

# Distributed Power and Channel Allocation for Cognitive Femtocell Network using a Coalitional Game in Partition Form Approach

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**Abstract**—The cognitive femtocell network (CFN) integrated with cognitive radio-enabled technology has emerged as one of the promising solutions to improve wireless broadband coverage in indoor environment for next-generation mobile networks. In this paper, we study a distributed resource allocation that consists of subchannel- and power-level allocation in the uplink of the two-tier CFN comprised of a conventional macrocell and multiple femtocells using underlay spectrum access. The distributed resource allocation problem is addressed via an optimization problem, in which we maximize the uplink sum-rate under constraints of intra-tier and inter-tier interferences while maintaining the average delay requirement for cognitive femtocell users. Specifically, the aggregated interference from cognitive femto users to the macrocell base station is also kept under an acceptable level. We show that this optimization problem is NP-hard and propose an autonomous framework, in which the cognitive femtocell users self-organize into disjoint groups (DJGs). Then, instead of maximizing the sum-rate in all cognitive femtocells, we only maximize the sum-rate of each DJG. After that, we formulate the optimization problem as a coalitional game in partition form, which obtains sub-optimal solutions. Moreover, distributed algorithms are also proposed for allocating resources to the CFN. Finally, the proposed framework is tested based on the simulation results and shown to perform efficient resource allocation.

**Keywords**—Cognitive femtocell network, resource allocation, power allocation, subchannel allocation, coalitional game, game theory.

## I. INTRODUCTION

In recent years, the number of mobile applications demanding high-quality communications have tremendously increased. For instance, high-quality video calling, mobile high-definition television, online gaming, and media sharing services always have connections with high-quality of services (QoS) requirements among devices and service providers [1]. In order to

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adapt to these requirements, the Third Generation Partnership Project (3GPP) Long-Term Evolution Advanced (LTE-Advanced) standard has been developed to support higher throughput and better user experience. Moreover, in order to accommodate a large amount of traffic from indoor environments, the next mobile broadband network uses the heterogeneous model, which consists of macrocells and smallcells [2], [3]. The smallcell model (such as femtocells) is one way of increasing coverage in dead zones in indoor environments, reducing the transmit power and the size of cells and improving spectrum reuse [4], [5].

In practice, a two-tier femtocell network can be implemented by spectrum-sharing between tiers, where a central macrocell is underlaid with several femtocells [6]. This network model is also called the cognitive femtocell network (CFN) [7], [8]. The CFN can be deployed successfully and cost-efficiently via two different spectrum-sharing paradigms: overlay and underlay [8]–[10]. The overlay access paradigm enables the cognitive femtocell user equipment (secondary user) to transmit their data only in spectrum holes where macrocell users (primary users) are not transmitting. A femtocell user equipment (CFUE) vacates its channel if it detects an occupancy requirement of a macro user equipment (MUE). In the underlay access, CFUEs are allowed to operate in the band of the macrocell network, while the overall interference from CFUEs occupancy on the same channel should be kept below a given threshold. Moreover, in this paradigm, entities in CFN are assumed to have knowledge of the interference caused by transmitters in the macrocell network [6], [9]. In this paper, we focus on the resource allocation in underlay CFN where the channel usages are based on the underlay cognitive transmission access paradigm [6], [8], [11].

In the CFN deployment, interference is a major challenge caused by overlapping area among cells in a network area and co-channel operations. The interference can be classified as: intra-tier (interference caused by macro-to-macro and femto-to-femto) or inter-tier (interference caused by macro-to-femto and femto-to-macro) [12], [13]. Specifically, the inter-tier interference, which is caused by using the underlay spectrum access, needs to be considered to protect the macrocell network [6]. In order to mitigate interference, some works have studied the downlink direction [12], [14]–[16]. Suppression of intra-tier interference using the coalitional game is studied in [12]. In [14], the authors employed frequency division multiple access

in terms of the area spectral efficiency and subjected to a sensible QoS requirement. The power and sub-carrier allocations for OFDMA femtocells based on underlay cognitive radios in a two-tier network are mentioned in [15]. A self-organization strategy for physical resource block allocation with QoS constraints to avoid co-channel and co-tier interference is investigated in [16]. However, the CFN uplink using the underlay paradigm is also an important challenge that needs to be considered [3]–[5]. In the uplink direction, the uplink capacity and interference avoidance of two-tier femtocell network were developed by Chandrasekhar *et al.* [7]. In [17], an interference mitigation was proposed by relaying data for macro users via femto users, based on the coalitional game approach and leasing channel. The power control under QoS and interference constraints in femtocell networks was studied in [18]. The distributed power control for spectrum-sharing femtocell networks using the Stackelberg game approach was presented in [6]. However, most of the above mentioned works only focus on single-channel operation and do not mention the channel allocation to the femto users. In [19], the uplink interference is considered in OFDMA-based femtocell networks with partial co-channel deployment without the femtocell users power control. Additionally, channel allocations are based on an auction algorithm for macrocell users and femtocell users. Clearly, the channel allocation in [19] is not efficient where users can reuse the channels by power control, as in [6], [18].

In this paper, we study an efficient distributed resource allocation for the CFN uplink in two-tier networks to overcome the drawbacks of the existing literature. The efficient distributed resource allocation in the multiple channel environment is represented by solving an optimization problem. The objective of this optimization problem is the uplink sum-rate. The intra-tier and inter-tier interference are considered with constraints in the optimization problem. Additionally, the guaranteed average delay requirements are at the minimum for the connected cognitive femtocell users, and the total interference at the MBS is kept under acceptable levels as well. We show that this optimization problem is an NP-hard optimization problem. Motivated by the design of self-optimization networks [4], [5], [8], [20], we propose a self-organizing framework in which CFUEs self-organize into disjoint groups (DJGs). By doing so, instead of maximizing the sum-rate in whole cognitive femtocells, we only maximize the sum-rate of each DJG where the computation of the original optimization is decomposed to the formed DJGs. Then, in order to solve the optimization problem at each DJG, we formulate this optimization problem as a coalitional game in the partition form, which obtains near-optimal solutions along with efficient resource allocation in a distributed way. The coalitional game is defined by a set of players who are the decision makers seeking to cooperate to form a coalition in a game [12], [21]. One kind of game expression is the coalitional game in the partition form that captures realistic inter-coalition effects in many areas, particularly in wireless communication networks [21], [22]. In this paper, CFUEs can join and leave a coalition to obtain the maximum data rate (denoted by individual payoff). The joining and leaving of CFUEs have to satisfy some constraints of the above mentioned optimization

problem. Specifically, the proposed game is solved based on the recursive core method [21], [22]. Throughout this method, the stability of the coalition formation is a result of the optimal channel and power allocation. The optimal power allocation to CFUEs corresponding to the network partition is obtained from sharing payoffs of CFUEs in a coalition. The geometric programming and dual-decomposition approaches, which are based on [18], [23]–[25], are proposed to determine the optimal power allocation in the coalition. Simulation results show that the proposed framework can be implemented in a distributed manner with an efficient resource allocation. Furthermore, the social welfare of the usable data rate in CFN under our solution is also examined via our simulation results. In addition, we also estimate the gap between the global optimal solution and the sub-optimal solution using proposed cooperative approach.

The main contributions of this paper are summarized as follows:

- We investigate an efficient resource allocation for the underlay CFN uplink that is addressed via a NP-hard optimization problem.
- The NP-hard optimization problem is simplified by dividing the network into DJGs. Then, it is solved by formulating the optimization problem as a coalition game in a partition form.
- We propose algorithms to allocate resources in a distributed way, in which the CFN implementation is self-organized and self-optimized.

The remainder of this paper is organized as follows: section II explains the system model and problem formulation. The optimization problem of the efficient resource allocation is formulated in section III, as is the DJGs formation. In section IV, we address the solutions to solve this optimization problem based on a coalitional game in the partition form approach. Section V provides simulation results. Finally, conclusions are drawn in section VI.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Firstly, we provide the system model followed by the problem formulation of primary network protection. Secondly, we consider the data transmission model in the uplink of CFUEs. Thirdly, we analyze a queuing model of CFUEs. Finally, we discuss some problems of licensed subchannel reuse among CFUEs in the CFN.

### A. System model

We consider an uplink CFN based on the underlay spectrum access paradigm, in which  $N$  CFBSs are deployed as in Fig. 1. These CFBSs are under-laid to the macrocell frequency spectrum and reuse the set of licensed subchannels of the uplink OFDMA macrocell. In the primary macrocell, there exist  $M$  subchannels which are correspondingly occupied by  $M$  macrocell user equipments (MUEs) in the uplink direction. Let  $\mathcal{N} = \{1, \dots, N\}$  and  $\mathcal{M} = \{1, \dots, M\}$  denote a set of all CFBSs and MUEs, respectively. A subchannel can contain one resource block or a group resource with the single carrier frequency division multiple access (SC-FDMA) technology in LTE system [26]. Every CFBS  $n \in \mathcal{N}$  is associated to the

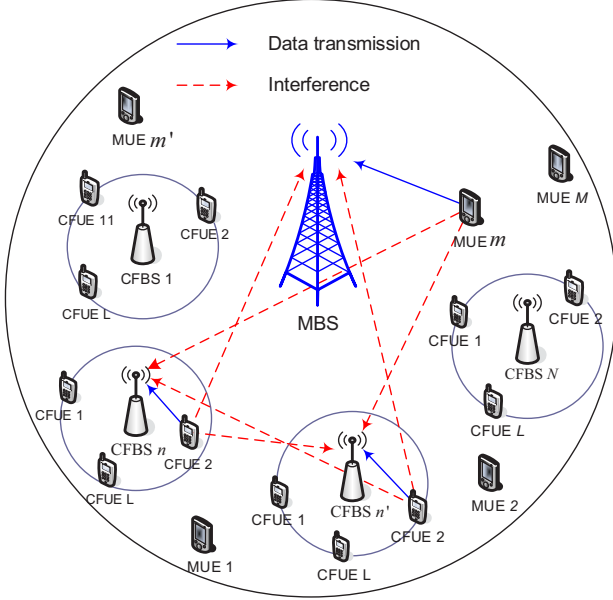


Fig. 1: System architecture of a cognitive femtocell network.

same  $L$  number of CFUEs. Let  $\mathcal{L}_n = \{1, \dots, L\}$  denote the set of CFUEs served by a CFBS  $n \in \mathcal{N}$ . Furthermore, cognitive modules are added to CFUEs and CFBSs to support self-organization, self-optimization as in [8]. Moreover, CFUEs and CFBSs exchange information via dedicated reliable feedback channels or wired back-hauls.

### B. Primary network protection

In the underlay CFN, the MBS of the macrocell needs to be protected against overall interference from CFUEs, as in [27]–[29]. The protection on subchannel  $m$  at the MBS is addressed as follows:

$$\sum_{l \in \mathcal{L}_n, n \in \mathcal{N}} \alpha_{ln}^m h_{ln,0}^m P_{ln}^m \leq \zeta_0^m, \quad \forall m \in \mathcal{M}, \quad (1)$$

where  $\alpha_{ln}^m$  is a subchannel allocation indicator defined as

$$\alpha_{ln}^m = \begin{cases} 1, & \text{if } l \in \mathcal{L}_n \text{ is allocated to subchannel } m, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

$h_{ln,0}^m$  denotes the channel gain between CFUE  $l \in \mathcal{L}_n$  and the primary MBS,  $P_{ln}^m$  is the power level of CFUE  $l \in \mathcal{L}_n$  using subchannel  $m$ , and  $\zeta_0^m$  is the interference threshold at the primary receiver MBS on subchannel  $m$ .

