the time duration to be $T = 1$. Each SBS must select its own strategy to optimize its utility considering its own traffic demand and offloaded traffic. On the other hand, based on the knowledge on behavior of SBSs, the MBS determines the amount of resource it must sacrifice and economic incentive in order to optimize its total profit.

III. Incentive Mechanism for Data Offloading

In this section, we consider the economic incentive aspect for SBSs to admit macrocell traffic.

A. Data Offloading: A Two-stage Stackelberg Game Approach

To offload traffic from heavily loaded MBS, it is necessary to design an efficient economic scheme that incentivizes each individual SBS to admit the offloaded traffic. This incentive issue is particularly important for scenarios in which SBSs suffer from both inter-tier and intra-tier interferences. The inter-tier interference can be mitigated using an ABS format for the subframes (LTE eICIC standard). In particular, each SBS m needs a fraction of resource α_m for transmission in order to mitigate inter-cell interference. Each SBS m must also determine its own demand traffic ($x_{m,i}$) as well as the offloaded traffic volume ($l_{m,i}$) it can admit in order to optimize its utility. We focus on the pricing incentives that the MBS needs to provide to SBSs in order to encourage cooperative data offloading. The challenge of the MBS is how to design a differentiated incentive scheme for heterogeneous SBSs whose own traffic, data rates, and QoS can be different.

The interactions between the MBS and SBSs can be characterized as a two-stage Stackelberg game model. The MBS publishes the resource partitioning scheme in a first stage and then the SBSs respond the admitted traffic amount in a second stage. All SBSs want to maximize their total utilities by optimizing the amount of offloaded traffic that they can admit according to the resource partitioning scheme. The MBS wants to maximize its utility by setting the right resource partitioning scheme to satisfy the admission abilities of SBSs. Next, we discuss in details the strategies and modelling of SBSs and MBS respectively.

B. Stage II: Followers Game - SBS Modeling

The strategy of each SBS m is to optimize its own traffic and the amount of offloaded traffic in order to maximize the total utility under the given fraction of resource and incentive proposed by the MBS. For the non-uniform economics incentive scheme, the MBS sets different economics incentives for different SBSs to encourage SBSs to offload traffic for MBS. We denote the economic incentive for SBS m as β_m. The revenue function and optimization problem of SBS m are given as follows:

$$\begin{aligned}
\mathbf{P_{SBS}} : \max_{\boldsymbol{x}_m, \boldsymbol{l}_m \geq 0} \; & P_m(\boldsymbol{x}_m, \boldsymbol{l}_m, \alpha_m, \beta_m) = \alpha_m \log\Big(\sum_{i \in \mathcal{K}_m} x_{m,i} \\
& + \sum_{i \in \mathcal{S}_m} l_{m,i}\Big) - \alpha_m \sum_{i \in \mathcal{S}_m} l_{m,i}^2 + \beta_m \sum_{i \in \mathcal{S}_m} l_{m,i}, \\
\text{s.t.} \quad & \sum_{i \in \mathcal{K}_m} \frac{x_{m,i}}{C_{m,i}} + \sum_{i \in \mathcal{S}_m} \frac{l_{m,i}}{C_{m,i}} \leq \alpha_m, \\
& x_{m,i} \geq x_{m,i}^{min}, \; \forall i \in \mathcal{K}_m, \\
& \sum_{i \in \mathcal{S}_m} l_{m,i} \leq l_m^{tot},
\end{aligned} \tag{4}$$

where $x_{m,i}^{min} \leq C_{m,i}, \; \forall i \in \mathcal{K}_m$ is the minimum traffic demand (QoS) required by SBS m's users. The choice of the first term of this objective function is motivated by its concave characteristic which reflects its diminishing return property with the total transmission rate of the SBS m. For tractability we choose a quadratic function to model the convex cost (second term) [2], which can be interpreted as the energy consumption of SBS m for servicing the offloaded data $l_{m,i}, \forall i$. In addition, we use a linear economic incentive function (the third term) which means serving an additional offloaded traffic unit results in an additional β_m unit of monetary or economic incentive. Since the objective function is strictly concave and the constraint set is compact and convex, there exists a unique solution $(\boldsymbol{x}_m^*, \boldsymbol{l}_m^*), \forall m$, for given α_m and β_m in Stage-II.

