Matching Theory for Distributed User Association and Resource Allocation in Cognitive Femtocell Networks

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Abstract—In this paper, a novel framework is proposed to jointly optimize user association and resource allocation in the uplink cognitive femtocell network (CFN). In the considered CFN, femtocell base stations (FBSs) are deployed to serve a set of femtocell user equipments (FUEs) by reusing subchannels used in a macrocell base station (MBS). The problem of joint user association, subchannel assignment, and power allocation is formulated as an optimization problem, in which the goal is to maximize the overall uplink throughput while guaranteeing FBSs' overloading avoidance, data rate requirements of the served FUEs, and MBS protection. To solve this problem, a distributed framework based on the matching game is proposed to model and analyze the interactions between the FUEs and FBSs. Using this framework, distributed algorithms are developed to enable the CFN to make decisions about user association, subchannel allocation, and transmit power. The algorithms are then shown to converge to a stable matching and exhibit a low computational complexity. Simulation results show that the proposed approach yields a performance improvement in terms of the overall network throughput and outage probability, with a small number of iterations to converge.

Index Terms—Cognitive femtocell network, resource allocation, power allocation, subchannel allocation, matching game, optimization problem.

I. INTRODUCTION

The use of small cell networks based on the pervasive deployment of low-power, low-cost femtocell base stations provides a promising solution to improve the capacity and enhance the coverage for indoor and cell edge users in next-generation wireless cellular networks [1]. In order to utilize the limited licensed spectrum efficiently, FBSs will need to reuse the same radio resources with the macrocell network in the current LTE wireless system, which is based on orthogonal frequency-division multiple access (OFDMA) [2]. This can lead to severe co-channel cross-tier interference, thus requiring a smart adaptive scheduling algorithm [2]. Cognitive radio (CR) can be a promising technology for realizing such flexible interference management. A femtocell network that reuses subchannels based on CR technology is commonly known as the cognitive femtocell network [3].

The goals of CFN deployment include the macrocell network protection and guaranteeing served FUEs’ quality of service (QoS) while maximizing the overall network throughput [2], [3]. To reap the benefits of CFN deployment, some technical challenges such as interference management, efficient spectrum usage, and cell association must be addressed [4]–[6]. To address these issues, there are some existing works on power allocation for the underlay CFN in the literature [7]–[9]. Moreover, the problems of subchannel allocation have been studied in [6], [10], [11]. In addition, the joint subchannel and power allocation issues have been addressed in [12]–[15]. Furthermore, user association design in the CFN presents another major challenge [2], [4], [5]. More recently, there are some studies on joint subchannel allocation and user association in the CFN [16], [17]. In general, the design of an efficient framework for joint user association, subchannel allocation, and power allocation for the underlay CFN must address various coupled problems, such as load-sharing among femtocells, MBS protection, FBSs’ overloading protection, and guaranteeing QoS for the served FUEs, and is still under explored in the current literature. Additionally, the uplink traffic model should be paid more attention to adapt the inevitable traffic explosion in future mobile networks [18]. For example, the emergence of Internet of Things (IoT) and machine type communications (MTC) change the bottleneck from downlink to uplink [19], [20].

The main contribution of this paper is to introduce a novel framework for joint user association, subchannel allocation and power allocation in the uplink underlay CFN, which is an NP-hard combinatorial optimization problem. This optimization problem pertains to finding an optimal solution for associating FUEs to FBSs, assigning subchannels to FUEs, and allocating transmit power levels for FUEs to maximize the uplink overall network throughput while considering intra-tier and inter-tier interference. Additionally, this problem formu-
allocation guarantees the data rate requirement of served FUE and provides FBS’ overloading protection and MBS protection. In summary, we make the following key contributions:

- We develop a distributed framework based on a matching game to solve the formulated optimization problem. The motivation behind this design is to model the competitive behaviors of FUEs, FBSs, and access controller, in interference management.
- We design distributed algorithms that enable to determine the association of FUEs to FBSs, assignment of subchannels to FUEs, and the power allocation for FUEs in an autonomous manner. In addition, the transmit power is optimized by employing geometric programming and dual-decomposition approaches. Then, we prove that the proposed algorithms converge to a group stable matching.
- Simulation results show that the proposed approach yields significant performance improvement in terms of the overall network throughput and outage probability in both the uniform and non-uniform user distribution scenarios, with a small number of iterations.

The rest of this paper is organized as follows: Section II discusses related work. Section III explains the system model and problem formulation. The NP-hard combinatorial optimization problem is solved based on the matching game in Section IV. We also study the convergence and stability of the proposed algorithms in Section IV. Section V provides simulation results. Finally, conclusions are drawn in Section VI.

II. RELATED WORK

Several recent studies have considered the resource allocation and user association problem in the uplink CFN [2], [4]–[13], [15]–[17]. These works, however, have only studied power control, subchannel allocation, or user association problems separately.

For power control, existing works focus on efficient sharing of a single channel through adaptively adjusting the power levels in uplink two-tier femtocell networks [7]–[9]. The studies in [7], [8] only considered access control and power control to minimize the number of secondary users to be removed and to maximize the overall network throughput for efficient sharing of a single channel. The distributed power control for spectrum-sharing femtocell networks using the Stackelberg game approach is also presented in [9]. However, the proposals in [7]–[9] do not consider the subchannel allocation issue.

There have been some existing works studying the subchannel assignment for uplink OFDMA-based femtocell networks, but they do not consider power control in their designs [6], [10]–[13], [15]. In [10], the authors proposed two approaches to mitigate the uplink interference for OFDMA femtocell networks. In the first approach, FUEs are only allowed to use dedicated subchannels if they produce strong interference to the MUEs. In the second approach, the channel assignment for both tiers is performed based on an auction algorithm. Moreover, some other works address the joint subchannel allocation and power control [11]–[13], [15]. A distributed power control and centralized matching algorithms for subchannel allocation were proposed in [12], which lead to fair resource allocation for uplink OFDMA femtocell networks. A distributed auction game is employed to design the joint power control and subchannel allocation in OFDMA femtocell networks [13]. Additionally, the authors in [15] investigated the joint uplink subchannel and power allocation problem in cognitive small cells using cooperative Nash bargaining game theory but ignoring interference among small cells. Nonetheless, the system models in [11]–[13], [15] are based on closed access models, where only the registered FUEs are allowed to communicate with FBSs.

There have also been some existing works on the user association design for the uplink CFN [16], [17]. In [16], the authors studied resource sharing and femtocell access control in OFDMA femtocell networks, in which incentive mechanisms are proposed to encourage FUEs to share their FBSs with MUEs. However, this work considers only resource sharing without power control. In [17], the authors propose a cross-layer resource allocation and admission control framework in a downlink CFN in which MUEs can establish connection with FBSs to mitigate the excessive cross-tier interference and achieve a better throughput.

Some ideas presented in this paper for the interference management are related to those works in [21], [22]. In [21], authors presented an uplink capacity analysis and interference avoidance strategy for a shared spectrum two-tier DS-CDMA network. In [22], authors proposed the non-collaborative intercell interference avoidance method in order to ensure fairness for cell edge users in the OFDMA network. The interference avoidance method in [22] is not applicable for the spectrum sharing network. Moreover, differently from these works, our paper studies the interference management strategy based on matching algorithm that captures on FUEs and FBSs’ behaviors to find the sub-optimal strategies of the proposed optimization problem in the two-tier network.

Recently, the application of the matching theory to engineer the future wireless network has received increasing attention [23]–[25]. A concise introduction and survey on matching theory applications is provided in [23]. A framework that jointly associates user equipment to the FBSs and allocates FBSs to the service provider using the matching game approach was proposed in [24]. This work does not, however, consider the underlay spectrum sharing approach between macro-tier and femto-tier. In addition, the algorithms developed in [24] do not ensure QoS guarantees for the served user equipments and the feasible solution of the formulated optimization problem in case of the resource limitation. An admission game for uplink user association in wireless small cell networks is addressed in [25]. The studies in [25] only considered the problem of user association based on a college admissions game by utilizing the matching and coalition game approach for the small cell network that does not consider the spectrum sharing with the macrocell network.

It can be seen that none of the existing works study joint user association, channel allocation and power control in the uplink CFN. This paper aims to address this joint design problem based on the matching theory.
III. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we present the system model and problem formulation.

A. System model

We consider the uplink of an OFDMA cognitive femtocell network, where a set \( \mathcal{M} = \{1, 2, ..., M\} \) of FBSs operates inside the coverage of a macrocell network and serves a set \( \mathcal{N} = \{1, 2, ..., N\} \) of FUEs as shown in Fig. 1. These FBSs adopt an open access mode which allows any FUEs to use the FBSs’ services [26]. FUEs are seen by indoor mobile users or extended capacity coverage of the existing macrocell to enhance the users’ QoS in an ultra-dense femtocells deployment [3]. We consider an OFDMA system with bandwidth \( B \) divided into a set \( \mathcal{K} = \{1, 2, ..., K\} \) of orthogonal subchannels, which are reused by the CFN using the underlay spectrum access model. These subchannels are correspondingly occupied by \( K \) macrocell user equipments (MUEs). Additionally, there is no interference among transmission on different subchannels. FBSs are connected to a cognitive femtocell management (CFM) controller that acts as a coordinator and spectrum manager. FBSs and FUEs are assumed to be selfish and rational entities that merely care about their own interests. Moreover, we assume that FBSs and MBS have knowledge about channel state information of FUEs. For notational convenience, we define \( \mathcal{K}_m \subseteq \mathcal{K} \) as the set of subchannels available for the FBS \( m \) as allocated by the CFM. Furthermore, we assume that each FUE is only permitted to access at most one subchannel. Additionally, let \( \mathcal{N}_m \) be the set of FUEs associated with the FBS \( m \), \( \mathcal{N}_m \subseteq \mathcal{N} \). We will use the index 0 to denote the MBS.