### C. Data transmission model in uplink

In our considered model, the data transmission of CFUEs is affected by the interference from the MUE and other CFUEs in other femtocells. Each CFUE is assumed to be assigned to one subchannel for a given time. The transmission rate of CFUE  $l \in \mathcal{L}_n$  on subchannel  $m$  follows the Shannon capacity as follows:

$$R_{ln}^m = B_w \log(1 + \Gamma_{ln}^m), \quad (3)$$

where  $B_w$  is the bandwidth of subchannel  $m$ ,  $\forall m \in \mathcal{M}$ , and  $\Gamma_{ln}^m$  is the Signal-to-interference-plus-noise ratio (SINR) of the CFUE  $l \in \mathcal{L}_n$  using subchannel  $m$  as follows:

$$\Gamma_{ln}^m = \frac{h_{ln}^m P_{ln}^m}{I_n^m + n_0}. \quad (4)$$

In (4),  $I_n^m$  denotes the total interference at CFBS  $n$  on subchannel  $m$ :

$$I_n^m = \sum_{l' \in \mathcal{L}_{n'}, n' \in \mathcal{N}} h_{l'n}^m P_{l'n}^m + h_{m,n}^m P_{m0}^m, \quad (5)$$

where  $n' \neq n$ ;  $h_{ln}^m$ ,  $h_{l'n}^m$  and  $h_{m,n}^m$  are the channel gains between CFUE  $l$  and CFBS  $n$ , CFUE  $l' \in \mathcal{L}_{n'}$  and CFBS  $n$ , and MUE  $m$  and CFBS  $n$ , respectively;  $n_0$  is the noise variance of the symmetric additive white Gaussian noise;  $h_{m,n}^m P_{m0}^m$  is the inter-tier interference at CFBS  $n$  from MUE  $m$ ; and  $\sum_{l' \in \mathcal{L}_{n'}, n' \in \mathcal{N}} h_{l'n}^m P_{l'n}^m$  is total intra-tier interference from CFUEs at the other CFBSs that use the same subchannel  $m$ .

In order to successfully decode the received signals at the CFBS of the CFUE transmission, the SINR at CFBS  $n$  from CFUE  $l \in \mathcal{L}_n$  has to satisfy [30]:

$$\Gamma_{ln}^m \geq \gamma, \quad (6)$$

where  $\gamma$  is the SINR threshold to decode received signals at the CFBS,  $\forall n \in \mathcal{N}, \forall m \in \mathcal{M}$ .

Because the transmission on the CFUE-CFBS link can be dropped due to a certain outage event, the successful transmission of CFUEs can be computed based on the probability of maintaining the SINR above a target level  $\gamma$ , given by:

$$\xi_{ln} = \Pr(\Gamma_{ln}^m > \gamma). \quad (7)$$

This outage value can be reduced by employment of the Hybrid Automatic-Repeat-Request protocol with Chase Combining at the medium access layer [31]. According to this protocol, packets will be re-transmitted if they have not been successfully received at the receiver. This re-transmission can occur up to  $K_{max}$  times until the successful data transmission. Hence, if arrivals to CFUE  $l \in \mathcal{L}_n$  follow a Poisson process with arrival rate  $\lambda_{ln}$ , the effective arrive rate  $\tilde{\lambda}_{ln}$  with a maximum of  $K_{max}$  re-transmissions is computed as follows:

$$\tilde{\lambda}_{ln} = \lambda_{ln} \sum_{k=1}^{K_{max}} \xi_{ln} (1 - \xi_{ln})^{k-1}, \quad (8)$$

where  $(1 - \xi_{ln})$  is the error packet transmission probability of the connected link CFUE  $l \in \mathcal{L}_n$  to CFBS  $n$ , which is calculated based on (7), and  $\sum_{k=1}^{K_{max}} \xi_{ln} (1 - \xi_{ln})^{k-1}$  is the successful transmission probability of a data packet of CFUE  $l$  with a maximum of  $K_{max}$  re-transmissions.

Clearly, through (8), congestion at the queue of the CFUE occurs when the departure rate or data rate on the CFUE-CFBS link is lower than the acceptable threshold. This congestion leads to delaying data packets in the queueing model of the CFUE data transmission. The queueing model for CFUEs will be discussed in the next subsection.

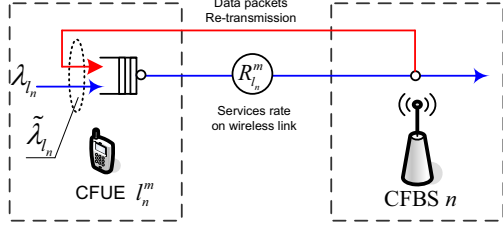


Fig. 2: The M/D/1 queuing model for data transmission of CFUEs.

#### D. Queuing model analysis for guaranteeing the CFUE demands

In this subsection, we address the data transmission of the CFUE using the M/D/1 queuing model [32], as shown in Fig. 2. In this queuing model, the arrival rate  $\lambda_{ln}$  depends on the data rate from the upper layer of CFUE  $l$ . Based on Little's law, the average waiting time of a packet in CFUE  $l$  can be calculated as follows:

$$D_{ln}^m = \frac{\tilde{\lambda}_{ln}}{2R_{ln}^m (R_{ln}^m - \tilde{\lambda}_{ln})}, \quad (9)$$

where  $R_{ln}^m$  is considered as the service rate in the M/D/1 queuing model determined by (3).

Assuming that, at the beginning of each time slot, the maximum delay requirement for each CFUE  $l \in \mathcal{L}_n$  is given by  $D_{ln}^m \leq D_{ln}^{\max}$ , the condition

$$R_{ln}^m \geq R_{ln}^{\text{th}} \quad (10)$$

has to be guaranteed. From (9) and the maximum delay value requirement  $D_{ln}^m = D_{ln}^{\max}$ , the data rate requirement  $R_{ln}^{\min}$  is calculated as follows:

$$R_{ln}^{\min} = \frac{\left( (D_{ln}^{\max} \tilde{\lambda}_{ln})^2 + 2D_{ln}^{\max} \tilde{\lambda}_{ln} \right)^{1/2} + D_{ln}^{\max} \tilde{\lambda}_{ln}}{2D_{ln}^{\max}}. \quad (11)$$

From (3), (4) and (10), we have the constraint of total interference to guarantee the minimum delay requirement of each CFUE as follows:

$$I_n^m + n_0 \leq h_{ln}^m P_{ln}^m \chi_{ln}, \quad (12)$$

where  $\chi_{ln} = \left( 2^{\frac{R_{ln}^{\min}}{B_w}} - 1 \right)$ .

Intuitively, from (3) and (10), in order to satisfy the minimum average delay requirement, the CFUE needs to increase its power greater than a power level threshold. However, this increase may produce harmful interference to other CFUEs, which leads to a reduced data rate of other CFUEs using the same subchannel, as in (3). Additionally, the increasing power level at the CFUEs using the same subchannel  $m$  will increase the overall interference at the MBS, as mentioned in (1). Therefore, when the power allocation to CFUEs cannot satisfy constraints (1), (3), and (10), CFUEs have an incentive

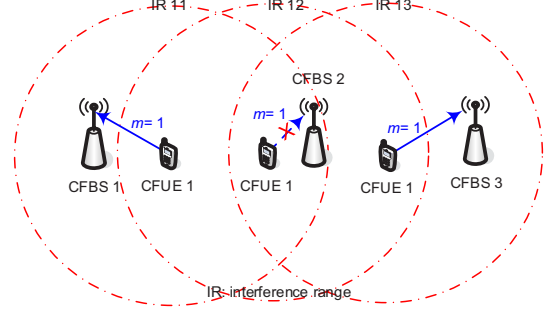


Fig. 3: The interference model for reusing a licensed subchannel among CFUEs.

to find another opportunity for selecting the subchannel from a set of subchannels.

#### E. Channel reuse in the CFN

In the CFN, a subchannel  $m$  that allocated to a CFUE  $n$  can be reused at other CFUEs if it overcomes the intra-tier interference constraints as considered in [16]. Certainly, in order to allocate a subchannel efficiently, the unlicensed subchannels need to be reused among CFUEs that are based on parameter  $\alpha_{l'n'}^m$  as follows:

$$\alpha_{l'n'}^m = \begin{cases} 0, & \text{if } l' \in \mathcal{L}_{n'}, n' \in T_{ln}^m, m \in \mathcal{M}, \\ 1, & \text{if } l' \in \mathcal{L}_{n'}, n' \notin T_{ln}^m, m \in \mathcal{M}, \end{cases} \quad (13)$$

where  $T_{ln}^m$  is a set of the CFBSs lying within the interference range of CFUE  $l \in \mathcal{L}_n$  on subchannel  $m$ . The CFBS  $n' \in T_{ln}^m$  if and only if:

$$IR_{ln,n'}^m \geq \gamma, \quad (14)$$

where the interference range  $IR_{ln,n'}^m$  is determined based on the SINR level from observing the surrounding CFBSs of CFUE  $l \in \mathcal{L}_n$  as follows:

$$IR_{ln,n'}^m = \frac{h_{ln'} P_{ln}^{m,\max}}{h_{mn'} P_{m0}^m + n_0}, \quad (15)$$

and  $P_{ln}^{m,\max}$  is the maximum transit power of CFUE  $l \in \mathcal{L}_n$  that can be allocated on subchannel  $m$ . In order to illustrate the reuse of subchannels among CFUEs and to form table  $T_{ln}^m$ , we present a simple example as follows.

**Example 1:** Let us consider reusing a licensed subchannel  $m = 1$  among three CFUEs as shown in Fig. 3, in which each CFBS serves one CFUE. The table  $T_{ln}^m$  of each CFUE is constructed by considering the interference range of the CFUEs based on (14), (15). Then, the CFBSs that belong to the table of CFUEs 11, 12 and 13 are  $T_{11}^1 = \{1, 2\}$ ,  $T_{12}^1 = \{2\}$  and  $T_{13}^1 = \{2, 3\}$ , respectively. From (13), if subchannel  $m = 1$  is allocated to CFUE 11, then  $\alpha_{11}^1 = 1$ ,  $\alpha_{12}^1 = 0$ , and  $\alpha_{13}^1 = 1$ . This means that subchannel 1 cannot be reused at CFUE 12 from CFUE 11 but CFUE 13 can reuse this subchannel. Similarly, we consider principles for CFUEs 12 and 13, respectively. The detail of the table  $T_{ln}^m$  formation is discussed in section III.B.

In order to illustrate the subchannel and power allocation efficiently and optimally, we address an optimization problem in the next section.

### III. OPTIMIZATION PROBLEM AND DJG FORMULATION

In this section, we first discuss an optimization problem that represents an efficient resource allocation for the underlay CFN uplink. Secondly, we address the network partition into DJGs to decompose the computation of the optimization problem into distributed computations at DJGs.

