A Lightweight Algorithm for Probability-based Spectrum Decision Scheme in Multiple Channels Cognitive Radio Networks

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***Abstract*—Compare with the sensing-based spectrum decision scheme, the probability-based spectrum decision scheme has been shown to yield a shorter queuing delay time in Cognitive Radio (CR) system that consists of many Primary Users (PUs) and Secondary Users (SUs). However, the former scheme had cumbersome algorithms and slowly converging speed. In this paper, by introducing Lagrange function, we propose a lightweight algorithm with the computational effort $O(N)$ to define the optimal distribution probability vector. Numerical results demonstrate a high degree of accuracy for the derived expressions.**

***Index Terms*—cognitive radio, dynamic spectrum access, multiple channels.**

I. Introduction

Recently, Cognitive Radio (CR) has been proposed as a way to improve spectrum efficiency by exploiting the unused spectrum in dynamically changing environments [**?**]-[**?**]. The spectrum decision process is an important process in CR system which helps the SU select the best channel to transmit data from candidate channels. There are two kinds of spectrum decision schemes: (i) the sensing-based spectrum decision scheme; and (ii) the probability-based spectrum decision scheme [1], [2]. For the probability-based spectrum decision method, which was proposed with the objective of minimizing the queueing delay time of the SU [1], the operating channel is selected based on the predetermined probabilities which are defined according to PU and SU traffic pattern.

However, the probability-based scheme needs to determine the optimal channel selection probability to minimize the queueing delay time. In [1], the analysis for general system integrated with sensing error was the main advantage. Nevertheless, it had complex expression of the SU queueing delay time. Therefore, in order to get the optimal solution, the authors used numerical optimization algorithm with exhausted search that is cumbersome and high complexity. In [2], the authors used "Best Reply Algorithm" to compute the optimal fraction of time lengths of all the slots occupied by the secondary device. The time complexity of the "Best Reply Algorithm" is $O(N\,logN)$.

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In this paper, the focus of our works is to propose an algorithm to determine the optimal channel selection probability to minimize the queueing delay time. Our main contribution includes:

- We obtain explicit and simple expressions for the expected queueing delays of SUs associated with using the PU band by employing the server-breakdown queueing model.
- We use convex optimization theory to propose a lightweight and fast algorithm to calculate the optimal distribution probability vector with the computational effort $O(N)$.

The remainder of this paper is organized as follows. The system model and problem statement are introduced in Section II. The queueing delay time of SU's packets is analyzed in Section III. In Section IV, the algorithm to compute the optimal distribution probability vector is presented. The numerical analysis and conclusion are shown in section V and VI respectively.

II. System Model and Problem Statement

We consider a CR system where an SU performs the spectrum sensing procedure before it transmits data. If the current operating channel is idle, the SU can transmit data. If the current operating channel is busy, the SU has to perform spectrum handoff procedures. The spectrum handoff procedures are initiated to help the SU return the channel to the PU and resume the SUs unfinished transmission at the same channel after the completion of the primary transmissions [1].

Unlike the previous methods that multiple SUs might contend for the same channel, the probability-based selection schemes can evenly distribute the traffic loads of SUs to multiple channels, thereby reducing the average queueing delay time. The queueing delay time of the SU's packets is affected by the multiple interruptions from the PUs because the PUs have higher priority. The interruptions of the SU's transmission consider as server breakdowns. The remaining transmission of the interrupted SU is placed into the head of the SU queue of the current operating channel. Furthermore, the interrupted SU can resume its unfinished transmission when the current channel becomes idle, instead of retransmitting the whole data.

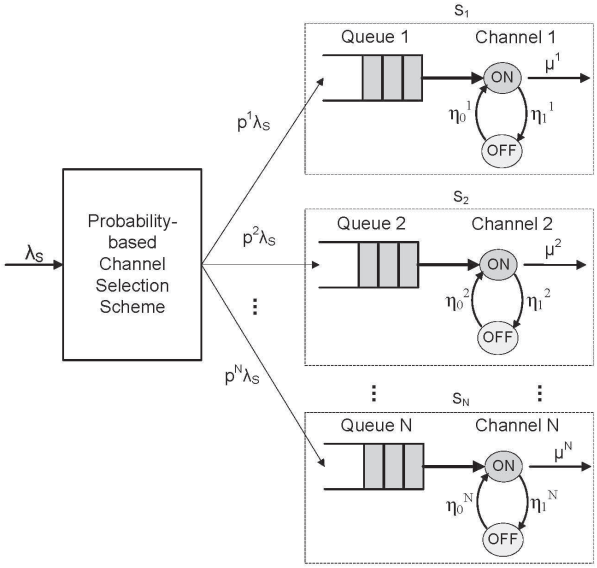

Fig. 1. The probability-based channel selection scheme where the channel usage behaviors are characterized by the server-breakdown queueing systems.