B. Problem formulation

We first describe all system and design constraints. After that, we formulate the problem of optimal user association, subchannel allocation, and power allocation.

**FBS protection.** The number of FUEs associated to each FBS is restricted due to the femtocell hardware limitations and the planned cellular networks of the femtocell network operator [4], [5], which is also taken into account in [24], [25], [27]. The number of FUEs that can be associated with the FBS \( m \) is constrained as follows:

\[
\sum_{n \in \mathcal{N}} x_{nm} \leq \tilde{N}_m, \forall m \in \mathcal{M},
\]

where \( x_{nm} \) is the binary variable representing the association status between the FUE \( n \) and FBS \( m \), and \( \tilde{N}_m \) is defined as a quota that represents the maximum number of FUEs that can be supported by the FBS \( m \). We further define \( \mathbf{X} = [x_{nm}]_{\mathcal{M} \times \mathcal{N}} \), where \( x_{nm} = 1 \) means that the FUE \( n \) is associated to FBS \( m \), and \( x_{nm} = 0 \) otherwise. **FUE QoS.** We consider the minimum data rate requirement of each FUE served by a certain FBS. When the FUE \( n \) is served by FBS \( m \) on subchannel \( k \) with transmit power \( P^m_n \), the data rate of FUE \( n \) will be given by

\[
R^k_{nm} = B_k \log_2 (1 + \Gamma^k_{nm}),
\]

where \( B_k \) is the bandwidth of subchannel \( k \), and \( \Gamma^k_{nm} \) is the signal-to-interference-plus-noise ratio (SINR) of FUE \( n \) associated with FBS \( m \) on subchannel \( k \), which can be written as

\[
\Gamma^k_{nm} = \frac{x_{nm} y_{nk} g^k_{nm} P^k_n}{\sum_{n' \in \mathcal{N} \setminus \{n\}, m' \in \mathcal{M} \setminus \{m\}} x_{n'm'} y_{n'k} g^k_{n'm'} P^k_{n'} + g^k_{km} P_k + \sigma^2},
\]

where \( \sum_{n' \in \mathcal{N} \setminus \{n\}, m' \in \mathcal{M} \setminus \{m\}} x_{n'm'} y_{n'k} g^k_{n'm'} P^k_{n'} \) is the total interference from other FUEs to the FBS \( m \) on subchannel \( k \); \( P^k_n \), \( P^k_{n'} \), and \( P_k \) denote the powers of FUE \( n \), FUE \( n' \), and MUE \( k \) on subchannel \( k \), respectively; \( g^k_{nm} \), \( g^k_{n'm'} \), and \( g^k_{km} \) are the channel power gains on subchannel \( k \) from FUE \( n \in \mathcal{N}_m \), from FUE \( n' \) to FBS \( m \), and from MUE \( k \) to FBS \( m \), respectively. For convenience, we define \( \mathbf{Y} = [y_{nk}]_{\mathcal{N} \times \mathcal{K}} \) as the subchannel allocation matrix, where \( y_{nk} = 1 \) means that subchannel \( k \) is assigned to FUE \( n \), and \( y_{nk} = 0 \) otherwise, and \( \mathbf{P} = [P^m_n]_{\mathcal{N} \times \mathcal{K}} \) is the power allocation matrix. Without loss of generality, the noise power \( \sigma^2 \) is assumed to be equal for all FBSs.

To maintain the minimum QoS of FUEs, we assume that the achievable rate of each FUE must be greater than or equal to a minimum rate as follows:

\[
\sum_{k \in \mathcal{K}} R^k_{nm} \geq R^\text{min}_n,
\]

where \( R^\text{min}_n \) is a predefined parameter of the FUE \( n \).

**MBS protection.** In our model, the total interference from FUEs to the MBS on each subchannel \( k \) is constrained to be below the threshold \( I^\text{th}_k \) to maintain the required QoS of the underlying MUE. This constraint can be expressed as

\[
\sum_{n \in \mathcal{N}} y_{nk} g^k_{n0} P^k_n \leq I^\text{th}_k, \forall k \in \mathcal{K},
\]

where \( \sum_{n \in \mathcal{N}} y_{nk} g^k_{n0} P^k_n \) is the total interference generated by all FUEs to the MBS on subchannel \( k \), and \( g^k_{n0} \) is the channel power gain on subchannel \( k \) from FUE \( n \) to MBS. The joint user association, subchannel allocation, and power control problem is formulated as an optimization problem.
that aims to maximize the overall network throughput as follows:

\[
\text{OPT : } \max_{(X,Y,P)} \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} R_{nm}^k
\]

s.t. (1), (4), (5),
\[
\sum_{m \in \mathcal{M}} x_{nm} \leq 1, \quad \forall n \in \mathcal{N},
\]
\[
\sum_{n \in \mathcal{N}_m} y_{nk} \leq 1, \quad \forall k \in \mathcal{K}, \forall m \in \mathcal{M},
\]
\[
\sum_{k \in \mathcal{K}} y_{nk} \leq 1, \quad \forall n \in \mathcal{N},
\]
\[
P_{nm} \leq P^k_n \leq P_{nm}^\text{max}, \quad \forall n \in \mathcal{N}, \forall k \in \mathcal{K},
\]
\[
x_{nm} = \{0, 1\}, \quad y_{nk} = \{0, 1\}, \forall m, n, k.
\]

Here, constraint (7) guarantees that each FUE can be associated with at most one FBS; constraints (8) and (9) imply that each subchannel can be allocated to at most one FUE in the FBS, and each FUE can be allocated at most one subchannel, respectively; constraint (10) guarantees that the transmit power of each FUE is adjusted within the desired range.

IV. MATCHING GAME BASED USER ASSOCIATION AND RESOURCE ALLOCATION

It is observed that the OPT is a mixed integer and non-linear optimization problem because it contains both binary variables \((X,Y)\) and continuous variables \(P\). Additionally, the considered joint user association, subchannel and power allocation problem is difficult to solve because of its coupled constraints: i) FUES’ QoS (constraint (4)) and ii) the macrocell base station protection (the constraint (5)). In order to solve the OPT, we iteratively solve three problems in three independent phases: user association (UA) phase, subchannel allocation (CA) phase, and power control (PC) phase as shown in Fig. 2. The three phases are run sequentially in each iteration until convergence. Firstly, given fixed transmit power in the PC phase and channel allocation in the CA phase, the FUEs are associated with the FBSs based on the one-to-many matching game (MATCH-UA algorithm) in the UA phase. Secondly, given fixed user association in the UA phase and transmit power in the PC phase, FUEs are assigned to subchannels based on the one-to-one matching game (MATCH-CA algorithm) in the CA phase. Thirdly, given fixed user association in the UA phase and subchannel allocation in the CA phase, the transmit powers are determined in the PC phase. Additionally, we consider an access control scheme to guarantee FUEs’ QoS, and MBS protection by utilizing the ELGRA algorithm [8] in the PC phase. Moreover, we determine the optimal transmit power by using geometric programming and decomposition approaches (the DIST-P algorithm). Finally, we propose the JUCAP algorithm that integrates the UA, CA, and PC phases.

A. User association as a matching problem (UA phase)

We consider the optimization of the user association solution \(X\) given the subchannel allocation \(Y\) and the transmit power allocation \(P\) by solving the following optimization problem:

\[
\text{OPT-UA: } \max_{X} \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} R_{nm}^k
\]

s.t. (1), (7),
\[
x_{nm} = \{0, 1\}, \quad \forall m, n,
\]
\[
P^m_n = P^\text{max}_m^k \left| \mathcal{K}_m \right|, \quad \forall n \in \mathcal{N}.
\]

In the OPT-UA, constraints (4) and (5) of the original problem OPT will be handled in the matching game formulation of the UA, CA, and PC phase. The overall network throughput is strongly impacted by the interference power at FBSs and MBS protection constraints captured in (2), (3), (4), and (5). In order to estimate the contribution to the overall network throughput of each FUE, the CFM requests FUEs that join the UA and CA phases are allocated power uniformly over the set of available subchannels, as represented in constraint (14).

In order to develop a distributed algorithm for the UA phase, we use the one-to-many matching game [23], [28] that can capture a local optimal solution for the OPT-UA problem. Under this design, each FUE will be matched to at most one FBS, while each FBS can be assigned to at most \(\bar{N}_m\) FUES, \(\forall m \in \mathcal{M}\).

1) Definition of a matching function for user association:

Formally, we can formulate the UA problem as a one-to-many matching game defined by a tuple \((\mathcal{M}, \mathcal{N}, \succ_{\mathcal{M}, \mathcal{N}}, \succ_{\mathcal{M}, \mathcal{UA}})\). Here, \(\succ_{\mathcal{M}, \mathcal{UA}} = \{\succ_{m, \mathcal{UA}}\}_{m \in \mathcal{M}}\) and \(\succ_{\mathcal{N}, \mathcal{UA}} = \{\succ_{n, \mathcal{UA}}\}_{n \in \mathcal{N}}\) denote the sets of the preference relations of FUES and FBSs, respectively. The matching game for user association \(\mu_{\mathcal{UA}}\) can be formulated as follows:

**Definition 1.** Given two disjoint finite sets of players \(\mathcal{N}\) and \(\mathcal{M}\), a matching \(\mu_{\mathcal{UA}}\) is defined as a function \(\mu_{\mathcal{UA}} : \mathcal{N} \rightarrow \mathcal{M}\),
such that:
1. \( m = \mu_{UA}(n) \Leftrightarrow n \in \mu_{UA}(m) \);
2. \( |\mu_{UA}(m)| \leq N_m \) and \( |\mu_{UA}(n)| \leq 1 \).