#### A. Optimization problem formulation

The objective is to maximize the uplink sum-rate of the whole CFN. The constraints include minimization of the intra-tier and inter-tier interference levels with similarly minimal average delay requirements for connected CFUEs. Specifically, the total interference at the MBS is also kept under acceptable levels. Moreover, the subchannels are efficiently reused among CFUEs. From the discussion of our considered problems in section II, the optimization problem is formulated as follows:

OPT1:

$$\underset{(\alpha_{ln}^m, P_{ln}^m)}{\text{maximize}} \quad \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}_n} \alpha_{ln}^m R_{ln}^m \quad (16)$$

subject to: (1), (12), (13),

$$0 \leq \sum_{m \in \mathcal{M}} \alpha_{ln}^m \leq 1, \quad n \in \mathcal{N}, l \in \mathcal{L}_n, \quad (17)$$

$$\alpha_{ln}^m = \{0, 1\}, \quad m \in \mathcal{M}, n \in \mathcal{N}, l \in \mathcal{L}_n, \quad (18)$$

$$P_{ln}^{m, \min} \leq P_{ln}^m \leq P_{ln}^{m, \max}, \forall m, n, l. \quad (19)$$

The purpose of OPT1 is to allocate the optimal subchannels and power levels for CFUEs in order to maximize the CFN uplink sum-rate. The constraints (1), (6), (12), (13) are addressed in section II. Moreover, some conditions of subchannel allocation indicator  $\alpha_{ln}^m$  are represented in (17), (18) and (19). Constraint (17) shows that each CFUE  $l \in \mathcal{L}_n$  is only assigned one subchannel at a given time, and (18) is represented as in (2). Constraint (19) represents the power range of each CFUE  $l \in \mathcal{L}_n$ , which has to be within the threshold range. The thresholds  $P_{ln}^{m, \max}$  and  $P_{ln}^{m, \min}$  indicate the limitations of the power range of CFUE  $l \in \mathcal{L}_n$  on each licensed subchannel  $m$ .

Clearly, OPT1 is an NP-hard optimization problem because, in order to find the optimal solution, we must allocate subchannels with mixed integer variable  $\alpha_{ln}^m$  and non-integer variable  $P_{ln}^m$  along with mixed linear and nonlinear constraints [33], [34]. The NP-hard optimization problem along with the huge number of CFUEs makes it infeasible to find an optimal solution. In order to solve OPT1, we propose a solution that is based on the DJGs' formation and coalitional game in the partition form approach. A sketchy summary of the proposed solution is illustrated in Fig. 4. Firstly, CFUEs in the network self-organize into DJGs using Algorithm 1 (to be discussed in section III.B), in which the interference from

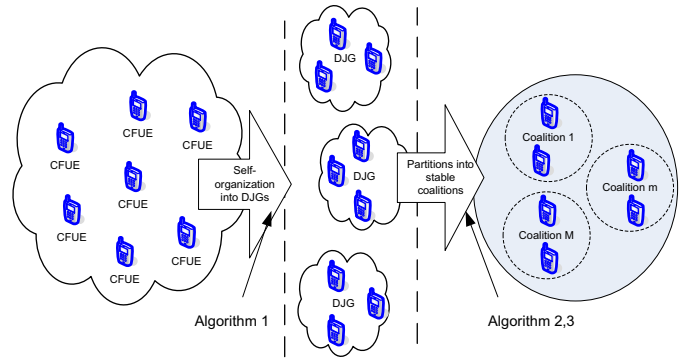


Fig. 4: The proposed structure for solving OPT1 based on the DJGs' formation and coalitional form in the partition form.

CFUEs transmission in a DJG is not affected by CFUEs transmission among other DJGs. The purposes of this division are to reduce feedback among network entities and decompose the computation in OPT1 into distributed computations at DJGs. Secondly, CFUEs in DJGs will be considered as players in the coalitional game. CFUE cooperates with other CFUEs to choose subchannel and power levels in order to form stable coalitions using Algorithms 2 and 3 (described in section IV.C). In the next subsection, we simplify the optimization problem OPT1 by addressing DJGs formation.

#### B. DJGs formation

In the CFN deployment, depending on the aims of network designers and mobile user equipments, the locations of CFBSs and CFUEs are distributed randomly in a network area. Some femtocells less be affected by interferences from others femtocells. Thus, CFUEs can self-organize into DJGs as addressed in Algorithm 1.

At the beginning of each period, the CFBS broadcasts a message that contains CFBS identification (ID) and interference from MUEs (line 1). The CFUE decodes the received messages (line 2,3) and detects the surrounding CFBSs within the interference range (IR) using (14) and (15). The detected CFBSs are stored in table  $T_{ln}^m$  and then form table  $T_{ln} = \{t_{ln'}\}_{M \times |\mathcal{N}_l|}$  (line 4); here,  $t_{ln'} = 1$  if  $n' \in T_{ln}^m$ , else  $t_{ln'} = 0$ , and  $\mathcal{N}_l$  is the set of CFBSs detected by CFUE  $l$ . Then, the CFUE sends its information  $T_{ln}$  to its CFBS  $n$  (line 5). The CFBS  $n$  collects information  $T_{ln}$  of all its CFUEs and constructs table  $T_n = \{t_{l,n'}^m\}_{M \times \max(|\mathcal{N}_l|) \times \mathcal{L}_n}$  (line 6). Here,  $\{t_{l,n'}^m\}$  equals to 1 if  $n' \in T_{ln}$ ; otherwise, it equals to 0. Simultaneously, the CFBSs exchanges information the table  $T_n$  with the CFBSs  $n' \in T_n$  to form a disjoint group  $g$  (line 7). Then, the CFBSs build a subchannel reuse table among CFUEs based on (13) (lines 8, 9). For convenience, let denote  $T_g = \{\alpha_{ln}^m\}_{(|\mathcal{L}_g| \times |\mathcal{L}_g| \times M)}$  the reuse table of CFUEs at DJG  $g$ . Here, denoting by  $\mathcal{L}_g = \cup_{n \in \mathcal{N}_g} \mathcal{L}_n$  is the set of CFUEs in DJG  $g$ , and  $\mathcal{N}_g$  is the set of CFBSs belonging group  $g$ . Clearly, some CFUEs can be self-organized into disjoint group  $g$ . After CFUEs form DJGs, CFUEs only exchange information for group formation if and only if the network has new events such



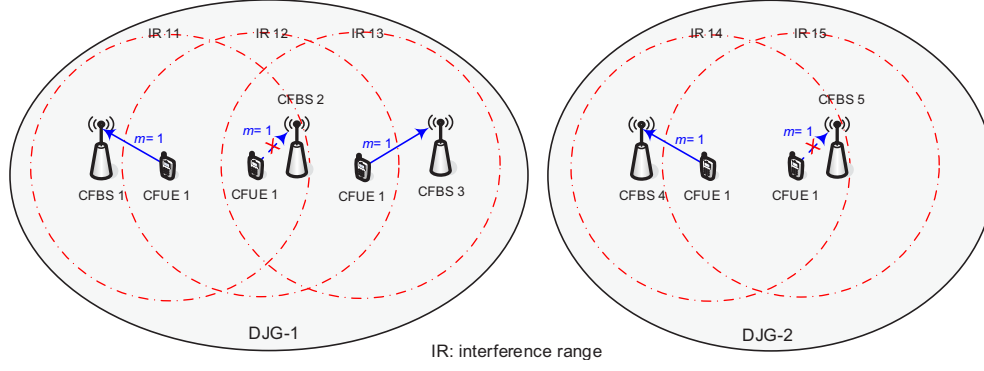


Fig. 5: The DJG formulation in example 2.

**Algorithm 1** : The self-organization of CFUEs into DJGs

- 1: Initially,  $T_{ln} = \emptyset$ ,  $P_{ln}^m = P_{ln}^{m,\max}$ ,  $\forall m \in \mathcal{M}$ ,  $\forall n \in \mathcal{N}$ ,  $l \in \mathcal{L}_n$ . The CFBSs broadcast TxCFemtoBS-ID messages based on pilot channels as discussed in [16].
- \* **At the CFUE**,  $\forall l \in \mathcal{L}_n, \forall n \in \mathcal{N}$ , **do**:
- 2: decodes TxCFemtoBS-ID message of the surrounding CFBSs.
- 3: estimates  $h_{ln'}$ ,  $n' \neq n$  based on received RSSIs.
- 4: constructs table  $T_{ln}$  based on (14).
- 5: sends table  $T_{ln}$  to its CFBS.
- \* **At the CFBS**  $n$ ,  $\forall n \in \mathcal{N}$ , **do**:
- 6: collects  $T_{ln}$  from its CFUEs.
- 7: constructs table  $T_n$  based on table  $T_{ln}, \forall l \in \mathcal{L}_n$ .
- 8: exchanges  $T_n \rightleftharpoons T_{n'}, \forall n' \in T_n$ .
- 9: self-organizes into groups.
- 10: constructs tables  $T_g$  based on condition (13), then sends it to the network coordinator.

as the CFUEs' location or new joining CFUEs. In addition, exchanging information among CFBSs in the DJG formation can be processed via asynchronous inter-cell signaling [35], [36]. A femtocell signals its status information to neighbor femtocells periodically and updates its CFUEs local information upon reception of the other femtocells signaling. For clear understanding of DJG formation, we provide Example 2 as below.

**Example 2:** Let us consider a CFN model consisting of five CFBSs, as shown in Fig. 5, in which each CFBS contains an CFUE. Assume that the interference ranges (IRs) of CFUEs are determined and exist as shown in Fig. 5 (Steps 1-3). Intuitively, table  $T_{ln}$  is constructed as in Table I (Step 4), where “1” indicates the CFBS belongs to the CFUE’s IR, “0” represents the CFBS does not belong to the IR of the CFUE, and  $\emptyset$  indicates that the CFUE does not receive the CFBS’s pilot signals. Because we only consider one subchannel, the tables  $T_1, T_2, T_3, T_4$  and  $T_5$  are also represented as  $T_{11}^1, T_{12}^1, T_{13}^1, T_{14}^1$  and  $T_{15}^1$  as in Table II, respectively. In order to obtain databases of tables of the surrounding CFBSs, the CFBS  $n$  exchanges table  $T_n$  with other CFBSs  $n' \in T_n$ . By doing so, the CFUEs {11}, {12} and {13} have the same database as in Table II and form a disjoint group, namely DJG-1. Moreover, DJG-2 is formed by CFUEs {14} and {15}. After finishing disjoint group formation, subchannel reuse tables for DJGs are

TABLE I: The table  $T_{ln}$  formulation of all CFUEs.

| Table $T_{ln}$ | CFemtoBS_ID |        |        |        |        |
|----------------|-------------|--------|--------|--------|--------|
|                | CFBS-1      | CFBS-2 | CFBS-3 | CFBS-4 | CFBS-5 |
| $T_{11}$       | 1           | 1      | 0      |        |        |
| $T_{12}$       | 0           | 1      | 0      |        |        |
| $T_{13}$       | 0           | 1      | 1      |        |        |
| $T_{14}$       |             |        |        | 1      | 1      |
| $T_{15}$       |             |        |        | 0      | 1      |

TABLE II: The subchannel reuse table  $T_g$  among CFUEs.

| Channel = 1 | CFUEs |    |    |    |    |
|-------------|-------|----|----|----|----|
|             | 11    | 12 | 13 | 14 | 15 |
| 11          | 1     | 0  | 1  |    |    |
| 12          | 0     | 1  | 0  |    |    |
| 13          | 1     | 0  | 1  |    |    |
| 14          |       |    |    | 1  | 0  |
| 15          |       |    |    | 0  | 1  |

formed based on (13) and exist as shown in Table II. Here, “1” denotes two CFUEs that can reuse subchannel 1, “0” denotes two CFUEs that cannot reuse subchannel 1, and the  $\emptyset$  denote two CFUEs belonging to different DJGs.