C. Stage I: Leader Game - MBS Modeling

The profit of MBS is given by the following revenue function:

$$P_M(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{l}) = \mathcal{U}_M\Big(d_0 + \sum_{m \in \mathcal{M}} \big(l_m^{tot} - \sum_{i \in \mathcal{S}_m} l_{m,i}\big)\Big) - \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{S}_m} \beta_m l_{m,i}, \tag{5}$$

where $\mathcal{U}_M(\cdot)$ represents the *utility* for serving remaining traffic $d_0+\sum_{m\in\mathcal{M}}(l_m^{tot}-l_m)$ using $(1-\sum_{m\in\mathcal{M}}\alpha_m)$ of the subframes. Even though we use a general form of the utility function, naturally, a function that is chosen arbitrarily may not lead to an equilibrium. Therefore, utility functions with well-defined properties must be chosen in order to achieve an unique equilibrium, e.g., a logarithmic utility or linear utility. The second term of (5) represents the *cost* MBS will pay for SBSs. The optimization problem for MBS can be formulated as

$$
\begin{aligned}
\mathbf{P_{MBS}} : & \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \quad P_M(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{l}) \\
\text{s.t.} \quad & 0 \le \sum_{m\in\mathcal{M}} \alpha_m \le 1-\delta, \quad \forall m \in \mathcal{M}, \\
& \beta_{min} \le \beta_m \le \beta_{max}, \quad \forall m \in \mathcal{M}, \\
& \frac{d_{0,i}}{C_{0,i}} \le (1-\sum_{m\in\mathcal{M}}\alpha_m)/|\mathcal{K}|, \forall i \in \mathcal{K}_0 \\
& \frac{d_i - l_{m,i}}{C_{0,i}} \le (1-\sum_{m\in\mathcal{M}}\alpha_m)/|\mathcal{K}|, \forall i \in \mathcal{S}_m, \forall m
\end{aligned}
\tag{6}
$$

where δ is a fixed threshold. In LTE-HetNets, with fixed eICIC pattern, the MBS could sacrifice up to 37.5% of its resource to small cells [3]. However, the fixed number of ABS usually leads to poor performance of network as shown in [3]. In our simulation mentioned in Section V, we set $\delta = 0.1$ in order to investigate the variation in optimal fraction of resource α.

IV. Two-Stage Stackelberg Game: Equilibrium and Algorithm

The objective of the proposed Stackelberg game is to find the Stackelberg equilibrium (SE), in which both MBS and APs have no incentive to deviate. Since the strategy of one stage will affect the other stage's strategy, we employ the backward induction method to analyze it.

A. Stackelberg equilibrium

Denoting a solution to the MBS's profit maximization by $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*)$, we have the following definition

Definition 1. $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{l}^*,\boldsymbol{x}^*)$ is a SE for the proposed game if it satisfies the following conditions for any values of $(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{l},\boldsymbol{x})$

$$
\begin{aligned}
P_M(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{l}^*) &\ge P_M(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{l}^*), \forall \alpha_m,\beta_m, \\
P_m(\boldsymbol{x}_m^*,\boldsymbol{l}_m^*,\alpha_m^*,\beta_m^*) &\ge P_m(\boldsymbol{x}_m,\boldsymbol{l}_m,\alpha_m^*,\beta_m^*), \forall x_{m,i}, l_{m,i}.
\end{aligned}
\tag{7}
$$

For the proposed game in this paper, the SE can be obtained as follow: the Stage-II problem is first solved to obtain $(\boldsymbol{x}_m^*,\boldsymbol{l}_m^*),\forall m$, which is then used to solve the Stage-I problem to obtain $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*)$.