The outcome of the matching game is the user association mapping \( \mu_{UA} \). If FUE \( n \) is matched to FBS \( m = \mu_{UA}(n) \), then FBS \( m \) is also matched to FUE \( n \). The condition \( |\mu_{UA}(m)| \leq N_m \) ensures that at most \( N_m \) FUEs will be matched to FBS \( m \) under the matching \( \mu_{UA} \). The condition \( |\mu_{UA}(n)| \leq 1 \) guarantees that at most one FBS can be matched to the FUE \( n \) under the matching \( \mu_{UA} \).

Let \( \phi_{UA}^m(n) \) and \( \phi_{UA}^n(m) \) denote the utility functions of FUE \( n \) for FBS \( m \) and FBS \( m \) for FUE \( n \), respectively. Given these utilities, we can say that the FUE prefers FBS \( m_1 \) to \( m_2 \), if \( \phi_{UA}^m(n_1) > \phi_{UA}^m(n_2) \), \( m_1, m_2 \in M \). This preference is denoted by \( m_1 \succ_{UA} m_2 \). FBS \( m \) prefers FUE \( n_1 \) to \( n_2 \), if \( \phi_{UA}^n(n_1) > \phi_{UA}^n(n_2) \), \( n_1, n_2 \in N \), denoted by \( n_1 \succ_{UA} n_2 \). For the problem OPT-UA, we build interference lists of FUEs and FBSs based on the utilities functions, as below:

**Utility function of the FUE.** For user association, we use the average received SINR over all subchannels, which is the most common criterion for user association in the wireless network [25], [29]. In particular, in particular, the utility archived by FUE \( n \) when it connects to the FBS \( m \) over \( K_m \) subchannels can be expressed as a function of the SINRs [24] as follows:

\[
\phi_{UA}^m(n) = \log_2 \left( 1 + \sum_{k \in K_m} \Gamma_{nm}^k \right),
\]

where \( \Gamma_{nm}^k \) is given by (3). We can clearly see that the utility of FUE \( n \) associated with each FBS \( m \) increases with the channel gains and decreases with the interference from the MUE \( k \in K_m \) and other FUEs.

**Utility function of the FBS.** To maximize the objective function (12), an efficient strategy must be designed for the FBS to accept a candidate FUE in the UA phase. Additionally, the FBS’s strategy must be designed to mitigate the violations of the constraints in (4) and (5) in the UA phase. To fulfill these requirements, we propose a utility function for the FBS that forms its preference relation to FUEs as follows:

\[
\phi_{UA}^n(m) = \varphi_{UA} \sum_{k \in K_m} \frac{g_{nm}^k P_k}{\Gamma_n^k} - C_{nm}^{(K_m)},
\]

where \( \varphi_{UA} \) is a weighting parameter capturing the benefit and the average direct channel power gain from the FUE to the FBS; \( \sum_{k \in K_m} \frac{g_{nm}^k P_k}{\Gamma_n^k} \) captures the channel gains of FBS \( m \) for FUE \( n \); \( \Gamma_n^k = 2^{c_{nm}^k} / B_k - 1 \) is the SINR threshold corresponding to the minimum required rate in (4); \( C_{nm}^{(K_m)} \) quantifies the aggregated relative interference that FUE \( n \) causes to the MBS and the other FBSs \( m' \) \( (m' \neq m) \) on all subchannels in a set \( K_m \) for a given transmit power \( P_n \) which is defined as follows:

\[
C_{nm}^{(K_m)} = \sum_{k \in K_m} (c^k_{0,0} g_{nm}^k P_n^k + \sum_{m' \in M \setminus \{m\}} c_{nm'k} P_k^k),
\]

where \( c^k_{0,0} g_{nm}^k P_n^k \) is the cost imposed by the MBS on sub-channel \( k \) to FUE \( n \), given the transmit power \( P_n \). \( \delta^k = \max(0, (\sum_{m \in N} g_{nm}^k P_k^k - I_{0}^k) / I_{0}^k) \) is defined to quantify the degree of violation of the constraint (5) at the MBS on subchannel \( k \); \( \sum_{m' \in M \setminus \{m\}} c_{nm'k} P_k^k \) describes the cost due to the interference that FUE \( n \) causes to the other FBS \( m' \) \( (m' \neq m) \) on subchannel \( k \); \( c^k_{0,0} \) and \( c_{nm'k} \) are the costs per unit of the interference power at the MBS and FBS \( m' \), respectively. Here, \( c^k_{0,0} \gg c^k_{m} \) is chosen in our design to guarantee that the CNF blocks user association solutions that cause harmful interference to the MBS. We can see that the utility function of the FBS increases with channel gains and decreases with aggregated interference from the FUE to the other base stations (FBSs and MBSs).

Due to the fact that \( R_{n}^{min} \) is constant, (12) \( \approx \max \sum_{n \in N} \sum_{m \in M} (R_{nm}^{(K_m)} - R_{n}^{min}) \). Moreover, by choosing these utility functions in (16) and (17), we aim to maximize the connected FUES’ satisfaction \( \max(R_{nm}^{(K_m)} - R_{n}^{min}) \) and minimize the possibility of an interference constraint violation for the MBS on each subchannel \( k \) \( \min(\sum_{n \in N} g_{nm}^k P_n^k - I_{0}^k) \) instead of strictly maintaining the constraints (4) and (5), respectively. Additionally, the explanation for the increment of the network throughput with direct channel gains is provided below in Remark 1.

**Remark 1.** Given a transmit power and subchannel allocation, the value \( R_{nm}^{(K_m)} - R_{n}^{min} \) of FUE \( n \) in FBS \( m \) increases with \( \sum_{k \in K_m} g_{nm}^k P_k^k \). This can be proved as follows.

**Proof.** See Appendix A in [30]

In order to compute the utility values in (16) and (17), the FBSs and MBSs should have the information on the interference of FUEs induced on all base stations. Typically, the FBS or MBS cannot directly measure these quantities [31]. FUEs can estimate the channel gains from the surrounding the base stations to themselves by exploiting the pilots of base stations and FUEs.

Next, we propose a matching game based user association algorithm, namely, the MATCH-UA algorithm.

2) Distributed user association algorithm based on the matching game: We now develop an algorithm to obtain a stable matching, which is one of the key solution concepts in matching theory [28]. Denote by \( \mu_{UA}(m,n) \) the subset of all possible matchings between \( M \) and \( N \). A stable matching is defined as follows:

**Definition 2.** A pair \( (m,n) \neq \mu_{UA} \) where \( m \in M, n \in N \) is said to be a blocking pair for the matching \( \mu_{UA} \) if it is not blocked by an individual FUE \( n \) and FBS \( m \), and there exists another matching \( \mu'_{UA} \subset \mu_{UA}(m,n) \) such that FUE \( n \) and FBS \( m \) can achieve a higher utility. This mathematically implies that \( \mu'_{UA} \succ_{UA} \mu_{UA} \) and \( \mu'_{UA} \succ_{UA} \mu_{UA} \). A matching \( \mu_{UA} \) is said to be stable if it is not blocked by an individual FUE \( n \) and FBS \( m \) or any pair.

The problem OPT-UA can be solved in a distributed manner based on a one-to-many matching game among the FUEs and FBSs. The details of this algorithm are presented.
Algorithm 1 MATCH-UA: Matching game for user association.

Initialization: $M, N, N_{mq} = \emptyset, A_{mq}^0 = \emptyset, P, \hat{N}_m, \forall m, n.$

Discovery and utility computation:
1. Each FBS $m$ broadcasts $K_m.$
2. Each FUE $n$ constructs $\succ_{n,UA}$ using (15).

Find stable matching $\mu_{UA}^*:\n$
3. while $\sum_{n,m} b_{UA}^{m,n}(t) \neq 0$ do
   4. For each unassociated FUE $n$:
      a. Find $m = \arg \max_{m \in \succ_{n,UA}} \phi_{UA}^m(n).$
      b. Send a request $b_{UA}^{m,n}(t) = 1$ to FBS $m.$
   5. For each FBS $m$:
      a. Update $n_{req}^m \leftarrow \{ n : b_{UA}^{m,n}(t) = 1, n \in N \}.$
      b. Construct $\succ_{m,UA}$ based on (16).
   6. if $|n_{req}| \leq \hat{N}_m$ then
      a. $\succ_{n,UA} \leftarrow n_{req}.$
      b. else
         a. repeat
            a. Accept $n = \arg \max_{n \in \succ_{m,UA}} \sum_{n \in N_{mq}} \phi_{UA}^m(n).$
            b. Update $n_{req} \leftarrow n_{req} \cup n.$
         b. until $|n_{req}| = \hat{N}_m.$
   7. end if
   8. Update $n_{req}^m \leftarrow \{ n_{req}^m \setminus N_{mq} \}.$
   9. Remove FBS $m \in \succ_{n,UA}, \forall n \in n_{req}.$
   10. end while

Results: A stable matching $\mu_{UA}^*.$

B. Sub-channel allocation as a matching problem

In this phase, we assume a given fixed variable $P.$ Then, the subchannel allocation $Y$ is determined by solving the following optimization problem:

$$\text{OPT-CA}_{(m)}: \max_Y \sum_{n \in N_m} \sum_{k \in K_m} R_{nm}^k$$

s.t. (8), (9), (14), $x_{nm} = x^*_m, \forall m, n.$ (19)

In the problem $\text{OPT-CA},$ the constraints (5) and (4), which will be considered in the PC phase, are also temporarily ignored as mentioned in the UA phase. Obviously, this optimization problem is still NP-hard. Moreover, we can see that in the CA phase, since FUEs are allocated with the transmit power as in (14), the interference at FBSs from other femtocells are fixed. Thus, we can decompose $\text{OPT-CA}$ into $M$ subproblems, where each subproblem corresponds to the subchannel allocation of FBS $m$ as follows:

$$\text{OPT-CA}_{(m)}: \max_Y \sum_{n \in N_m} \sum_{k \in K_m} R_{nm}^k$$

s.t. (8), (9), (14). (20)

Here, we consider only FUEs that are matched to FBS $m$ in the UA phase, $x_{nm} = x^*_m, \forall n \in N_m \equiv \mu_{UA}(m).$ $\text{OPT-CA}_{(m)}$ is a combinatorial optimization problem with binary variables $y_{nk}$ that can be solved in a centralized fashion. However, in the considered model, the FUEs and FBS selfishly and rationally interact in a way that maximizes their utilities. Therefore, in order to model competition among the FUEs and FBSs, we solve $\text{OPT-CA}_{(m)}$ using a one-to-one matching game [28], [32], which helps us to find the subchannel allocation in a distributed manner.