After establishing DJGs, without loss of generality, we find the local optimal solution of OPT1 by finding an optimal solution of OPT1<sub>g</sub> in each DJG  $g$ , which is taken from OPT1 as follows:

OPT1<sub>g</sub> :

$$\max_{(\alpha_{ln}^m, P_{ln}^m)} \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}_g} \sum_{l \in \mathcal{L}_n} \alpha_{ln}^m R_{ln}^m \quad (20)$$

s.t.

$$\sum_{l \in \mathcal{L}_n, n \in \mathcal{N}_g} \alpha_{ln}^m h_{ln,0}^m P_{ln}^m \leq c_0^m, \quad m \in \mathcal{M}, \quad (21)$$

$$I_{n,g}^m + n_0 \leq h_{ln}^m P_{ln}^m \chi_{ln}, \forall n, m, l, \quad (22)$$

$$\alpha_{l'n'}^m = \begin{cases} 0, & \text{if } l' \in \mathcal{L}_{n'}, n' \in T_{ln}^m, m \in \mathcal{M}, \\ 1, & \text{if } l' \in \mathcal{L}_{n'}, n' \notin T_{ln}^m, m \in \mathcal{M}, \end{cases} \quad (23)$$

$$0 \leq \sum_{m=1}^M \alpha_{ln}^m \leq 1, \quad n \in \mathcal{N}_g, l \in \mathcal{L}_n, \quad (24)$$

$$\alpha_{ln}^m = \{0, 1\}, \quad n \in \mathcal{N}_g, l \in \mathcal{L}_n, m \in \mathcal{M}, \quad (25)$$

$$P_{ln}^{m,\min} \leq P_{ln}^m \leq P_{ln}^{m,\max}, \quad \forall n, m, l, \quad (26)$$

where let  $\mathcal{N}_g$  denote the set of CFBSs that belong to the DJG  $g$ . Constraint (23) is taken from (13), and  $n, n' \in \mathcal{N}_g$ . Herein, the network size is decreased, but OPT1 $_g$  is still an NP-hard optimization problem. In the next section, we discuss in detail how to find the optimal solution of OPT1 $_g$ .

We note that, the intra-tier interference  $I_{n,g}^m$  in (22) is determined based on (3) as follows:

$$I_{n,g}^m = Z_{n,g}^m + Z_{n,g'}^m + h_{m,n}^m P_{m0}^m, \quad (27)$$

where  $Z_{n,g}^m = \sum_{l' \in \mathcal{L}_{n'}, n' \in \mathcal{N}_g} h_{l'n}^m P_{l'n'}^m$  is the intra-tier interference from CFUEs inside DJG  $g$  to CFBS  $m$  on subchannel  $m$ ;  $Z_{n,g'}^m = \sum_{l'' \in \mathcal{L}_{n''}, n'' \in \mathcal{N} \setminus \mathcal{N}_g} h_{l''n}^m P_{l''n''}^m$  is the intra-tier interference from CFUEs outside DJG  $g$  to CFBS  $n$  on subchannel  $m$ .

#### IV. RESOURCE ALLOCATION BASED ON COALITIONAL GAME IN PARTITION FORM.

Herein, the problem OPT1 is solved based on coalition game approach where CFUEs are players as follows. Firstly, the OPT1 $_g$  of each DJG  $g$  is formulated as a coalitional game in partition form. Secondly, we present the recursive core method to solve the proposed game. Thirdly, we address an implementation of the recursive core method to determine the optimal subchannel and power allocation in a distributed way. Finally, we consider the convergence and existence of the Nash-stable coalitions in the game.

##### A. Formulation OPT1 $_g$ as a coalitional game in partition form

The coalitional game is a kind of cooperative game that is denoted by  $(\mathcal{L}_g, U_{\mathcal{L}_g})$ , in which individual payoffs of a set of players  $\mathcal{L}_g$  are mapped in a payoff vector  $U_{\mathcal{L}_g}$ . The players have incentives to cooperate with other players, in which they seek coalitions to achieve the overall benefit or worth of the coalitions. The coalitional game in partition form is one such game expression, which is studied and applied in [21], [37], [38]. The worth of coalitions depend on how the players outside of the coalition are organized and on how the coalitions are formed. In the coalitional game, the cooperation of players to form coalitions is represented as the *non transferable utility* (NTU) game which is defined as follows [38]:

**Definition 1:** A coalitional game in partition form with NTU is defined by the pair  $(\mathcal{L}_g, U_{\mathcal{L}_g})$ . Here,  $U_{\mathcal{L}_g}$  is a mapping function such that every coalition  $\mathcal{S}_{m,g} \subset \mathcal{L}_g$ ,  $U_{\mathcal{L}_g}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  is a closed convex subset of  $\mathfrak{R}^{|\mathcal{S}_{m,g}|}$ , which contains the payoff vectors available to players in  $\mathcal{S}_{m,g}$ .

The mapping function  $U_{\mathcal{L}_g}$  is defined as follows:

$$U_{\mathcal{L}_g}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) = \{x \in \mathfrak{R}^{|\mathcal{S}_{m,g}|} | x_{ln}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) = R_{ln}^m(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})\}, \quad (28)$$

where  $x_{ln}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  is the individual payoff of player  $l \in \mathcal{L}_n$ , which corresponds to the benefit of a member in  $\mathcal{S}_{m,g}$  in partition form  $\phi_{\mathcal{L}_g}$  of group  $g$ . The CFUE  $l \in \mathcal{L}_n$  belongs to coalition  $\mathcal{S}_{m,g}$  depending on the partition  $\phi_{\mathcal{L}_g}$  in a feasible set  $\Phi_{\mathcal{L}_g}$  of players joining coalitions.

**Remark 1:** The singleton set  $U_{\mathcal{L}_g}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  is closed and convex [23].

In summary, the players make individual distributed decisions to join or leave a coalition to form optimal partitions that maximize their utilities and bring the overall benefit of coalitions. Based on the characteristics and principles of this game, we model the OPT1 $_g$  as a coalitional game in partition form. Instead of finding the global optimal that cannot be solved directly, CFUEs will cooperate with other CFUEs to achieve sub-optimal solution of the optimization problem OPT1 $_g$ .

**Proposition 1:** The optimization problem OPT1 $_g$  can be modeled as a coalitional game in partition form  $(\mathcal{L}_g, U_{\mathcal{L}_g})$ .

**Proof:** CFUE  $l \in \mathcal{L}_n$  and its data rate  $R_{ln}^m$  in a certain DJG  $g$  are considered as player  $l \in \mathcal{L}_n$  and individual payoff  $x_{ln}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  in the game, respectively. A set of CFUEs that belong to DJG  $g$  is represented as  $\mathcal{L}_g$ . The data rate  $R_{ln}^m$  is mapped in a payoff vector  $U_{\mathcal{L}_g}$  as in (28). In order to address formation of a certain coalition  $\mathcal{S}_{m,g}$ , we assume that there are only  $M + 1$  candidate coalitions  $\mathcal{S}_{m,g}$  that CFUEs can join,  $m \in \mathcal{M} \cup \{0\}$ . Here,  $\mathcal{S}_0$  means that CFUEs in this coalition are not allocated to any subchannel. Furthermore, each joining or leaving coalition of CFUEs has to satisfy the constraints of the optimization problem OPT1 $_g$ . The total data rate of CFUEs using the same subchannel bring the overall benefit or worth of a coalition. In order to find a sub-optimal value in OPT1 $_g$ , CFUEs have incentives to cooperate with other CFUEs. The cooperation information consists of the subchannels and power levels allocated to CFUEs. Intuitively, if CFUEs do not exchange their information with other CFUEs, the system performance will be degraded due to unsatisfied constraints (21)-(26), as mentioned in III.A. Moreover, from (27), the individual payoff of each CFUE depends on CFUEs belong to  $\mathcal{L}_g$  using the same subchannels. In addition, the individual payoff of CFUEs depend on using subchannels of CFUEs at other DJGs. Hence, in order to improve the individual payoff value of CFUEs, incentives to cooperate among CFUEs are necessary [12], [38], [39]. Therefore, the OPT1 $_g$  can be solved based on modeling as a coalitional game in partition form. ■

In order to solve this game, we simplify the coalition formation by assuming the value of the coalition depends on the outside coalitions, which is intra-tier interference from other DJGs. Then, we apply the recursive core method that is introduced in [22], [37] to solve this proposed game. Different from the core of Shapley value in the characteristic form, recursive core allows modeling of externalities for games in partition form [22]. The details of the solution are discussed in the following subsection.

##### B. Recursive core solution

As discussed in [22], [37], the NTU game in partition form is very challenging to solve. However, we can use the concept

of a recursive core to solve the proposed game [22]. Normally, the recursive core is defined for games with transferable utility (TU), where a real function captures the benefit of a coalition instead of mapping [22], [37]. Moreover, since the mapping function in (28) is a singleton set, we can define an adjunct coalition game as  $(\mathcal{L}_g, v)$  for the proposed game in which the benefit of each coalition  $\mathcal{S}_{m,g}$  is captured over a real line  $v(\mathcal{S}_{m,g})$ . By doing so, the original game  $(\mathcal{L}_g, U_{\mathcal{L}_g})$  is solved via the adjunct coalition game  $(\mathcal{L}_g, v)$  that is similar to games with transferable utility as studied in [12], [17], [39].

Whenever a CFUE detects a coalition  $\mathcal{S}_{m,g}$  that it can join, it compares its payoff in the current coalition and payoff in coalition  $\mathcal{S}_{m,g}$ . If the payoff in  $\mathcal{S}_{m,g}$  is greater than the current then CFUE will join it; otherwise will stay in the current coalition. In the NTU game, payoffs are a direct by product of the game itself due to power allocation of CFUEs on subchannel  $m$  to avoid violation of MBS protection (21) and providing guaranteed QoS to CFUEs in coalition as in (22). However, the payoff values of players are not determined solely by the data rate that CFUEs can achieve because of the CFUEs are the subscribed users of the wireless service providers. Meanwhile, the wireless service providers are the operators of the networks, so they can control the payoffs of the players via network coordinator from two aspects. First, service providers can physically provide different services to cooperative and non-cooperative users using rewards and punishment. Second, the CFUEs are stimulated to act cooperatively and improve the overall performance of the formed coalition  $\mathcal{S}_{m,g}$  or network while guaranteeing the MBS protection and CFUEs' QoS, which can be obtained via division of the single TU value  $v(\mathcal{S}_{m,g})$  [22], [37]. Whenever  $\mathcal{S}_{m,g}$  belongs to  $\phi_{\mathcal{L}_g}$ , the function value  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) \in v$  of the our game is determined as follows:

$$v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) = \begin{cases} \sum_{ln \in \mathcal{S}_{m,g}} x_{ln}, & \text{if (21), (22), and } |\mathcal{S}_{m,g}| \geq 1, \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

We can see that the mapping vector of the individual payoff value of CFUEs in (28) is uniquely given from (29) and the core in TU game is non-empty [37]. Thus, we are able to exploit the recursive core as a solution concept of the original game  $(\mathcal{L}_g, U_{\mathcal{L}_g})$  by solving the game  $(\mathcal{L}_g, v)$  while restricting the transfer of payoffs according to the unique mapping in (28). Here, the value  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  is the sum-rate of CFUEs allocated to the same subchannel  $m$  in partition  $\phi_{\mathcal{L}_g}$ . Through cooperating and sharing the payoff among CFUEs in the coalition  $m$ , CFUEs achieve their optimal power allocation to maximize each coalition  $\mathcal{S}_{m,g}$  to which they belongs (details are discussed in Algorithm 2 of section IV.C). Then, based on the results in each coalition, the optimal subchannel allocations are determined by finding the core of the game using the recursive core definition.