B. Optimal Solution at Stage II:

The Lagrangian of the SBS m problem (4) can be written as

$$
\begin{aligned}
L(\boldsymbol{x}_m,\boldsymbol{l}_m,\nu,\eta,\boldsymbol{\lambda},\boldsymbol{\zeta}) &= \alpha_m \log(\sum_{i\in\mathcal{K}_m} x_{m,i} + \sum_{i\in\mathcal{S}_m} l_{m,i}) \\
&- \alpha_m \sum_{i\in\mathcal{S}_m} l_{m,i}^2 + \beta_m \sum_{i\in\mathcal{S}_m} l_{m,i} - \nu(\sum_{i\in\mathcal{K}_m}\frac{x_{m,i}}{C_{m,i}} + \sum_{i\in\mathcal{S}_m}\frac{l_{m,i}}{C_{m,i}} - \alpha_m) \\
&+ \sum_{i\in\mathcal{K}_m}\lambda_i(x_{m,i}-x_{m,i}^{min}) - \eta(\sum_{i\in\mathcal{S}_m} l_{m,i} - l_m^{tot}) + \sum_{i\in\mathcal{S}_m}\zeta_i l_{m,i},
\end{aligned}
\tag{8}
$$

where ν, η, λ, and ζ are the nonnegative Lagrange multipliers associated with the constraints. By using KKT conditions [6], we have the following result.

Algorithm 1 Branch and Bound Method

input: $\epsilon > 0$
initialize: $k=0$; $\boldsymbol{Q} = \{\mathcal{Q}_{init}\}$; $L_0 = \Phi_{lb}(\mathcal{Q}_{init})$; $U_0 = \Phi_{ub}(\mathcal{Q}_{init})$;
repeat
  $k \leftarrow k+1$;
  $\mathcal{Q}_k = \{\mathcal{Q} \in \boldsymbol{Q} | m = \text{argmin}(|z_m^* - 1/2|)\}$;
  $\mathcal{Q}_k^{(0)} = \{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{z}|z_m = 0\}$; $\mathcal{Q}_k^{(1)} = \{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{z}|z_m = 1\}$;
  $\boldsymbol{Q} = \{\boldsymbol{Q} \setminus \mathcal{Q}_k\} \cup \{\mathcal{Q}_k^{(0)}, \mathcal{Q}_k^{(1)}\}$;
  for $\mathcal{Q}_k^{(i)}, i \in \{0,1\}$ **do**
    Calculate $\Phi_{lb}(\mathcal{Q}_k^{(i)})$ and $\Phi_{ub}(\mathcal{Q}_k^{(i)})$;
  $U_k = \min(U_k, \Phi_{ub}(\mathcal{Q}_k^{(i)}), i = 0,1)$;
  $L_k = \min(L_k, \Phi_{lb}(\mathcal{Q}_k^{(i)}), i = 0,1)$;
  $\mathcal{Q}^{(pru)} = \{\mathcal{Q} \in \boldsymbol{Q} | \Phi_{lb}(\mathcal{Q}) \ge U_k)\}$;
  $\boldsymbol{Q} = \{\boldsymbol{Q} \setminus \mathcal{Q}^{(pru)}\}$;
until $U_k - L_k \le \epsilon$;

Theorem 1. *If a optimal solution exists in Stage-II, then it is symmetric, i.e.,* $l_{m,i}^* = l_{m,j}^*, \forall i,j \in \mathcal{S}_m$, *and* $x_{m,i}^* = x_{m,j}^*, \forall i,j \in \mathcal{K}_m, \forall m \in \mathcal{M}$.

Proof: We can prove this theorem by contradiction similar to Theorem 4 in [5]. □

Theorem 2. *For given* α_m *and* β_m*, the unique optimal solution for Stage-II is:*

$$
l_{m,i}^* = \begin{cases} \frac{l_m^{tot}}{|\mathcal{S}_m|}, & \text{if } \beta_m \ge \frac{2\alpha_m l_m^{tot}}{|\mathcal{S}_m|}; \\ \frac{\beta_m}{2\alpha_m}, & \text{if } \beta_m < \frac{2\alpha_m l_m^{tot}}{|\mathcal{S}_m|}, \end{cases}
\tag{9}
$$

and

$$
x_{m,i}^* = \frac{\alpha_m - l_{m,i}^* \sum_{i\in\mathcal{S}_m}\frac{1}{C_{m,i}}}{\sum_{i\in\mathcal{K}_m}\frac{1}{C_{m,i}}}.
\tag{10}
$$

Proof: The proof is omitted here for brevity. □

Remark. It is observed from (9) that l_m^* is a piecewise function of the fraction resource α_m and economics incentive β_m. If economics incentive is higher than a threshold, i.e., $2\alpha_m l_m^{tot}/|\mathcal{S}_m|$, then SBS m will offload all traffic demand l_m^{tot} from MUEs. Otherwise SBS m will offload a portion of traffic demand from MUEs. If the economics incentive equals 0 then the SBS does not admit any traffic demand from MUEs.