1) Definition of matching game for subchannel allocation in the CA phase: The problem in (20) is formulated as a matching game, which is defined by a tuple $(N_m, K_m, \succ_{m,CA}, \succ_{m,CA}).$ Here, $\succ_{m,CA} = \{ \succ_{m,CA} \}_{n \in N_m}$ and $\succ_{m,CA} = \{ \succ_{m,CA} \}_{k \in K_m}$ denote the preference relations of the FUEs and subchannels in FBS $m,$ respectively. We define the problem as a one-to-one matching game, as follows:

Definition 3. Given two disjoint finite sets $N_m$ and $K_m,$ a matching game for subchannel allocation is defined as a function $\mu_{m,CA}: N_m \mapsto K_m$ such that:
1. $n = \mu_{m,CA}(k) \leftrightarrow k = \mu_{m,CA}(n), \forall n \in N_m, k \in K_m;$
2. $|\mu_{m,CA}(k)| \leq 1$ and $|\mu_{m,CA}(n)| \leq 1,$ $n \in N_m, k \in K_m, m \in M.$

The conditions $|\mu_{m,CA}(k)| \leq 1$ and $|\mu_{m,CA}(n)| \leq 1$ in Definition 3 correspond to the constraints (8) and (9), respectively. In the matching $\mu_{m,CA},$ we define $\phi_{CA}^m(n)$ and $\phi_{CA}^m(n)$ as the preference relations of utility values of FUE $n$ in evaluating subchannels in FBS $m$ and the utility value of FBS $m$ in subchannel $k$ for FUE $n,$ respectively. Similar to the matching definition in the UA phase, the FUE $n$ associated to FBS $m$ preferring subchannel $k_1$ to $k_2$ and a subchannel $k$ in FBS $m$ preferring FUE $n_1$ to $n_2$ are denoted by $k_1 \succ_{nm,CA} k_2$ and $k_1 \succ_{nm,CA} k_2$ respectively.

in Algorithm 1 (MATCH-UA). After initialization, each FUE constructs the preference relations $\succ_{n,UA}$ based on (15) (line 2). In order to find a stable matching $\mu_{UA},$ each FUE $n$ sends a bid request $b_{UA}^{m,n}$ to FBS $m,$ which has the highest utility in its preference relation $\succ_{n,UA}$ (line 5). The bid value $b_{UA}^{m,n} = 1$ when FUE $n$ prefers to associate with FBS $m,$ otherwise it is equal to zero. At the FBS side, each FBS $m$ inserts the requested FUEs into a set $\succ_{n,UA}^m.$ Then, FBS $m$ updates its preference relation $\succ_{m,UA}$ based on (16) (lines 8 and 9). FUEs are updated according to matched list $\succ_{n,UA}^m$ by FBS $m$ under the matching $\mu_{UA}(m)$ if they guarantee a limited quota $\hat{N}_m$ and maximize the total utility in the matched list $\mu_{UA}(m)$ (lines 14 and 15). FUEs in the rejected list $\succ_{n,UA}^m$ remove FBS $m$ in their preference relation of the UA phase (line 19).

The convergence of the MATCH-UA algorithm can be verified by observing the preference formulation of players, i.e., FUEs and FBSs in the game. Preference relations of the FUEs and FBSs are fixed given subchannel allocation and power allocation of FUEs. Hence, given fixed preference relations of FUEs and FBSs, the MATCH-UA algorithm is known as the deferred acceptance algorithm in the two-sided matching which converges to a stable matching $\mu_{UA}^*$ [32].

Lemma 1. The stable matching $\mu_{UA}^*$ captures a local optimal solution for the OPT-UA problem.

Proof. See Appendix B in [30]

After finishing the UA phase, the FUEs that are associated to FBSs will be matched to subchannels, which is described as follows.
Algorithm 2 MATCH-CA: Matching game for allocating subchannels in a single FBS

Initialization: $N_m, K_m, N^\text{eq}_{m,k} = \emptyset, A^\text{eq}_{m,k} = \emptyset, \forall n, k, m \in M$.

Discovery and utility computation:
1: Each FUE $n \in N_m$ constructs its preference relation $\succ_{n,m,\text{CA}}$ by estimating $\phi^\text{CA}_{n,m}(k)$ on each subchannel $k \in K_m$ based on (21).

Find stable matching $\mu^*_{m,\text{CA}}$:
2: while $\sum_{k=1}^{K_m} b_{n,k}^A(t) \neq 0$ do
3: Each FUE $n \in N_m$:
4: Find $k = \arg \max_{k \in K_m} \phi^\text{CA}_{n,m}(k), \forall k \in K_m$.
5: Send a bid $b_{n,k}^A(t) = 1$ to FBS $m$.
6: Each subchannel $k \in K_m$:
7: Update bidders list on each subchannel $k$: $\Lambda^\text{eq}_{k,n}^m = \{ n : b_{n,k}^A(t) = 1, n \in N_m \}$.
8: Construct preference relation $\succ_{k,m,\text{CA}}$ based on (22).
9: Assign subchannel $k$ to FUE $n^* = \arg \max_{n \in \Lambda^\text{eq}_{k,m}^m} \phi^\text{CA}_{n,m}(k)$.
10: Update reject list: $\Lambda_{k,n}^\text{eq}^m = \{ N^\text{eq}_{m,k}^n \}_{m,n} \rightarrow \Lambda_{k,n}^\text{eq}^m$.
11: Remove subchannel $k$ from $\Phi^\text{CA}_{m,n}$, $\forall n \in N^\text{eq}_{m,k}$.
12: end while
until: Convergence to stable matching $\mu^*_{m,\text{CA}}$.

$(k_1, k_2 \in K_m)$ and $n_1 \succ_{k,m,\text{CA}} n_2$ ($n_1, n_2 \in N_m$), respectively. Next, we define the utility function of both FUE and FBS.

Utility function of the FUE. After each FUE is associated with an FBS, the FUE obtains the corresponding utility as:

$$\phi^\text{CA}_{n,m}(k) = R^k_{n,m},$$

where $n \in N_m$ estimates its utility on each subchannel $k$ based on the data rate achieved on subchannel $k$. By using the utility function in (21), FUEs have to bid to occupy each subchannel that maximizes their utility function.

Utility function of the FBS for each subchannel. In response to the requests from the FUEs for occupying certain subchannels, each FBS wishes to maximize a utility function on each subchannel, which is proposed as follows:

$$\phi^\text{CA}_{k,m}(n) = \varphi^\text{CA}_{k,m}(n),$$

where $R^k_{n,m} - R^\text{min}_{n,m}$ describes the reduction ratio of the effective interference required for UE $n$ on subchannel $k$, and $C^k_n = C^k_0 + \sum_{m' \in \mathcal{M} \setminus \{ m \}} R^k_{m',m}P_k$, $\varphi^\text{CA}_{k,m}$ is a weighted parameter. Obviously, for a given fixed power and subchannel allocation, the value $(R^k_{n,m} - R^\text{min}_{n,m})$ of FUE $n$ in FBS $m$ increases with $R^k_{n,m} - R^\text{min}_{n,m}$. In our proposed matching game, each FBS $m$ prefers to assign its subchannel to the FUE that maximizes the FUE’s satisfaction, but minimizes the impact on the macrocell network and aggregate interference to other FBSs on each subchannel.

2) Distributed algorithm for subchannel allocation based on the matching game: For the proposed one-to-one matching game, our goal is to find a stable matching which is similarly defined as the Definition 2. A matching $\mu^*_{\text{CA}}$ is said to be stable if it is not blocked by individual FUE $n$ and FBS $m$ with subchannel $k$ or any pair.

The distributed algorithm to solve the $\text{OPT-CA}_{(m)}$ is presented in the MATCH-CA algorithm. In this algorithm, each FUE constructs its preference relation based on (21) (line 1). In the swap matching phase, each FUE sends a bid request $b_{n,k}^A(t) = 1$ to access the subchannel $k$ that has the highest utility value (lines 2, 3 and 4). At the FBS side, the FBS collects all bidding requests and constructs a preference list on each subchannel (lines 7, 8 and 9). Based on the preference relation of the subchannels, the FBS assigns subchannels to FUEs which bring the highest utility value (line 10). The FUE removes the subchannel which is rejected by FBS $m$ in its preference (line 11). In the formulated game, preference relations of FUEs and subchannels in a single FBS are determined based on broadcast information in the network, as in the matching game in the UA phase. Hence, for a given subchannel allocation and power allocation of the FUEs, the formulated preference relations of the FUEs and subchannels are fixed. Moreover, the process of acceptance or rejection of applicants is performed in a manner similar to the conventional deferred acceptance algorithms [28], [32]. Thus, the MATCH-CA algorithm in single FBS $m$ converges to the stable matching $\mu^*_{m,\text{CA}}, \forall m \in M$.