Before describing the recursive core definition, we define a residual game that is an important intermediate problem. The residual game  $(\mathcal{R}, v)$  is a coalitional game in partition form that is defined on a set of CFUEs  $\mathcal{R} = \mathcal{L}_g \setminus \mathcal{S}_{m,g}$ . CFUEs outside of  $\mathcal{R}$  are deviators, while CFUEs inside of  $\mathcal{R}$  are residuals [22], [37]. The residual game is still in partition

form and can be solved as an independent game, regardless of how it is generated [38]. For instance, when some CFUEs are deviators that reject an existing partition, they have incentives to join another coalition that satisfy (23), (24), (25) and subchannel reuse table  $T_g$ . Naturally, their decisions will affect the payoff values of the residual CFUEs. Hence, the residual game of CFUEs forms a new game that is a part of the original game. CFUEs in the residual game still have the possibility to divide any coalitional game into a number of residual games which, in essence, are easier to solve. The solution of a residual game is known as the residual core [22], [37], which is a set of possible game outcomes, i.e. possible partitions of  $\mathcal{R}$ . The recursive core solution can be found by recursively playing residuals games, which are defined as follows (mentioned in [22], definition 4):

**Definition 2:** The recursive core  $C(\mathcal{L}_g, v)$  of a coalitional formation game  $(\mathcal{L}_g, v)$  is inductively defined as follows:

1) *Trivial Partition.* The core of a game with  $\mathcal{L}_g$  is only an outcome with the trivial partition.

2) *Inductive Assumption.* Proceeding recursively, consider all CFUEs belonging to the DJG  $g$ , and suppose the residual core  $C(\mathcal{R}, v)$  for all games with at most  $|\mathcal{L}_g|-1$  CFUEs has been defined. Now, we define  $A(\mathcal{R}, v)$  as follows:  $A(\mathcal{R}, v) = C(\mathcal{R}, v)$ , if  $C(\mathcal{R}, v) \neq \emptyset$ ;  $A(\mathcal{R}, v) = \Omega(\mathcal{R}, v)$ , otherwise. Here, let  $\Omega(\mathcal{R}, v)$  denote a set of all possible outcomes of game  $(\mathcal{R}, v)$ .

3) *Dominance.* An outcome  $(\mathbf{x}, \phi_{\mathcal{L}_g})$  is dominated via coalition  $\mathcal{S}_m$  if at least one  $(\mathbf{y}_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}, \phi_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}) \in A(\mathcal{L}_g \setminus \mathcal{S}_{m,g}, v)$  there exists an outcome  $((\mathbf{y}_{\mathcal{S}_{m,g}}, \mathbf{y}_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}), \phi_{\mathcal{S}_{m,g}} \cup \phi_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}) \in \Omega(\mathcal{L}_g, v)$ , such that  $(\mathbf{y}_{\mathcal{S}_{m,g}}, \mathbf{y}_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}) \succ_{\mathcal{S}_{m,g}} \mathbf{x}$ . The outcome  $(\mathbf{x}, \phi_{\mathcal{L}_g})$  is dominated if it is dominated via a coalition.

4) *Core Generation.* The recursive core of a game of  $|\mathcal{L}_g|$  is a set of undominated partitions, denoted by  $C(\mathcal{L}_g, v)$ .

In Step 1, the core of a trivial partition is initialized with CFUEs belonging to coalition 0. Step 2 is an inductive assumption that establishes the dominance for a game of  $|\mathcal{L}_g|-1$  CFUEs through inductive steps of the formed coalitions. For instance, subchannel allocation permits assigning subchannels to CFUEs to bring dominance. Step 3 is the main step for checking and finding dominant coalitions, which captures the value of a coalition depending on partitions. We define  $\mathbf{x}$  as the payoff vector of players and  $\phi_{\mathcal{S}_{m,g}}$  as the partition of the user set  $\mathcal{L}_g$ . The payoff vector  $\mathbf{x}$  is an undominated coalition if there exists a way to partition that brings an outcome  $((\mathbf{y}_{\mathcal{S}_{m,g}}, \mathbf{y}_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}}), \phi_{\mathcal{S}_{m,g}} \cup \phi_{\mathcal{L}_g \setminus \mathcal{S}_{m,g}})$  that achieves greater reward to CFUEs of  $\mathcal{S}_{m,g}$ , compared to  $\mathbf{x}$ . Corresponding to each DJG partition, the individual payoffs of all CFUEs in the game are uniquely determined and undominated. Furthermore, the coalitions in the recursive core are formed to provide the highest individual payoffs or data rates of CFUEs, as detailed in Step 4.

### C. Implementation of the recursive core at each coalitional game formation in partition form at DJGs

We address implementation of the recursive core method to solve the proposed game, which is sketched in Fig. 6.



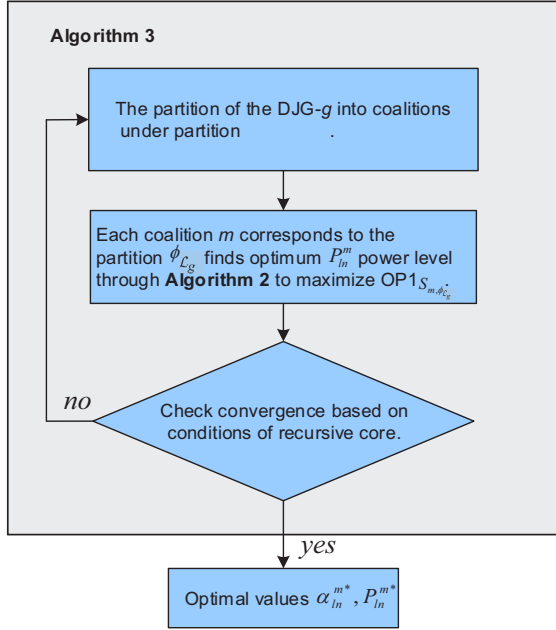


Fig. 6: The determination of the optimal solution of OPT1<sub>g</sub> based on the coalitional game in partition form.

As discussed in the above subsection, the game  $(\mathcal{L}_g, U_g)$  is solved via the game  $(\mathcal{L}_g, v)$ . According to this alternative, the coalition  $\mathcal{S}_{m,g}$  in a partition  $\phi_{\mathcal{L}_g}$  is represented by a real function  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  as in (29). Corresponding to the sub-channel allocation of CFUEs, some CFUEs can be allocated into the same subchannel  $m$ , which forms a coalition  $\mathcal{S}_{m,g}$ . Then, CFUEs optimize their individual payoffs by sharing with other CFUEs in the same coalition  $\mathcal{S}_{m,g}$ . In this case, CFUEs cooperate with others in coalition  $m$  to maximize the individual payoff and value  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$ . Sharing is achieved by finding optimum power values of each CFUE in the following optimization problem:

$$\text{OPT1}_{\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}} : \quad \underset{P_{ln}^m}{\text{maximize}} \quad v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) \quad (30)$$

$$\text{subject to:} \quad \sum_{ln \in \mathcal{S}_{m,g}} h_{ln,0}^m P_{ln}^m \leq \zeta_0^m, \quad (31)$$

$$Z_{n,g}^m + Z_{n,g'}^m + h_{m,n}^m P_{m0}^m + n_0 \leq h_{ln}^m P_{ln}^m \chi_{ln}, \quad l \in \mathcal{L}_n, ln \in \mathcal{S}_{m,g}, \quad (32)$$

$$P_{ln}^{m,\min} \leq P_{ln}^m \leq P_{ln}^{m,\max}, \quad ln \in \mathcal{S}_{m,g}, l \in \mathcal{L}_n. \quad (33)$$

The constraint (32) is taken from (22) and (27). When CFUE  $l \in \mathcal{L}_n$  belongs the coalition  $\mathcal{S}_{m,g}$ ,  $\alpha_{ln}^m$  is set to 1, otherwise is set to 0. Therefore, without loss of generality, we ignore parameter  $\alpha_{ln}^m$  in OPT1<sub>g</sub>. By finding the optimal power allocation to CFUEs, they will achieve an optimal individual payoff value that maximizes the worth of coalition  $\mathcal{S}_{m,g}$ . The optimal solution of OPT1<sub>g</sub> can be

found in a centralized or distributed way. We find the optimal solution in a distributed way. We solve the optimization problem by modeling as a geometric convex programming problem [18], [24], [29]. Then, the optimum values can be found using Karush–Kuhn–Tucker (KKT) conditions [18], [23], [24], [40], as follows:

$$P_{ln}^m = e^{y_{ln}^m} = \frac{1 + \mu_{ln}}{\beta h_{ln,0}^m + \eta_{ln} - \varsigma_{ln}}, \quad (34)$$

where  $[a]^+ = \max\{a, 0\}$ ; the Lagrange multipliers  $\beta$ ,  $\mu_{ln}$ ,  $\eta_{ln}$ ,  $\varsigma_{ln}$  and the consistency price  $\vartheta_{ln}$  for all CFUE  $ln \in \mathcal{S}_{m,g}$  are updated as (35), (39), (36), (37) and (38), respectively.

$$\beta(t) = \left[ \beta(t-1) + s_1(t) \left( \sum_{\forall ln \in \mathcal{S}_{m,g}} h_{ln,0}^m e^{y_{ln}^m} - \zeta_0^m \right) \right]^+, \quad (35)$$

$$\eta_{ln}(t) = [\eta_{ln}(t-1) + s_3(t) (y_{ln}^m(t) - \log P_{ln}^{m,\max})]^+, \quad (36)$$

$$\varsigma_{ln}(t) = [\varsigma_{ln}(t-1) + s_4(t) (-y_{ln}^m(t) + \log P_{ln}^{m,\min})]^+, \quad (37)$$

$$\vartheta_{ln}(t) = [\vartheta_{ln}(t-1) + s_5(t) (Z_{n,g}^m - e^{z_{n,g}^m})]^+. \quad (38)$$

We use the changing logarithm of the variables  $y_{ln}^m = \log P_{ln}^m$ . The parameter  $s_i(t)$  represents the step size satisfying

$$\sum_{t=0}^{\infty} s_i(t)^2 < \infty, \quad \text{and} \quad \sum_{t=0}^{\infty} s_i(t) = \infty, \quad \forall i = 1, 2, 3, 4, 5, \quad (40)$$

which leads to the convergence of algorithms [23]. Additionally, the variable  $z_{n,g}^m$  is also calculated from the KKT necessary conditions as follows:

$$e^{z_{n,g}^m} = \frac{\vartheta_{ln} (Z_{n,g'}^m + h_{m,n}^m P_{m0}^m + n_0)}{1 - \vartheta_{ln} + \mu_{ln}}, \quad (41)$$

where  $z_{n,g}^m = \log(Z_{n,g}^m)$ .

The details of finding the optimum value  $P_{ln}^m$  and  $e^{z_{n,g}^m}$  in OPT1<sub>g</sub> is expressed in [41]. The updating of values  $e^{z_{n,g}^m}$  and  $P_{ln}^m$  are expressed in Algorithm 2.