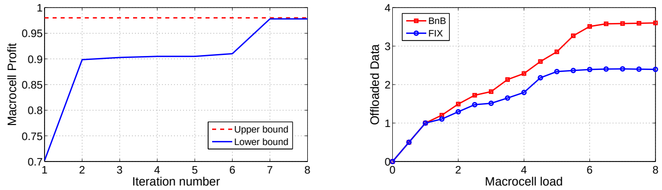
C. Optimal Solution at Stage I:

We now characterize the optimal solution of the Stage-I based on the optimal solution of Stage II. For each SBS m, we introduce the following indicator variable

$$
z_m = \begin{cases} 1, & \text{if } \ l_{m,i} = \frac{\beta_m}{2\alpha_m}, \beta_m < \frac{2\alpha_m l_m^{tot}}{|\mathcal{S}_m|}; \\ 0, & \text{otherwise.} \end{cases}
\tag{11}
$$

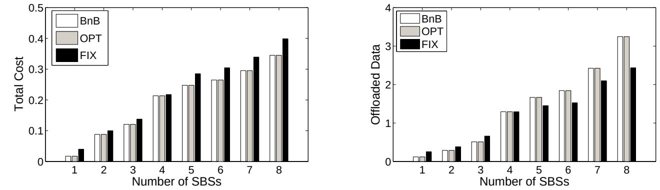
substituting (9) into (6), the MBS problem in Stage-I can be reformulated as

$$
\begin{aligned}
\mathbf{P'_{MBS}} : \max_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{z}} \; & \mathcal{U}_M(d_0 + \sum_{m\in\mathcal{M}}(l_m^{tot} - |\mathcal{S}_m|\frac{\beta_m}{2\alpha_m})z_m) \\
& - \sum_{m\in\mathcal{M}}\beta_m(l_m^{tot}(1-z_m) + |\mathcal{S}_m|\frac{\beta_m}{2\alpha_m}z_m), \\
\text{s.t.} \quad & 0 \le \sum_{m\in\mathcal{M}}\alpha_m \le 1-\delta, \quad \forall m \in \mathcal{M}, \\
& \beta_{min} \le \beta_m \le \beta_{max}, \quad \forall m \in \mathcal{M}, \\
& \frac{d_{0,i}}{C_{0,i}} \le (1-\sum_{m\in\mathcal{M}}\alpha_m)/|\mathcal{K}|, \forall i \in \mathcal{K}_0 \\
& \frac{d_i - \frac{\beta_m}{2\alpha_m}}{C_{0,i}} \le (1-\sum_{m\in\mathcal{M}}\alpha_m)/|\mathcal{K}|, \forall i \in \mathcal{S}_m, \forall m \\
& z_m \in \{0,1\}, \quad \forall m \in \mathcal{M}.
\end{aligned}
\tag{12}
$$



(a) (b)

Fig. 1: a) Bounds with 5 SBSs, b) Performance comparison



(a) (b)

Fig. 2: a) Cost for offloading traffic, b) Amount of offloaded data (l_m).

We see that problem (12) is a mixed-boolean programming, which requires exponential computation efforts to obtain the optimal solution through the exhaustive search. This motivates us to propose a practically efficient algorithm in Algorithm 1, to solve this problem.

We now develop an algorithm based on BnB method [7] to find the global optimal solution for Stage-I. Let $\mathcal{Q}_{init} = \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{z}\}$ be the original search space, including all possible combinations of indicator variable $\boldsymbol{z}$. The proposed algorithm maintains a set of subdomains $\boldsymbol{Q} = \{\mathcal{Q}_k \subset \mathcal{Q}_{init}, k = 1, 2, ...\}$ where k represents the iteration of the algorithm. For any $\mathcal{Q}_k$, consider $\Phi_{ub}(\cdot)$ and $\Phi_{lb}(\cdot)$ as the upper and lower bounds. We refer to $\Phi_{ub}(\mathcal{Q}_k)$ and $\Phi_{lb}(\mathcal{Q}_k)$ as the local upper and local lower bounds, respectively which correspond to subdomain $\mathcal{Q}_k$.