Lemma 2. The stable matching $\mu^*_{\text{CA}}$ is a local optimal solution for the $\text{OPT-CA}_{(m)}$ problem.

Proof. The proof is similar to that for Lemma 1 so it is omitted.

C. Power allocation in the PC phase

Since each subchannel can be allocated to multiple FUEs associated with different FBSs, there is multi-cell interference among different femtocells. To mitigate the multi-cell interference among different femtocells and improve the spectrum utilization, subchannel and power allocation among femtocells needs to be coordinated.

In order to coordinate different femtocells, each FBS $m$ can send its proposed solutions $\forall n \in \mu^*_{m,\text{CA}}(k)$ to the CFM on subchannel $k, \forall k \in K_m$. The CFM collects information from FBS and then makes decisions about the subchannel and power allocation to the proposed femtocells, in which the coordinated problem is decomposed into $K$ sub-problems. Each sub-problem is given by

$$\text{OPT-PC}_{k \in K, \text{max.}} \sum_{(m,n) \in \mathcal{G}_k} R^k_{n,m}(P_{(n,m)})$$

s.t. (5), (4), (10).

Here, we only consider the FUE-FBS pairs which are assigned the same subchannel $k$, denoted by the set $\mathcal{G}_k = \{(m,n)|n = \mu_{UA}(m), n = \mu_{m,\text{CA}}(k), \forall n \in N, \forall m \in M, k \in K\}$. OPT-PC$_{k \in K}$ is commonly known as the problem of joint power and admission control of the FUEs based on spectrum underlay, which aims to find and admit a subset of FUEs to optimize different objectives [8], [9]. These objectives include maximizing the number of admitted FUEs in the set $\mathcal{G}_k$ and maximizing the total throughput of (23). The problem of maximizing the number of active FUEs on subchannel $k$ is well-studied in the literature, with many...
existing schemes for finding the optimal solutions [8], [9]. In our work, we utilize an algorithm called an effective link gain radio removal algorithm (ELGRA), which is proposed in [8]. ELGRA is proved to obtain the globally optimal solution of the minimum outage problem stated in the OPT-PC $\tilde{G}_k, k \in K$ with a computational complexity of $O(|\tilde{G}_k| \log |\tilde{G}_k|)$.

Once a maximal feasible subset $\tilde{G}_k$ is found that is defined by $\tilde{G}_k$, what remain is to adapt the transmit-power $P_{\tilde{G}_k}$ of the admitted FUEs so that the sum rate in OPT-PC $\tilde{G}_k, k \in K$ problem is maximized. Different from previous works, we solve OPT-PC $\tilde{G}_k, k \in K$ using geometric programming and dual decomposition [33]–[35].

Toward this end, we transform OPT-PC $\tilde{G}_k, k \in K$ into a convex optimization problem. When $\Gamma_{nm} >> 1$, we employ the approximation $\bar{R}_{nm} \approx B_k \log(P_{nm})$. Additionally, we introduce a new variable $\bar{R}_{nm} = \log P_{nm}$ with a new feasible set $\bar{V}_n = \{P_n | P_{nm} \in [\log P_{min}, \log P_{max}], \forall n \in \tilde{G}_k\}$. Moreover, we define an auxiliary variable to estimate the intra-tier interference $Z_{nm}^{k} = \sum_{n' \in \tilde{G}_k, n' \neq n} g_{nm} P_{nm}$ and a new variable $\bar{Z}_{nm}^{k} = \log(Z_{nm}^{k}), \forall(n, m) \in \tilde{G}_k$. The OPT-PC $\tilde{G}_k, k \in K$ then becomes:

$$\begin{align*}
\text{OPT-PC'}^{(k), k \in K}: \\
\min_{(P, Z)} \sum_{(n, m) \in \tilde{G}_k} \log \left[ \frac{e^{-R_{nm}^{k}}}{P_{nm}^{k}} (e^{Z_{nm}^{k}} + g_{nm} P_{nm}^{k} + \sigma^2) \right] \\
\text{s.t.} \\
\sum_{n \in \tilde{G}_k} g_{nm} \bar{R}_{nm}^{k} - t_0^{k} \leq 0, \\
\log \left[ \frac{e^{-R_{nm}^{k}}}{P_{nm}^{k}} (e^{Z_{nm}^{k}} + g_{nm} P_{nm}^{k} + \sigma^2) \right] - \log(\lambda_n) \leq 0, \forall(n, m) \in \tilde{G}_k, \\
\bar{Z}_{nm}^{k} = \log(Z_{nm}^{k}), \forall(n, m) \in \tilde{G}_k, \\
\bar{P}_{nm}^{k} \in \bar{V}_n^{k}, \forall n \in \tilde{G}_k, \\
in which \lambda_n = 2^{-\frac{\nu_{nm}^{k}}{\theta_n}}, \forall n \in \tilde{G}_k.
\end{align*}$$

**Proposition 1.** The problem OPT-PC' $(k), k \in K$ is a convex optimization problem in $(P, Z)$-space. Then, based on the KKT condition and sub-gradient method, the optimal transmit power levels are determined as follows:

$$P_{nm}^{k*} = e^{-\bar{R}_{nm}^{k} \left[ \frac{1 + \nu_{nm}^{k}}{\lambda_n g_{nm}^{k}} \right] P_{nm}}, \forall(n, m) \in \tilde{G}_k,$$

where $[a]^{P_{nm}}$ is the projection of $a$ onto the set $P_{nm} = [P_{nm}^{min}, P_{nm}^{max}]$. Moreover, the auxiliary variable $Z_{nm}^{k*}$ is determined as follows:

$$\bar{Z}_{nm}^{k*} = \left[ \frac{(\frac{\nu_{nm}^{k}}{P_{nm}^{k}} + \sigma^2) \nu_{nm}^{k}}{1 - \nu_{nm}^{k} + \kappa_{nm}^{k}} \right]^+, (30)$$

in which the Lagrange multipliers $\lambda_n, \kappa_{nm}^{k}$, and consistency price $\nu_{nm}^{k}$ are updated as in (31), (32), and (33) with step sizes $s_1, s_2$, and $s_3$, respectively.

**Algorithm 3** DIST-P: Distributed power allocation on sub-channels

**Input:** $G_k, k \in K$, $t = 0$, $P_n^{k} \in P_n$, $\lambda_n^k(0) > 0, \kappa_{nm}^{k}(0) > 0, \nu_{nm}^{k}(0) > 0, \forall(n, m) \in \mathcal{M}$. Each FBS $m (m \in \mathcal{M})$:

1. Estimate $Z_{nm}^{k}(0)$.
2. Calculate $e^{-\bar{R}_{nm}^{k}(t + 1)}$ based on (30).
3. Update $\nu_{nm}^{k}(t + 1)$ and $\nu_{nm}^{k}(t + 1)$ using (32) and (33), respectively.
4. Transmit $\nu_{nm}^{k}(t + 1)$ to FUE $n$.

Each FUE $n (n \in \mathcal{N}_n)$:

5. Receive the updated value $\nu_{nm}^{k}(t + 1)$.
6. Update the Lagrange multipliers $\lambda_n^k(t + 1)$ from (31).
7. Calculate the power value $P_{nm}^{k}(t + 1)$ as in (29).
8. Broadcast $g_{nm}^k$ and $P_{nm}^k(t + 1)$.

**Output:** Convergence to optimal power $P_{nm}^{k*}$.

Proof. See Appendix C in [30]

Then, we employ the sub-gradient method to update the Lagrange multipliers and find the optimal power allocation as in Algorithm 3, namely DIST-P.

In the DIST-P algorithm, the information exchange among FUEs and FBSs can be realized by exploiting feedback, such as ACK/NACK. Note that the messages in the DIST-P algorithm are broadcast to coupled FUEs and FBSs in the set $G_k$ through the coordination of CFM. Moreover, $g_{nm}^k$ required in the DIST-P algorithm can be estimated at FUE $n$ in femtocell $m$ by using any of the available channel estimation methods [31].

**Lemma 3.** If the optimization problem OPT-PC' $(k), k \in K$ is feasible, then the DIST-P algorithm converges to the optimal solution $P_{nm}^{k*}$.

Proof. See Appendix D in [30]

If Lemma 3 is not true, FUEs that sent proposals to CFM would not be guaranteed to achieve their QoS. Moreover, MBS may not be protected without the DIST-P algorithm.

Moreover, we see that in the PC phase, by using distributed ELGRA and DIST-P algorithms, the access controller prefers to serve a set of FUEs on each subchannel $k$ that satisfies all of the constraints in OPT-PC' $(k), k \in K$. It can be verified that, in order to converge with $T_{G_k}$ iterations for the $G_k$ pair FUE-FBSs, the exchange overhead is $|G_k| T_{G_k}$. 

Algorithm 4 JUCAP: Join UA, CA, and PC

Inputs: $\mathcal{N}, \mathcal{M}, \mathcal{K}$.

Initialization: $\tau = 0$, $\mathcal{G}_k = \emptyset$; FBSs broadcast the set $\mathcal{K}_m$, and announce the set of available subchannels. Initialize the set of $\mathcal{G}_k$ for each subchannel $k, k \in \mathcal{K}$ as an empty set.