In Algorithm 2, the information being exchanged among CFUEs and CFBSs is based on feedback, such as ACK/NACK. This information can be exchanged in forms such as wired back-hauls, dedicated control channels, or pilot signals. The CFBS measures the intra-tier interference and inter-tier interference on subchannel  $m$  (Step 2). Then, the CFBS updates the value  $e^{z_{n,g}^m}(t+1)$  (Step 3). The Lagrange multiplier  $\mu_{ln}(t+1)$  and consistency price  $\vartheta_{ln}(t+1)$  are updated (Step 4). After that, the CFBS transmits  $\mu_{ln}(t+1)$  to CFUE  $l$  (Step 5). CFUE  $l \in \mathcal{L}_n$  estimates channel gain  $h_{ln,0}^m$  and the aggregated interference at the MBS (Step 6). Simultaneously, CFUE  $l \in \mathcal{L}_n$  gets updated values of  $\beta(t+1)$ ,  $\vartheta_{ln}(t+1)$  from CFBS  $n$ . We note that the value threshold  $\xi_0^m$  is updated from MBS via a weighted interference vector depending on the formed coalition. Then, the remaining Lagrange multipliers

$$\mu_{ln}(t) = \left[ \mu_{ln}(t-1) + s_2(t) \log \left( \frac{e^{-y_{ln}^m}}{h_{ln}^m} \left( e^{z_{n,g}^m} + Z_{n,g'}^m + h_{m,n}^m P_{m0}^m + n_0 \right) \right) - \log(\chi_{ln}) \right]^+, \quad (39)$$

**Algorithm 2** Distributed power allocation for CFUEs in the coalition  $\mathcal{S}_{m,g} \subseteq \phi_{\mathcal{L}_g}$

- \* Initialization:**
- 1: Initialize  $t = 0$ ,  $\beta(0) > 0$ ,  $\mu_{ln}(0) > 0$ ,  $\eta_{ln}(0) > 0$ ,  $\varsigma_{ln}(0) > 0$ ,  $\vartheta_{ln}(0) > 0$ ,  $P_{ln}^m(0) \in [P_{ln}^{m,\min}, P_{ln}^{m,\max}]$ ,  $\chi_{ln}$ ,  $\forall ln \in \mathcal{S}_{m,g}$ .
  - \* At CFBS  $n$ ,  $\forall n \in \mathcal{S}_{m,g}$ :**
  - 2: Measures the interference  $I_{n,g}^m$ .
  - 3: Calculates the variable  $e^{z_{n,g}^m}$  as in (41).
  - 4: Updates the Lagrange multiplier  $\mu_{ln}(t+1)$  and consistency price  $\vartheta_{ln}(t+1)$  using (39) and (38), respectively.
  - 5: Transmits  $\mu_{ln}(t+1)$  to CFUE  $l \in \mathcal{L}_n$ .
  - \* At CFUE  $l \in \mathcal{L}_n$ ,  $ln \in \mathcal{S}_{m,g}$ :**
  - 6: Estimates channel gain  $h_{ln,0}^m$  and compute the total interference at the MBS; Receives the updated value  $\mu_{ln}$ ,  $\vartheta_{ln}$ .
  - 7: Updates the Lagrange multipliers  $\beta$ ,  $\eta_{ln}$ , and  $\varsigma_{ln}$  from (35), (36), and (37), respectively.
  - 8: Calculates the power value  $P_{ln}^m(t+1)$  as in (34).
  - 9: Sends power value  $P_{ln}^m(t+1)$  and  $h_{ln,0}^m(t+1)$  to other CFUEs in the coalition  $\mathcal{S}_{m,g}$ .
  - \* Output:** Optimal transmit power level  $P_{ln}^{m*}$  and optimal value  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})^{m*}$  of the formed coalition  $m$ .

are updated via (35), (36), (37) (Step 7). After that, the CFUE updates the power value at time  $t+1$ , as in Step 8. Then, the CFUE sends its the newest power value  $P_{ln}^m(t+1)$  and newest channel gain  $h_{ln,0}^m(t+1)$  to other CFUEs that belong to coalition  $\mathcal{S}_{m,g}$  (Step 9).  $\text{OPT}_{\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}}$  is transformed to the convex optimization problem, the optimal duality gap is equal to zero, and step-sizes satisfy (40). Therefore, the solution  $P_{ln}^*$  will converge to the optimal solution under Algorithm 2.

After finishing Algorithm 2, in general, coalition  $\mathcal{S}_{m,g}$  guarantees the optimal sharing payoffs among members CFUEs. Simultaneously, we also find the optimum worth  $v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  of coalition  $\mathcal{S}_{m,g}$ . Based on the steps in the Definition 2, we propose Algorithm 3 to find recursive core which leads to the distributed subchannel and power allocation.

To obtain a partition in the recursive core, the CFUEs in  $\mathcal{L}_g$  use Algorithm 3. In the initial step, the information on subchannel reuse table  $T_g$  of DJG  $g$  is formed at the network coordinator (Step 1). The network coordinator makes decision to allocate subchannel to CFUEs with the assurance to protect MBS and provide guaranteed QoS to CFUEs. The individual payoff values of CFUEs in (28) are mapped to the formed coalitions via (29). Then, in the coalition formation, the value of whole game in group  $g$  ( $\sum_{m \in \mathcal{M} \cup \{0\}} v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$ ) is captured at the network coordinator. Network partitions of DJG  $g$  are controlled by network coordinator that makes a decision to assign CFUEs into coalition  $\mathcal{S}_{m,g}$ ,  $\forall m \in \mathcal{M} \cup \{0\}$  (Step 2). Additionally, network partitions of DJG  $g$  have to satisfy the principles in the subchannel reuse table  $T_g$  and Steps 3, 4 and 5. After that CFUEs find the optimal transmit power and individual payoff value based on Algorithm 2 in order to

**Algorithm 3** Distributed algorithm for subchannel and power allocation in cognitive femtocell network.

- \* Initialization:**
- 1: CFUEs and CFBSs form DJGs  $\leftarrow$  Algorithm 1; Forms table  $T_g$ ;  $\phi_{\mathcal{L}_g}^{(0)} = \{\{1\}, \{2\}, \dots, \{|\mathcal{M}|\}\}$  in which CFUEs are randomly allocated subchannel and transmit power with non-cooperative among FUEs.
  - \* Coalition formation at each DJG  $g$ :**
  - 2: CFUEs operate in cooperative mode and join into potential coalitions  $\phi_{\mathcal{L}_g} = \{\{0\}, \{1\}, \dots, \{|\mathcal{M}|\}\}$  that satisfy the table  $T_g$ .
  - 3: **for** player  $\{nl\} \in \mathcal{L}_g$  **do**
  - 4:     **for**  $\mathcal{S}_{m,g} \in \{\phi_{\mathcal{L}_g}^{(k-1)*} \setminus \{nl\}\}$  **do**
  - 5:         Set  $\phi_{\mathcal{L}_g}^{(k)} := \{\phi_{\mathcal{L}_g}^{(k-1)*} \setminus \mathcal{S}_{m,g}, \mathcal{S}'_{m,g} = \mathcal{S}_{m,g} \cup \{nl\}\}$ .
  - 6:         Find  $v(\mathcal{S}'_{m,g}, \phi_{\mathcal{L}_g}^{(k+1)}) \leftarrow$  based on (29) and Alg.2.
  - 7:         **if**  $\sum_{m \in \mathcal{M} \cup \{0\}} v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}^{(k)}) > \sum_{m \in \mathcal{M} \cup \{0\}} v(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}^{(k-1)*})$  **then**
  - 8:             Set  $\phi_{\mathcal{L}_g}^{(k)*} = \phi_{\mathcal{L}_g}^{(k)}$ .
  - 9:             Update  $\alpha_{ln}^{m*}, P_{ln}^{m*}$ .
  - 10:         **end if**
  - 11:     **end for**
  - 12: **end for**
  - \* Output:** Output the stable core of game  $(\mathcal{L}_g, v)$  consisting of both the final partition  $\phi_{\mathcal{L}_g}^*$ , subchannel allocation decision  $\alpha_{ln}^{m*}$ , and transmit power level  $P_{ln}^{m*}$ .

adopt their delay requirement and the MBS protection (Step 6). The network coordinator updates the undominance partition via Step 3 of Definition 2 (Steps 7 and 8). The algorithm is repeated until it converges to the stable partition  $\phi_{\mathcal{L}_g}^{(k)*}$ , which results in an undominated partition in the recursive core. Whenever undominated partition  $\phi_{\mathcal{L}_g}^{(k)*}$  is updated at time  $k$ , the network coordinator updates subchannel allocation to CFUEs (Step 9). CFBS of each femtocell shares the resource usage information among each other when they get the updated information from its CFUEs. Sharing of these confirmation causes overhead in the system. However, we have mitigated the amount of messages exchange by forming DJGs. By doing this, only CFUEs inside a DJG are permitted to exchange information. In addition, observation of the value  $v(\mathcal{S}_m, \phi_{\mathcal{L}_g})$  is done by network coordinator such as the femtocell gateway [39]. We note that the subchannel and power allocation of CFUEs are updated whenever a network partition is transferred from partition  $(k-1)$  to partition  $(k)$ , which produces Pareto dominates  $\mathcal{S}_{m,g}^{(k)}$ . The convergence and Nash-stable coalitions in Algorithm 3 are discussed in the next subsection.

#### D. Convergence and stable analysis of the proposed game

Convergence of the proposed game through four steps of the recursive core method is guaranteed as follows:

**Propriety 1:** Starting from any initial partition  $\phi_{\mathcal{L}_g}$ , using the Algorithm 3, coalitions of CFUEs merge together by Pareto dominance, which results in a stable network partition and lies in the non-empty recursive core  $C(\mathcal{L}_g, v)$ .

*Proof:* Let  $\phi_{\mathcal{L}_g}^{(k)}$  be the formed partition at iteration  $k$  that is based on principles of residual game (Steps 4 and 5 of Algorithm 3). The individual payoff of CFUE  $l \in \mathcal{L}_n$  via a function  $v^{(k)}(\mathcal{S}_{m,g}^{(k)}, \phi_{\mathcal{L}_g}^{(k)})$  as (29) is denoted by  $x^{(k)}(\mathcal{S}_{m,g}^{(k)}, \phi_{\mathcal{L}_g}^{(k)})$ . Therefore, each distributed decision made by the CFUE in Algorithm 3 can be seen as a sequential transformation of the composition of the network partition as follows:

$$\phi_{\mathcal{L}_g}^{(0)} \rightarrow \phi_{\mathcal{L}_g}^{(1)} \rightarrow \phi_{\mathcal{L}_g}^{(2)} \rightarrow \dots \rightarrow \phi_{\mathcal{L}_g}^{(k)} \rightarrow \dots, \quad (42)$$

where the decision of making a partition is managed by the network coordinator; and  $\phi_{\mathcal{L}_g}^{(k)}$  is the network partition in DJG  $g$  after  $k$  transfers. Every transfer operation from partition  $(k-1)$  to partition  $(k)$  is an inductive step, which produces Pareto dominates  $\mathcal{S}_{m,g}^{(k)}$  as follows:

$$\begin{aligned} \phi_{\mathcal{L}_g}^{(k-1)} \rightarrow \phi_{\mathcal{L}_g}^{(k)} \Leftrightarrow \\ \sum_{\mathcal{S}_{m,g}^{(k)} \in \phi_{\mathcal{L}_g}^{(k)}} v(\mathcal{S}_{m,g}^{(k)}, \phi_{\mathcal{L}_g}^{(k)}) > \sum_{\mathcal{S}_{m,g}^{(k-1)} \in \phi_{\mathcal{L}_g}^{(k-1)}} v(\mathcal{S}_{m,g}^{(k-1)}, \phi_{\mathcal{L}_g}^{(k-1)}). \end{aligned} \quad (43)$$

We note that, each CFUE gradually selects the coalition based on reuse table  $T_g$  and conditions (23), (24) and (25). Hence, the value of coalition will be set to zero if any condition of the formed coalition is violated, and the value of other coalitions remains unchanged. Therefore, in Algorithm 3, when any two successive steps  $k-1$  and  $k$  are successful, then we have  $v(\phi_{\mathcal{L}_g}^{(k)}) = \sum_{\mathcal{S}_{m,g}^{(k)} \in \phi_{\mathcal{L}_g}^{(k)}} v(\mathcal{S}_{m,g}^{(k)}, \phi_{\mathcal{L}_g}^{(k)})$  is Pareto dominated by  $\phi_{\mathcal{L}_g}^{(k)}$ .