We consider a scenario composed of PUs and SUs sharing a given portion of the spectrum consisting of N channels. We assume all SUs can dynamically select their operating channels with suitable probability which can balance the traffic loads of SUs in multiple channels. The system model can be illustrated by Fig. 1 where each SU's packet can select one of N candidate channels for its operating channel. The distribution probability vector $\mathbf{p} = (p^1, p^2, ..., p^N)$ represents the set of probabilities for selecting all the candidate channels, in which p^i denotes the probability of selecting channel i. We assume that SU's packets arrive to the network according to a Poisson process as modeled in [1] with rate λ_S. Thus, the effective arrival rate of SU's packet at channel i is $\lambda^i = p^i \lambda_S$. When the packet of SUs arrives at the system, it can be directly connected to the selected channel based on the predetermined distribution probability vector $\mathbf{p}$. Then, we aim to find the optimal distribution probability vector $\mathbf{p}^*$ to minimize the expected queueing delay time of the SU's packet. Formally, we have Problem I as follows:

$$\mathbf{p}^* = \arg\min_{\forall p} E[\bar{T}] = \arg\min_{\forall p} \sum_{i=1}^{N} p^i E[T^i], \tag{1}$$

$$\begin{array}{c} 0 \le p^i \le 1, \forall i \\ \sum_{i=1}^{N} p^i = 1, \end{array} \tag{2}$$

where $E[T^i]$ is the expected queueing delay time of SU's packets at channel i and $E[\bar{T}]$ is the expected queueing delay time of the SU's packet over N channels. $E[T^i]$ consists of the waiting time and the extended service time at channel i. The expression of $E[T^i]$ will be discussed in the next section based on SU's and PU's statistical information which are assumed to be estimated by existing methods [4].

III. QUEUEING DELAY TIME ANALYSIS

In this section, we use a M/M/1 queueing model subject to the breakdown server to analyze the expected queueing delay time $E[\bar{T}]$ (waiting time + serving time). The breakdown server model is considered as a special case of the queueing with priorities model which was discussed by White and Christiek in [6], [3]. Based on the queueing delay analysis, the spectrum decision scheme for multiple PU channels system is analyzed in the next sections.

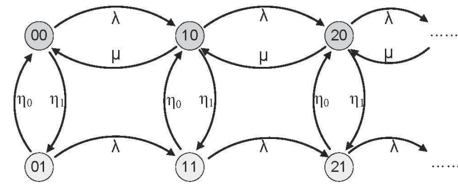

Fig. 2. Transition-rate diagram.

A. The breakdown queueing system in equilibrium

We introduce the server breakdown model and then derive a set of global balance equations of the system in equilibrium. Each PU channel oscillates between two feasible states ON/OFF which can be modeled by using Markov ON/OFF channel model [5]. The PU channel can be considered as a SU server who serves the SU's packet. Due to the higher priority of PU, when PU emerge, the SU server stops serving SU's packet, i.e. it has a breakdown. Then, if the SU server functions at state 0 (i.e. when the PU is absent on channel i) then it tends to jump randomly to the alternative state 1 (i.e, when the PU is present on channel i) with Poisson intensity η_1^i. And the reverse is also a Poisson process with intensity η_0^i. We assume that the SU's packets are served at channel i with the service times which are exponentially distributed with rate μ^i with the absence of PUs. For simplicity expression, we consider a single PU channel in general case in order to eliminate the index i. We represent the state of the system at time t by a pair $(K(t), L(t))$, where $K(t)$ denotes the number of packets in the queueing system and $L(t)$ denotes the working state of the SU server $(0 : \text{ON}, 1 : \text{OFF})$. The process $(K(t), L(t))$ is a continuous time Markov chain with non-zero transition rates. Fig. 2 shows the Markov process corresponding to the system evolution. The steady-state probability of the server working at the state ON is $P_0 = \eta_0/(\eta_0 + \eta_1)$, and at the state OFF is $P_1 = \eta_1/(\eta_0 + \eta_1)$. Then the average service rate of the SU's packet is $\bar{\mu} = \mu P_0$. The Markov process is a positive recurrent if the average arrival rate is less than the average service rate. Therefore, the steady-state condition of the system is $\lambda - \bar{\mu} < 0$. We have a set of global balance equations as