While $\mathcal{G}_k, \forall k \in \mathcal{K}$ remain unchanged for two consecutive iterations.

1: $\tau = \tau + 1$.

UA Phase:

2: FUEs determine their preference ordering for FBSs $m \in \mathcal{M}$ using (15).
3: FBSs calculate utility of each FUE applicant (16).
4: FUEs apply for FBSs $m \in \mathcal{M}$ and get accepted or rejected via the MATCH-UA algorithm.

CA Phase:

5: FUEs that are accepted in FBS $m \in \mathcal{M}$ apply for subchannel $k \in \mathcal{K}_m, \forall m \in \mathcal{M}$ using (21).
6: FBSs calculate utility of FUEs applicant on subchannel $k \in \mathcal{K}_m, \forall m \in \mathcal{M}$ using (22).
7: FUEs get accepted or rejected by FBSs on subchannels via the MATCH-CA algorithm.

PC Phase:

8: FBSs send FUE’s proposal on subchannels to the access controller.
9: FUEs get accepted or rejected by CFM on subchannels $k \in \mathcal{K}$ via the ELGRA algorithm.
10: Update $\mathcal{G}_k, \forall k \in \mathcal{K}$.
11: FUEs update power using (34).
12: FUEs and FBSs update user association and subchannel assignment information.
13: repeat UA phase, CA phase, and PC phase.

end

Output: Convergence to group stable $\mathcal{G}_k, \forall k \in \mathcal{K}$.

Then, the transmit power of FUEs in the set $\mathcal{G}_k$ will be updated as follows:

$$P_n^k = \begin{cases} P_n^k*, & \text{if } n \text{ is accepted by CFM on subchannel } k, \\ 0, & \text{if } n \text{ is rejected by CFM on subchannel } k. \end{cases}$$  \hfill (34)

In (34), $P_n^k*$ is updated as in (29), which means that the FUE $n$ is permitted to transmit on subchannel $k$ with power level $P_n^k*$. In addition, the transmit power $P_n^k*$ of FUE $n$ on subchannel $k$ is maintained during the next iteration. However, these FUEs will not be included at the UA and CA phases in the next iteration. On the other hand, when $P_n^k$ is set to be zero, which mean that subchannel $k$ is not allocated to FUE $n$ at FBS $m$ by the CFM, then in next iteration, subchannel $k$ will not be considered in the preference relations of FUE $n$ at FBS $m$ in the CA phase. However, the FUEs that are rejected in the PC phase will continue joining the UA and CA phases. Next, the transmit power of the rejected FUEs will be determined, as discussed in (14).

Obviously, after finishing the PC phase, the transmit power and subchannel allocations of the FUEs are updated, which affects the preference of players in the UA and CA phases. Next, we propose a framework for joint user association, subchannel assignment, and power allocation.

D. Joint user association, subchannel allocation, and power control

In this subsection, we propose a framework for joint UA, CA, and PC phases, shown in Fig. 2. The proposed framework is summarized in Algorithm 4, which is referred to as the JUCAP algorithm. The proposed algorithm comprises three main phases: the UA phase, CA phase, and PC phase, operating in separate time scales. The UA phase matches FUEs to FBSs. The CA phase focuses on the matching of FUEs to subchannels in the associated FBS. The PC phase performs admission controls, updating subchannels, and transmit power allocation via the CFM.

Initialization. FBSs use candidate control channels to send proposals, which are composed of Femto-ID and their available subchannels to its surrounding FUEs. FUEs accept or reject the proposals. Then, the channel gain states of the FUEs are estimated and sent back to FBSs that accepted the proposals. Next, the preference relations of FUEs and FBSs in the UA and CA phases are estimated at the beginning of each iteration. Utility values in the preference relation of FUEs and FBSs will be maintained in whole matching processes in the UA and CA phases of an iteration.

UA phase. After initialization, FUEs join the UA phase. The FUEs first determine their preference orders for FBSs using (15) (Step 2). Each FUE applies for an FBS based on the estimated utility value in (15) (Step 3). Then, each FUE accepts the most preferred FUE and rejects other proposals based on the utilities defined in (16) and FBSs’ quota. The FUEs in the UA phase get accepted and rejected by FBSs, as in the MATCH-UA algorithm (Step 4). Once FUEs are accepted by an FBS or rejected by all its preferred FBSs, the MATCH-UA algorithm is terminated. The matching in the UA phase remains unchanged until the FUEs start new iterations.

CA phase. When the MATCH-UA algorithm is terminated in the UA phase of the current iteration, the FUEs accepted by FBSs will join the CA phase. The FUEs that are associated to FBS $m$ apply for subchannel $k \in \mathcal{K}_m$ based on the utilities defined in (21). Each FBS $m \in \mathcal{M}$ handling the subchannels $\mathcal{K}_m$ accepts the FUE that gives the highest utility based on (22). In addition, the FBSs $m \in \mathcal{M}$ reject all other applicants. The MATCH-CA algorithm terminates when every FUE is accepted by a subchannel or rejected by all subchannels in the associated FBS.

PC phase. After finishing the CA phase, FUEs join the PC phase, where the minimum data rate requirement and MBS protection have to be guaranteed. The FBSs send proposals on subchannels to the access controller in the CFM (step 8). Next, FUEs get accepted or rejected by the CFM on subchannels via the ELGRA algorithm to guarantee a feasible solution of OPT-PC$_{\tau}(\mathcal{G}_k), k \in \mathcal{K}$, as discussed in Section IV.C (step 9). The FUEs and FBSs accepted in the ELGRA algorithm or allocated by the CFM will be inserted into the groups $\mathcal{G}_k, \forall k \in \mathcal{K}$. Then, the transmit power of FUEs belonging to the groups $\mathcal{G}_k, \forall k \in \mathcal{K}$ is updated by using (34) (step 11). Given a set of proposals of FBSs on subchannels $k \in \mathcal{K}$, the PC phase is terminated when both the ELGRA and DIST-P algorithms converge to the optimal solutions, which are discussed in Section IV.C. At the end of the iterations, the information about remaining quota, subchannel availability, user association, subchannel allocation, and transmit power are updated in the next iteration.

In our proposal, once $(m, n)$ pairs are rejected by the subchannel $k$ in the PC phase, these FUEs are not served...
by any FBS. Then, these FUEs will be considered as new users. After that, these FUEs are moved to the new iteration, which again performs the UA, CA, and PC phases. In order to start new iterations, preference relations of the FUEs and FBs in both the UA and CA phases will be updated in the last step of the previous iterations. However, FUEs rejected by the CFM on the proposed subchannel will not be considered in the new preference relation in the CA phase at the next iteration. Additionally, when the FUE is rejected by the CFM on all subchannels in the matched FBS, this FBS will not be considered in the preference relation of the UA phase in the next iteration. Specifically, the FUEs that are not rejected by the CFM in the previous iterations will not join the next matchings in the UA and CA phases. However, in order to optimize a reused subchannel in the CFM, new proposals from FBs on subchannels at the CFM are still processed in the PC phase. Obviously, social welfare is maximized on each subchannel given optimal proposals from the FBs. In addition, when the JUCAP algorithm proceeds to ELGRA and DISt-P, these algorithms enable the feasibility of two constraints (4) and (5), respectively. Subsequently, the JUCAP algorithm terminates once the groups $G_{k}, \forall k \in K$ do not change for two consecutive iterations (step 12). This means that there is no further new requests from FUEs in three UA, CA, and PC phases for two consecutive iterations. The convergence of the JUCAP algorithm and groups stability are analyzed in Section E.

E. Convergence and stability of the proposed algorithm

In this subsection, we prove convergence of the JUCAP algorithm. Let us consider group $G_{k}$, $k \in K$, which is formed as a result from the UA, CA, and PC phases. Then, we introduce the following definition:

**Definition 4.** Given the interrelationship between FUEs, FBs, subchannels, and CFM in the JUCAP algorithm, the group $G_{k}$, $k \in K$, is said to be stable if it is not blocked by any group which can be represented by two conditions as follows:

1) No FUE $n'$ outside the group $G_{k}$ can join it.

2) No FUE $n$ inside the group $G_{k}$ can leave it.

**Proof.**

From Definition 4 and Theorem 1, we can state the convergence of the JUCAP algorithm in the following.

**Theorem 2.** A group stable $G_{k}$ is formed in a finite number of iterations and, thus, the JUCAP algorithm is guaranteed to converge.

**Proof.**

Because the numbers of FBs and subchannels are finite, the numbers of preference relations $\succ n, UA$, $\succ n, CA$, and $\succ n, CA$ of FUEs and FBs are also finite. Moreover, the number of preference relations $\succ n, UA$, $\succ n, CA$, and $\succ n, CA$ are reduced after each iteration due to the rejected operations in the UA, CA, and PC phases. Furthermore, the accepting or rejecting decision in the JUCAP algorithm is based on stable matchings at each iteration, as stated in Theorem 1. Therefore, each group stable $G_{k}$ that is defined in Definition 4 is formed after a finite number of iterations.