Therefore, Algorithm 3 ensures that the overall network utility sequentially increases by Pareto dominance. In addition, the sum of values of the coalitions in each group  $g$  increases without decreasing the payoffs of the individual CFUEs and the whole network as well. Since the number of partitions of  $\mathcal{L}_g$  CFUEs into  $M+1$  coalitions is a finite set given by the Bell number [37], thus the number of transmission steps in (42) is finite. Hence, the sequence in (42) will terminate after a finite number of inductive steps and will converge to a final partition. ■

After DJG  $g$  partition converges to a final partition  $\phi_{\mathcal{L}_g}$ , it is still not guaranteed analytically that the partition is Nash-stable. A partition  $\phi_{\mathcal{L}_g}$  is Nash-stable if no player can get benefit in transferring from its coalition  $(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g})$  to another existing coalition  $\mathcal{S}_{m'}$ , which can be mathematically formulated based on [42] as follows:

**Definition 3:** The partition  $\phi_{\mathcal{L}_g}$  is Nash-stable with Pareto dominance if  $\forall ln \in \mathcal{L}_g$ , such that  $ln \in \mathcal{S}_{m,g}, \mathcal{S}_{m,g} \in \phi_{\mathcal{L}_g}$ ; thus,  $(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}) \succeq_{ln} (\mathcal{S}_{m'} \cup \{ln\}, \phi'_{\mathcal{L}_g})$  for all  $\mathcal{S}_{m'} \in \phi_{\mathcal{L}_g} \cup \{\emptyset\}$  with  $\phi'_{\mathcal{L}_g} = (\phi_{\mathcal{L}_g} \setminus \{\mathcal{S}_{m,g}, \mathcal{S}_{m'}\}) \cup \{\mathcal{S}_{m,g} \setminus \{ln\}, \mathcal{S}_{m'} \cup \{ln\}\}$ .

Hence, the stability of partition  $\phi_{\mathcal{L}_g}$  in the proposed game can be considered as below.

**Proposition 2:** Any final partition  $\phi_{\mathcal{L}_g}$  belongs to the core  $C(\mathcal{L}_g, v)$  of the DJG in Algorithm 3 and always converges to a Nash-stable partition.

*Proof:* Consider a partition  $\phi_{\mathcal{L}_g}^c$  belongs to core  $C(\mathcal{L}_g, v)$ , that is found according to the four steps in Definition 2. If this partition is not Nash-stable, then there exists a CFUE  $ln \in \mathcal{L}_g$  with  $ln \in \mathcal{S}_{m,g}, \mathcal{S}_{m,g} \in \phi_{\mathcal{L}_g}^c$ , and a coalition  $\mathcal{S}_{m'} \in \phi_{\mathcal{L}_g}^c$  such that  $\mathbf{y}(\mathcal{S}_{m'} \cup \{ln\}, \phi'_{\mathcal{L}_g}) \succ_{ln} \mathbf{x}(\mathcal{S}_{m,g}, \phi_{\mathcal{L}_g}^c)$ , and CFUE  $l \in \mathcal{L}_n$  can move to coalition  $\mathcal{S}_{m'}$ . Here,  $\phi'_{\mathcal{L}_g} = (\phi_{\mathcal{L}_g}^c \setminus \{\mathcal{S}_{m,g}, \mathcal{S}_{m'}\}) \cup \{\mathcal{S}_{m,g} \setminus \{ln\}, \mathcal{S}_{m'} \cup \{ln\}\}$ . However, this contradicts with the final partition  $\phi_{\mathcal{L}_g}$  in Propriety 1. On the other hand, after finishing the recursive core formation, we can see that CFUEs have no incentive to abandon their coalitions, because any deviation can be detrimental. As a result, a partition  $\phi_{\mathcal{L}_g}$  in the recursive core is also stable since it ensures the highest possible payoff for each CFUE with no incentive to leave this partition, as studied in [43]. Thus, any partition  $\phi_{\mathcal{L}_g}$  that belongs to the core of Algorithm 3 is Nash-stable. ■

With respect of computational complexity, related to centralized solution, it is worth mentioning that finding a partition is strongly challenged by the exponentially growing number of required iterations and the signaling overhead traffic which would rapidly congest the backhaul and dedicate channels. Moreover, femtocell does not have reliable centralized control due to unreliable backhaul [9]. Due to these characteristics, femtocell deployment needs distributed solutions with automatic channel selection, power adjustment for autonomous interference coordination and coverage optimization. In our distributed solution, the complexity can be significantly reduced by considering follows aspects. First, in our game, the network partition is managed by network coordinator of each DJG. Moreover, cooperation is established only among those CFUEs who are using the same subchannel in their DJG is often small. Further, the network partition formation does not depend on the order in which the CFUEs in coalition are evaluated, the number of iterations is further reduced. Second, the network partition is obtained by running residual games in each DJG with checks in subchannels reusing table, significantly reducing the search space and amount of exchanged information.

Obviously, the recursive core method applied to our proposed game always converges to a final DJG partition. Moreover, the network partition based on residual game always converges to a Nash-stable partition.

## V. SIMULATION RESULTS

As shown in Fig. 7, we simulate an MBS and 16 CFBSs with the coverage radii of 500 m and 30 m, respectively. In order to allocate subchannels to the femtocells, we utilize three SC-FDMA licensed subchannels, which are allocated to uplink transmission of three MUEs, each with bandwidth  $B_w = 360$  kHz (by using two sub-carriers for each licensed subchannel) and a fixed power level of 500 mW. Moreover, the interference threshold at the MBS for each licensed subchannel equals to -70 dbmW. Each CFBS has two CFUEs, a pilots signal with power equals to 500 mW. Each CFUE has an arrival

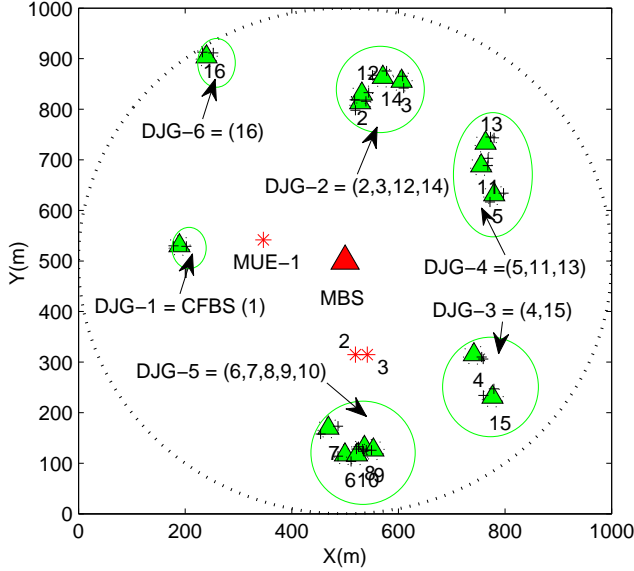


Fig. 7: Self-organization of CFUEs in the CFN to six DJGs according to Algorithm 1. DJG-1, DJG-2, DJG-3, DJG-4, DJG-5 and DJG-6 are composed of CFUEs belonging to the groups CFBS {1}, {2, 3, 12, 14}, {4, 15}, {5, 11, 13}, {6, 7, 8, 9, 10} and {16}, respectively.

rate equals to 1.5 Mb/s, and the delay must be less than or equal to 10 ms. In addition, each CFUE has a maximum power level constraint ( $P^{max}$ ) of 100 mW. We assume that distance-dependent path loss shadowing according to the 3GPP specifications [44] affects the transmissions.

After the network is initialized, the CFBS and MBS periodically broadcast the pilot signal to CFUEs. CFUEs measure RSSI of the pilot signals and estimate channel gain to the surrounding CFBSs. Additionally, the CFUE also estimates its channel gain to the MBS based on the messages broadcast from the MBS. Independently, the CFUE estimates the maximum power level for each licensed subchannel based on its own observed channel gain to the MBS and (1), in which the maximum power level on each licensed subchannel is determined by  $P_{ln}^{m,max} = [\min(P^{max}, \zeta_0^m / h_{ln}^m)]^+$ ,  $\forall m \in \{1, 2, 3\}$ . The maximum power level of CFUEs on licensed subchannels are shown in Fig. 8. CFUEs in CFBS-16 have the highest maximum power level because the distance to MBS is too far from CFUEs where the MBS lies outside the interference threshold range.

In order to find DJGs, we run Algorithm 1. The self-organization DJGs are shown in Fig. 7, in which CFUEs self-organize into six DJGs. Simultaneously, the subchannel reuse tables among CFUEs of DJGs are also formed. The subchannel reuse tables of all DJGs are depicted in Fig. 9, in which value “0” means two CFUEs cannot be reused, else it has a value equal to “1”.

In Fig. 10, we compare our proposal to a random subchannel allocation scheme. Specifically, we consider DJGs 1 and 5. Intuitively, by using Algorithm 3, DJG-5 needs 20 time steps, and DJG-1 needs 5 time steps to converge to the optimum value.

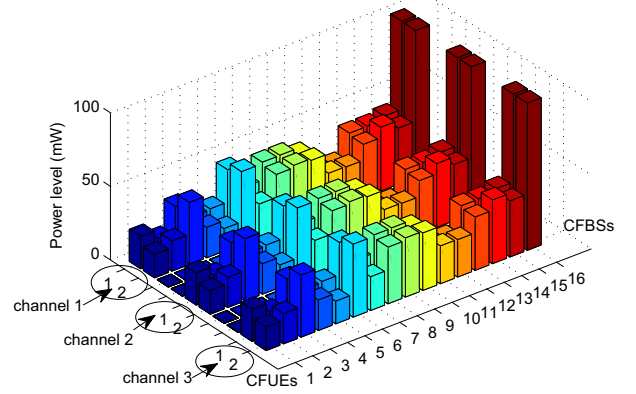


Fig. 8: The maximum power levels of CFUEs on three licensed subchannels.

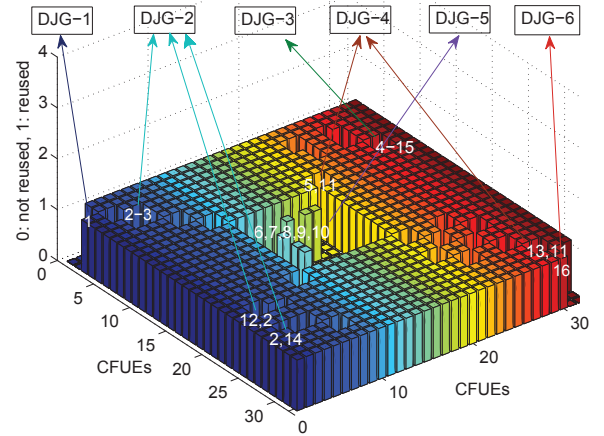


Fig. 9: The subchannel reuse tables among CFUEs in DJGs of the CFN. The value “0” means two CFUEs cannot be reused the subchannels, else it has a value equal to “1”.