The algorithm starts by relaxing the boolean variable z_m i.e., $0 \le z_m \le 1, \forall m \in \mathcal{M}$, and solve the relaxed problem to obtain lower bound $L_0 = \Phi_{lb}(\mathcal{Q}_{init})$ for the original problem (12). Then, we round the optimal relaxed variables z_m^* to 0 or 1, $\forall m \in \mathcal{M}$, and solve (12) again with these fixed values of z_m^* to obtain the upper bound $U_0 = \Phi_{ub}(\mathcal{Q}_{init})$ (line 2). At each iteration k, we split the search space into two subspaces $\mathcal{Q}_k^{(0)}$ and $\mathcal{Q}_k^{(1)}$ by picking $\mathcal{Q}_k \in \boldsymbol{Q}$ such that $m = \text{argmin}(|z_m^* - 1/2|)$, then update $\boldsymbol{Q}$ by removing $\mathcal{Q}_k$ (lines 5-7). We then calculate the lower and upper bounds for each subspace and choose the one with the smallest lower bound (lines 8-11). Finally, the subspaces that satisfy $\Phi_{lb}(\mathcal{Q}) \ge U_k$ are removed, since every point in such space leads to the performance lower than the current upper bound (lines 12-13). If $U_k - L_k \le \epsilon$, the algorithm terminates.

Lemma 1. *The Algorithm 1 converges to the optimal solution of Stage-I of the proposed game.*

Proof: The proof is similar to the one provided in [7]. □

Remark.

1) Although the worst-case complexity of such a procedure is exponential, the actual running time could be fast when all partition variables are integers, which is the case in this paper.
2) At each iteration of our algorithm, we solve the relaxation of subproblems using interior-point algorithms [6]. However, the optimal relaxed solution may not be feasible. Then the lower bound and upper bound are set to be ∞. And the subproblem which is infeasible can be eliminated.
3) The optimal solution of Stage-II and the optimal solution of Stage-I achieved by Algorithm 1 represent a Stackelberg equilibrium of the proposed game.

V. Numerical Analysis

We consider a network with varying number of SBSs for performance evaluation. The MBS's utility is chosen to be logarithmic. The MBS's capacity $C_{0,i}$ is chosen to be uniformly distributed in [0,2]. The MUEs' traffic that cannot be offloaded $d_{0,i}$ is chosen from a uniform distribution over [0,0.5], and the offloaded traffic demand of MUEs which is randomly located in a SBS coverage area is uniformly distributed in [0,1]. The SBSs transmission rate $C_{m,i}$ is chosen to be uniformly distributed in [0,1].

Fig. 1a shows the evolution of the global lower and upper bounds. It can be seen that the globally optimal value is close to the upper bound for the first iteration (rounded result of the relaxation of problem (12)); however, it takes 7 more iterations (in case 5 SBSs) to be sufficiently close to the optimal value.

We compare the performance of Algorithm 1 (BnB) with two baselines. The first baseline, named OPT, is the optimal solution of problem (12) which is obtained by using the exhaustive search. The second baseline, called FIX, employs the fixed eICIC pattern [3], which represents a simple but inefficient scheme. It is observed that while Alg. 1 and OPT achieve the same performance, the scheme FIX is not as efficient as the others. In Fig. 1b, it is observed that by increasing macrocell load, our proposal outperforms the FIX scheme by up to 50% in terms of the offloaded traffic when the offloaded data reach the maximum value. This performance gain is observed because of the limitation of ABS of FIX scheme. In Fig. 2a, it can be seen that our proposed approach outperforms the FIX scheme by achieving 14% lower average cost for offloading traffic. Fig 2b illustrates the improvement of our proposal by 7% in term of average amount of offloaded data compared to the FIX scheme.

VI. Conclusion

In this paper, we have developed a Stackelberg game model for the coordinated data offloading and resource partitioning problem in co-channel two-tier heterogeneous networks. We have shown that the problem in Stage-I at MBS is a mixed-boolean nonlinear program, which is NP-hard. A low complexity solution method, based on the branch and bound technique has been proposed to solve the combinatorial nonconvex problem globally. We have shown that the proposed game achieves a unique Stackelberg equilibrium. Numerical results have confirmed that the proposed algorithm converges fairly fast in all considered setups and outperforms the conventional design.

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