To analyze the complexity and overhead of the JUCAP algorithm, we find an upper bound on the maximum number of requested messages at each outer iteration $\tau$. We abuse the notation $\tau$ as the slot time duration at the $\tau$-th outer iteration of the JUCAP algorithm. The requested messages at each iteration $\tau$ is determined by the number of requested messages that must be exchanged in the UA, CA, and PC phases at each outer $\tau$-th iteration. By considering the worst case for each phase at the outer iteration $\tau$, an upper bound on the number of requested messages of a slot time $\tau$ can be determined by $N_{\text{UA}}(\tau) \times M$, $N_{\text{CA}}(\tau) \times M$, and $\max_{k \in K}(\vert G_{k}(\tau)\vert) \times K$, respectively. (See Appendix C in [30]). Here, we define $N_{\text{UA}}(\tau)$ and $N_{\text{CA}}(\tau)$ as the number of FUEs join into the UA and CA phases at the iteration $\tau$, respectively. Additionally, $|G_{k}(\tau)|$ is the number of FUE-FBS pairs join into the subchannel $k$ in the PC phase at the iteration $\tau$, where $|G_{k}(\tau)| \leq M, \forall k$. Hence, the upper bound on the number of requested messages during a slot time $\tau$ is $N_{\text{UA}}(\tau) \times M + (N_{\text{CA}}(\tau) \times M + \max_{k \in K}(\vert G_{k}(\tau)\vert)) \times K$.

V. Simulation results

This section presents simulation results to evaluate the proposed algorithms. We first present our setup and then the results.

**A. Simulation setup**

In order to evaluate our framework, we use the following simulation setup. We simulate an MBS and 5 FBs ($M = 5$) with the coverage radii of 500 m and 25 m, respectively. The FBs are deployed in a small indoor area of 250 $\times$ 250 m.$^{2}$
to serve $N = 20$ FUEs. Each FBS has the quota equal to $4 \hat{N}_m = 4(\forall m)$ [4]. In the CFN, we consider 10 subchannels, which are allocated to 10 MUEs in the macrocell network. The bandwidth of each subchannel is 360 kHz and MUEs have a fixed power level of 100 mW. The power channel gains are assumed to be i.i.d Rayleigh fading with the mean value of one. The path loss model is followed by the log-distance path loss model [37], [38]. In the MUE-to-MBS path-loss for distance $d$, $L_d = 15.3 + 37.6 \log_{10}(d)$. In the FUE-to-MBS path-loss for distance $d$, $L_d = 15.3 + 37.6 \log_{10}(d) + \rho$. The wall penetration loss $\rho$ equals to 10 dB. In the FBS-to-same-cell-FUE path-loss for distance $d$, $L_d = 38.46 + 20 \log_{10}(d)$. The maximum interference power on each subchannel at the MBS is -70 dBm. The noise power is set to -114 dBm. Each FUE has a maximum transmit power of 100 mW. We set the values of $\varphi_{UA}$, $\varphi_{CA}$, $c_k$, and $\bar{c}_m(\forall m, k)$ equal to 100, 100, 10, and 0.1, respectively. Moreover, the MUEs are randomly distributed outside the area $250 \times 250$ m$^2$.

B. Simulation results

In the following, we present the results based on the above settings. We first show single snapshot results from executing the algorithms only once. Then, the results over multiple time intervals will be presented.

1) Evaluation of the proposed algorithms in a single snapshot: We now present a snapshot resulting from the proposed algorithms in the UA, CA, and PA phases with $N = 20$ FUEs, $M = 5$ FBSs, $K = 10$ subchannels, and $\hat{N}_m = 4$. The results of the distributed user association based on the MATCH-UA algorithm for given network settings are presented in Fig. 3. A load-sharing has been achieved in the MATCH-UA algorithm to avoid FBS overloading, as shown in Fig. 3a. We can see that the allocated subchannel (Fig. 3b) and power level (Fig. 3c) for the FUEs can meet the constraints on the minimum data rate, as presented in Fig. 3d.

2) Evaluation of the proposed algorithms over multiple snapshots: We evaluate our proposals by considering average throughput and outage probability. In addition, we evaluate the convergence time of the proposed algorithms via the total number of requests, which are requested in the UA, CA, and PC phases, to meet the JUCAP algorithm convergence as discussed in Theorem 2. All statistical results are averaged over a large number of independent simulation runs. Moreover, we compare the average throughput and outage probability against three other schemes: “max-SINR without JUCAP”, “without ELGRA”, “without DIST-P” and “max-SINR with JUCAP”. Basically, the compared schemes are based on the JUCAP algorithm. For the “max-SINR without JUCAP” scheme, the UA phase is executed by removing step (19) in the MATCH-UA, which is known as the “max-SINR algorithm”. Additionally, the CA phase is performed based on the “greedy algorithm” by removing the step (11) in the MATCH-CA algorithm. For the “max-SINR with JUCAP” scheme, the UA phase is executed based on the “max-SINR algorithm”.

For the “without ELGRA” scheme, the ELGRA algorithm is removed in the PC phase. Then, applicants to the PC phase are processed directly by the DIST-P algorithm. By doing this, the CFM rejects applicants that would not guarantee the FUEs’ QoS and MBS protection as discussed in Lemma 3. For the “without DIST-P” scheme, the PC phase ignores the DIST-P algorithm and the transmit powers of FUEs are set equal to the maximum power on the assigned subchannel. By doing so, the minimum data rate requirements of the FUEs and interference power constraint at the MBS may not be guaranteed. When these constraints are violated, the FUEs are not permitted to transmit data, which are controlled by the CFM.

Fig. 4a and Fig. 4b present the average aggregate throughput and outage probability following the quota values of FBSs with 20 FUEs, 5 FBSs, 10 subchannels, and $I_0^{\text{th}} = -70$ dBm.
∀k, respectively. The FUEs are deployed uniformly inside the FBS's coverage. As shown, the average throughput in our proposed scheme increases with the quota value of SBSs since the number of served FUEs increases. However, this value is saturated as the quota value becomes sufficiently large. Moreover, we can see from Fig. 4a that the average network throughput of the “proposed approach” scheme can reach up to 30.72%, 646.71%, 110.37, and 2.34% gain over the “max-SINR without JUCAP”, “without ELGRA”, “without DIST-P”, and “max-SINR with JUCAP” schemes with a quota value of 4, respectively. Additionally, Fig. 4b shows that the average outage probability of our proposal decreases as the quota value increases. This is because the higher quota value increases the number of FUEs that are associated in the UA, CA, and PC phases. Clearly, the “proposed approach” scheme outperforms the other schemes by increasing the average network throughput for large network sizes since the SBSs’ quota and subchannels are limited. Moreover, Fig. 5a compares the average network throughput for the “proposed approach” scheme and the other three schemes as the number of FUEs varies. In Fig. 5a, we can see that the “proposed approach” scheme outperforms the other schemes by increasing the average network throughput for a different number of FUEs. Specifically, Fig. 5a shows that the average network throughput of the “proposed approach” scheme can reach up to 30.04%, 596.51%, 113.15%, and 4.86% gain over to the “max-SINR without JUCAP”, “without ELGRA”, “without DIST-P”, and “max-SINR with JUCAP” schemes for the network size of 16 FUEs, respectively.

Moreover, given that requests from the FBSs in the PC phase are limited. Moreover, Fig. 5b compares the average outage probability for the “proposed approach” and other three schemes. The outage probability determines how many of FUEs on average could be served by the femtocell network. The average outage probability increases with the number of FUEs because more FUEs will be rejected in the UA, CA, and PC phases. This is because the competition among FUEs in the UA, CA, and PC phases increases when number of FUEs increases and the network resources become more scarce. Moreover, Fig. 5b shows that the “proposed approach” scheme can achieve the smallest average outage probability.

Fig. 5c evaluates the average number of requests and iterations versus the number of FUEs in the network from the JUCAP algorithm. As shown in Fig. 5c, the average total number of requests in the UA phase increases with the number of FUEs. This is because the number of requests in the UA phase depends on the number of FUEs that bid to associate FBSs and rejected FUEs in the MATCH – UA algorithm. Moreover, Fig. 5c shows that the number of requests in the CA phase increases with number of FUEs from 0 to 20, since FBSs can still have resources to serve. The number of requests in the CA phase does not increase much when the number of FUEs is greater than 20. This is because the maximum number of FUEs which can be associated with FBSs is 20 with network parameters $M = 5$ and $\hat{N}_m = 4, \forall m \in \mathcal{M}$ for each FBS. Moreover, given that requests from the FBSs in the PC phase...
different schemes. The FUEs are deployed uniformly inside of all femtocells versus the maximum power of FUEs for the uniform FUEs distribution scenario. The average throughput of the “proposed approach” in the non-“without DIST-P”, and “max-SINR with JUCAP” schemes, can reach up to 28.25%, 470.39%, 91.66%, 21.16% gain over to the “max-SINR without JUCAP”, “without ELGRA”, “without DIST-P” and “max-SINR with JUCAP” schemes, respectively for the network size of 30 FUEs. Clearly, the average network throughput of the “proposed approach” scheme also outperforms the other schemes by increasing the average network throughput for a different number of FUEs. Moreover, Fig. 7 shows that the average network throughput of the “proposed approach” can reach up to 28.25%, 470.39%, 91.66%, 21.16% gain over to the “max-SINR without JUCAP”, “without ELGRA”, “without DIST-P”, and “max-SINR with JUCAP” schemes, respectively for the network size of 30 FUEs. Clearly, the average throughput of the “proposed approach” in the non-uniform FUEs distribution scenario is higher than that of the uniform FUEs distribution scenario.

In Fig. 8, we show the average total network throughput of all femtocells versus the maximum power of FUEs for the different schemes. The FUEs are deployed uniformly inside the FBSs’s coverage. In order to estimate optimality gap of the proposed approach, we also compare the “proposed approach” scheme with “centralized approach” that obtains an optimal solution, in which user association, subchannel and power allocations are searched exhaustively. However, the comparison is limited for the small number of FUEs (N = 8, M = 4, K = 4, and |Nm| = 2) due to the complexity of the “centralized approach” scheme. As shown, when we increase the value of the maximum power for each FUE, the average throughput of the “without DIST-P” scheme decreases when the maximum power is greater than 100 mW due to the increasing violation of the MBS protection and minimum rate requirement of each FUE. Additionally, the “without DIST-P” scheme reaches zero when the transmit powers of FUEs are equal to 1 W. This is because the data transmission of all FUEs in the PC phase violated constraints (4) and (5). Hence, all proposals from UA and CA phases to the PC phase are rejected and the data transmission of all FUEs are interrupted by the CFM. The comparison shows that the “proposed approach” scheme yields the solution close to that of the “centralized approach” scheme, with a gap of 7.74% for a network with a maximum transmit power of 100 mW.