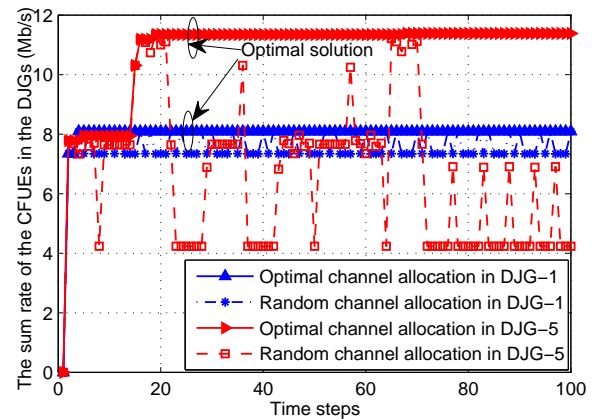


Fig. 10: The optimal subchannel allocation in DJG-1 and DJG-5.

On the other hand, by randomly allocating the subchannels, the sum-rate of each DJG-1 and DJG-5 is not stable and are

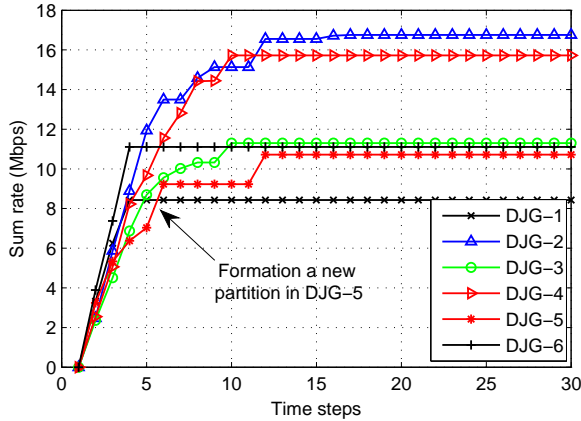


Fig. 11: The convergence of each disjoint group.

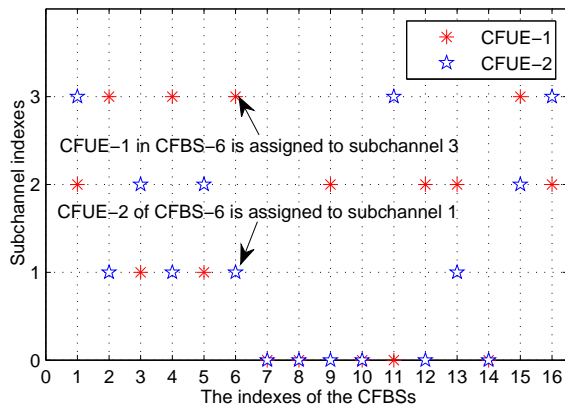


Fig. 12: The optimal subchannel allocation of CFUEs.

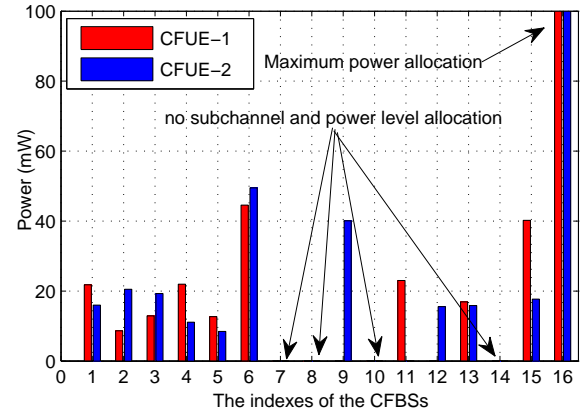


Fig. 13: The optimal power allocation of CFUEs.

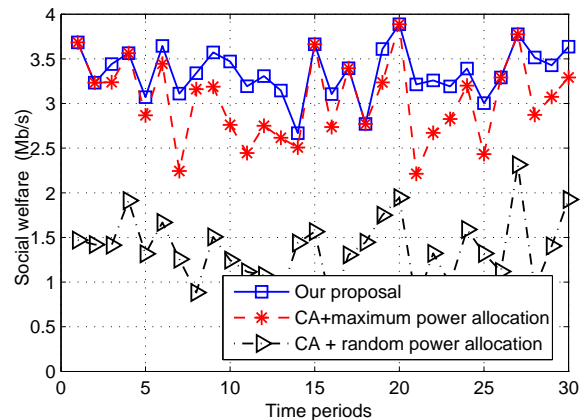


Fig. 14: The social welfare of CFUEs in OPT1 with different power allocation schemes.

lower than the optimum value. This examination is the same for all DJGs in the network. We also see that the sum-rate in each DJG using our scheme is greater than that of the random subchannel allocation scheme. Therefore, our scheme is more efficient than the random subchannel allocation scheme.

The convergence of DJGs after applying coalitional game approach via Algorithm 3 is shown in Fig. 11. By using the coalitional game, in each time step, all CFUEs will cooperate with other CFUEs in its formed coalition and form DJGs with joining and leaving principles to maximize the sum-rate of DJGs. Clearly, using the coalitional game approach, we can achieve a local optimum. We also see that the convergence of Algorithm 3 is achieved after around 18 time steps, as shown in Fig. 11.

The results of the subchannel and power allocation based on the core of the game are shown in Figs. 12 and 13, respectively. In each group, some CFUEs may not be allocated to any subchannel because these allocations do not satisfy the constraints of the minimum delay and protection at the MBS. Additionally, the power of CFUEs in CFBS-16 are allocated with the maximum power level because the MBS is outside of the interference range of CFUEs in CFBS 16. Furthermore, this group is not affected by interference from other CFUEs

in other DJGs or by interference from the MUEs.

To leverage subchannel variations and network stochastic realizations, the results are averaged over a large number of simulations. We consider our problem in 30 periods, and we estimate the social welfare of CFUEs by the utilitarian measure  $\left(\frac{1}{N \times L} \sum_{l=1}^L \sum_{n=1}^N R_{ln}\right)$  for each period. There are three schemes considered: our proposal given in Algorithm 3, subchannel allocation using Algorithm 3 with the fixed maximum power level of CFUE (CA + maximum power allocation), and subchannel allocation using Algorithm 3 with random power level allocation to each CFUEs (CA + random power allocation). The results are shown in Fig. 14. Intuitively, our proposed scheme always achieves a higher social welfare utilities for all CFUEs compared to the other methods. Hence, sharing the individual payoff among CFUEs using Algorithms 2 is necessary to find the optimal solution of OPT1.

Next, we estimate the social welfare of CFUEs  $\left(\sum_{l=1}^L \sum_{n=1}^N R_{ln}\right)$  with respect to different interference thresholds of the MBS. Fig. 15 shows that for any interference threshold at the MBS, the social welfare of the proposed approach is always higher than those of the (CA + maximum



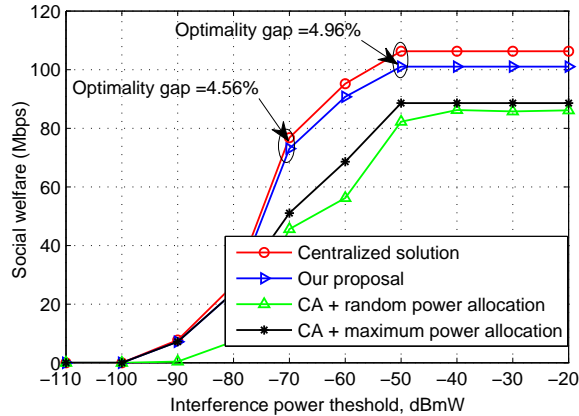


Fig. 15: The social welfare of CFUEs in OPT1 versus the interference threshold at the MBS.

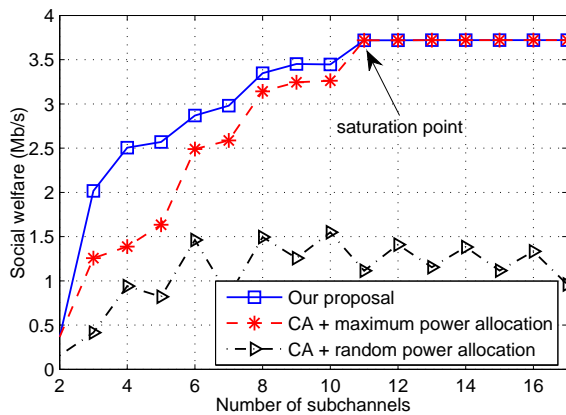


Fig. 16: The average data rates of CFUEs follow number of the subchannels.

power allocation) and (CA + random power allocation) schemes. Further, in Fig. 15, we have numerically compared the proposed approach with the optimal solution, in which CFUEs are allocated subchannel and transmit power in a centralized fashion. The social welfare of all the schemes grows with the increase in interference thresholds at the MBS. The comparison shows that performance of the proposed is close to centralized solution. In addition, gaps between the proposed approach and centralized solution are 4.56% and 4.96% when the interference thresholds are of -70 dBmW and -40 dBmW, respectively.

In order to see the social welfare versus the numbers of licensed subchannels, we fix the positions of CFUEs and CFBSs. Then, we increase the number of licensed subchannels that are allocated to CFN in the uplink direction. We examine OPT1 with different methods, as shown in Fig. 16. When the number of subchannels is less than or equal to 11, the social welfare of CFUEs in our scheme is higher than that of the other two schemes. With increasing number of licensed

subchannels, the social welfare is increased because more CFUEs are allocated to subchannels. The saturation point is achieved when the number of subchannels is greater than or equal to 11 because, at this point, all CFUEs are allocated to the optimal subchannel and power level formed in the core of the proposed game. The scheme “Optimal CA + random power allocation” always has the smallest value because some CFUEs which are allocated with random power level do not satisfy the conditions to protect MBS or the minimum delay requirement of each CFUE. In such cases, the subchannel is not allocated to these CFUEs, which dramatically decreases the sum rate of all CFUEs, as well as the the social welfare in the network.

## VI. CONCLUSIONS

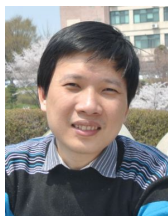
In this paper, we investigated an efficient distributed resource allocation scheme for uplink underlay CFN. The efficient resource allocation is characterized via an optimization problem. We identified the optimal subchannels and power levels for CFUEs to maximize the sum-rate. The optimization problem guaranteed the inter-tier and inter-tier interference thresholds. Specifically, the aggregated interference from femtocell users to the MBS and the maximum delay requirement of the connected CFUE are kept under the acceptable level. In order to solve the optimization problem, we simplified the CFN by forming DJGs and suggested a formulation optimization problem as a coalitional game in partition form in each formed disjoint group. The convergence of algorithms was also carefully investigated. Simulation results showed that the CFUEs can be self-organized into DJGs. Additionally, the sum-rate in the proposed framework is achieved by the optimal subchannel and power allocation policy with all CFUEs’ average delay constraints being satisfied. Moreover, the efficient resource allocation is tested, with the sum-rate of the proposed framework always being close to optimal solution and better than those of the other frameworks.

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