VI. CONCLUSIONS

In this paper, a novel framework has been proposed to jointly optimize user association and resource allocation in the uplink cognitive femtocell network. In the considered CFN, FBSs have been deployed to serve a set of FUEs by reusing subchannels in a macrocell. The joint for the user association, subchannel assignment, and power allocation have been formulated as an optimization problem that maximizes the overall uplink throughput while guaranteeing FBS overloading avoidance, data rate requirements of the served FUEs, and MBS protection. To solve this problem, a distributed
framework based on the matching game has been proposed to model and analyze the competitive behaviors among the FUEs and FBSs. Using this framework, distributed algorithms have been implemented to enable the CFN to make decisions regarding user association, subchannel allocation, and transmit power. The developed algorithms have been shown to converge to stable matchings and exhibit a low computational complexity. Simulation results have shown that the proposed approach yields a performance improvement in terms of the overall network throughput and outage probability with a low computational complexity.

For the future work, the proposed framework in the scenario without distinction between MUEs and FUEs can be considered. However, it leads to more coupled problems compared to those in our considered scenario. In this scenario, the network access modes needs to regulate how subscribed users can associate with the macrocell network or CFN. Also, the network access mode selection depends on strategies of the network managers and mobile users (such as cost-based and load balancing-based strategies).

**APPENDIX**

**A. Proof of the Theorem 1**

In the initialized state of each iteration \( \tau \), the FUEs that have not yet joined any group \( G_k(\tau - 1), \forall k \in \mathcal{K} \) are going to new subsequent matchings at the UA, CA, and PC phases. Given group formations \( G_k(\tau - 1), \forall k \in \mathcal{K} \), the interference at FBSs on all subchannels is fixed. Hence, new preference relations \( \succ^{(\tau)}_{m,\text{UA}}, \succ^{(\tau)}_{n,\text{UA}}, \succ^{(\tau)}_{nm,\text{CA}} \) and \( \succ^{(\tau)}_{km,\text{CA}} \) in both the UA and CA phases are determined.

**Definition 5.** A matching \( \succ_{\text{UA}} \) is stable if it is individually rational and there is no blocking pair or any \((m, n)\) in the set of acceptable pairs such that \( n \) prefers \( m \) to \( \succ_{\text{UA}} (n) \) and \( m \) prefers \( n \) to \( \succ_{\text{UA}} (m) \).

**Lemma 4.** The association performed in the MATCH-UA algorithm follows the preference relations \( \succ^{(\tau)}_{m,\text{UA}} \) and \( \succ^{(\tau)}_{n,\text{UA}} \), and leads to a stable matching in each iteration \( \tau \).

**Proof.** In any iteration \( \tau \), new preference relations of the FUEs and FBSs are given by \( \succ^{(\tau)}_{m,\text{UA}} \) and \( \succ^{(\tau)}_{n,\text{UA}} \), respectively. Additionally, the quota and available subchannels at each FBS in \( \tau \) are given by \( \vec{N}_m \) and \( \vec{K}_m \), respectively. We note that only FUEs that are rejected by the CFM and UA phase in iteration \( \tau - 1 \) will be included in the UA phase at iteration \( \tau \). Then, these FUEs will be processed in the MATCH-UA algorithm. The FUEs that are matched by the CFM in iteration \( \tau - 1 \) will be kept in the preference relation \( \succ^{(\tau)}_{m,\text{UA}} \) of FBSs. The MATCH-UA algorithm design is based on basic principles of the deferred-acceptance algorithm and college admissions model with responsive preferences [32], in which it is proved that does not exist any blocking pair when the algorithm terminates. Hence, the MATCH-UA algorithm produces a stable matching \( \mu_{\text{UA}} \), where they are not blocked by any FUE-FBS pair. Hence, the matching in the UA phase is a stable matching.

**Lemma 5.** The allocation performed in the MATCH-CA algorithm follows the preference relations \( \succ^{(\tau)}_{nm,\text{CA}} \) and \( \succ^{(\tau)}_{km,\text{CA}} \) and leads to a stable matching in each iteration \( \tau \).

**Proof.** Following the stable matching in the UA phase, new FUEs matched to the FBSs will join the CA phase at iteration \( \tau \). Given the new preference relations \( \succ^{(\tau)}_{nm,\text{CA}} \) and \( \succ^{(\tau)}_{km,\text{CA}} \) of FUEs and subchannels in the FBS, we consider the stability of matchings based on the MATCH-CA algorithm at iteration \( \tau \). In this phase, we also note that only the subchannels that are unmatched by the CFM at iteration \( \tau - 1 \) will be included in the CA phase at iteration \( \tau \) based on the MATCH-CA algorithm. In addition, the FUEs that are matched by CFM in iteration \( \tau - 1 \) on the subchannel of the FBS will be kept in the preference relation \( \succ^{(\tau)}_{km,\text{CA}} \) of the FBSs. We prove Proposition 5 by contradiction. We define \( \mu_{n,\text{CA}} \) as a matching obtained by the MATCH-CA algorithm at any iteration \( \tau \) of JUCAP algorithm. Let us assume that FUE \( n = \mu_{\text{UA}}(m) \) is not allocated to subchannel \( k \) of FBS \( m \), but it has a higher order in the preference relations. Hence, the \((n, k)\) pair will block \( \mu_{n,\text{CA}} \). However, since \( n \succ^{(\tau)}_{km,\text{CA}} n' \), in which \( n' = \mu_{m,\text{CA}}(k) \), subchannel \( k \) must select FUE \( n \) before the algorithm terminates. As a result, the pair \((n, k)\) will not block \( \mu_{m,\text{CA}} \), which contradicts our assumption. Therefore, matchings in the CA phase are stable when there are no blocked pairs \((n, k)\) at all FBSs \( m \), \( \forall m \in M \) or any pair.

Hence, given a stable matching in the CA phase, the number of new proposals from the FBSs for accessing subchannels is fixed at iteration \( \tau \). Then, the subchannel assignment in the PC phase can be considered as matching FUE-FBS \((m, n)\) pairs to subchannels \( k \in \mathcal{K} \), in which matching operations are based on the ELGRA and DIST-P algorithms.

**Definition 6.** A matching for resource allocation in the PC phase is weak Pareto optimal if there is no other matching that can achieve a better sum-rate, where the inequality is component-wise and strict for one pair \((m, n)\).

**Lemma 6.** Given a proposal from the CA phase, the resource allocation in the PC phase on each iteration \( \tau \) is weak Pareto optimal [39] under the proposals offered from the FBSs.

**Proof.** Let \( \mu_{\text{PC}} \) be a matching obtained by the ELGRA and DIST-P algorithms at any iteration \( \tau \) of the JUCAP algorithm. Let \( R_{nm}(\mu_{\text{PC}}) \) be the data rate achieved by pair \((n, m)\) for a matching \( \mu_{\text{PC}} \) given a set of proposals offered from FBSs on subchannel \( k \), where \( n = \mu_{\text{UA}}(m) \) and \( n = \mu_{m,\text{CA}}(k) \). Additionally, we define 
\[
\phi_{\text{PC}}(\mu_{\text{PC}}) = \sum_{(n, m) \in G_k, k \in \mathcal{K}} R_{nm}(\mu_{\text{PC}}(k))
\]

as the sum rate of all FUE-FBS pairs at iteration \( \tau \). On the contrary, we define \( \mu_{\text{PC}}' \) as an arbitrary unstable outcome better than \( \mu_{\text{PC}} \). Hence, we consider two scenarios:

1) **Matching \( \mu_{\text{PC}} \) is lack of individual rationality:** If subchannel \( k \) is not individually rational, then the CFM can remove the pair \((n, m)\) to \( \mu_{\text{PC}}'(k) \) to improve the utility on subchannel \( k \), or 
\[
\sum_{(n, m) \in G_k, k \in \mathcal{K}} R_{nm}(\mu_{\text{PC}}'(k)) < \sum_{G_k} R_{nm}(\mu_{\text{PC}}(k)).
\]

2) **Matching \( \mu_{\text{PC}} \) is blocked:** Whenever the matching \( \mu_{\text{PC}} \) is blocked by any pair \((n, m)\), the CFM strictly prefers the FUE-FBS pair \((n, m)\) to \( \mu_{\text{PC}}(k), (n, m) \in G_k, \) and the
pair \((n, m)\) strictly prefers subchannel \(k\) to \(\mu_{PC}(n, m) = \emptyset\). In this case, the CFM will add the pair \((n, m)\) to improve the utility on subchannel \(k\), or \(\sum_{(n,m): \mu_{PC}(n, m) = \emptyset} \sum_{k} R_{nm}(\mu_{PC}(k)) > \sum_{k} R_{nm}(\mu_{PC}(k))\). Hence, the matching \(\mu_{PC}(k)\) is replaced by the new matching \(\mu_{PC}(k)\).

Obviously, there is no outcome \(\mu_{PC}'\) better than the matching \(\mu_{PC}\) for both scenarios (1) and (2). Based on Definition 6, \(\mu_{PC}\) is a stable outcome or an optimal allocation and the proof follows.

Hence, we obtain Lemmas 4, 5, and 6 that prove Theorem 1

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