$$(\lambda + \eta_0) P_{01} = \eta_1 P_{00}, \tag{3}$$

$$(\lambda + \eta_0) P_{k1} = \lambda P_{k-11} + \eta_1 P_{k0},\ k \ge 1, \tag{4}$$

$$(\lambda + \eta_1) P_{00} = \mu P_{10} + \eta_0 P_{01}, \tag{5}$$

$$(\lambda + \eta_1 + \mu) P_{k0} = \lambda P_{k-10} + \eta_0 P_{k1} + \mu P_{k+10},\ k \ge 1. \tag{6}$$

Following the conservation of flow in Fig. 2, we obtain

$$\lambda P_{k0} + \lambda P_{k1} = \mu P_{k+10}, k \ge 0, \tag{7}$$

where P_{kl} denotes the limiting probability of the system in state (k, l).

B. The expected queueing delay time

In order to get the expected queueing delay time in the single PU channel system, we start to derive the average number of SU's packets in the system as follows. By summing (7) over k, we get

$$\lambda\sum_{k=0}^{\infty}P_{k0}+\lambda\sum_{k=0}^{\infty}P_{k1}=\mu(P_0-P_{00}), \tag{8}$$

or $\lambda P_0+\lambda P_1=\mu(P_0-P_{00})$ since $P_l=\sum_{k=0}^{\infty}P_{kl}, l=0,1$. But $P_0+P_1=1$, then we have the relationship as follows

$$P_{00}=P_0-\frac{\lambda}{\mu}. \tag{9}$$

We define the partial generating function of the constant l as

$$G_l(z)=\sum_{k=0}^{\infty}z^kP_{kl}, l=0,1, \tag{10}$$

with $0\le z\le 1$ to insure convergence. Multiplying each equation (3), (4), (5) and (6) by z^k and sum over k, we obtain

$$(\lambda+\eta_0)G_1(z)=\lambda zG_1(z)+\eta_1G_0(z), \tag{11}$$

$$\begin{aligned}(\lambda+\eta_1+\mu)G_0(z)&=\lambda zG_0(z)+\eta_0G_1(z)\\&\quad+\frac{\mu}{z}\left[G_0(z)-P_{00}\right]+\mu_0P_{00}.\end{aligned} \tag{12}$$

Combining (11) and (12), we have

$$G_0(z)=\frac{P_{00}\mu\left[\eta_0+\lambda(1-z)\right]}{g(z)}, \tag{13}$$

$$G_1(z)=\frac{P_{00}\eta_1\mu}{g(z)}, \tag{14}$$

where a polynomial of the second degree $g(z)$ is defined as:

$$g(z)=\lambda^2z^2-(\lambda^2+\mu\lambda+\eta_1\lambda+\eta_0\lambda)z+(\mu\lambda+\eta_0\mu). \tag{15}$$

If we define $E[L_l]=\sum_{k=0}^{\infty}kP_{kl}$ as the contribution of server state l to the mean number of SU's packets in the system. Using (10), then it becomes

$$E[L_l]=(d/dz)G_l(z)|_{z=1}. \tag{16}$$

From (9), (13), (14), (15) and (16), the (unconditional) expected number of packets in the system $E[L]$ is

$$\begin{aligned}E[L]&=E[L_0]+E[L_1]\\&=(d/dz)G_0(z)|_{z=1}+(d/dz)G_1(z)|_{z=1}\\&=\frac{\lambda(\eta_0+\eta_1+P_1\mu)}{(\eta_0+\eta_1)(\mu P_0-\lambda)}.\end{aligned} \tag{17}$$

By applying Little's formula to $E[L]$, we get the expected queueing delay time of an SU's packets on one channel as

$$E[T]=\frac{E[L]}{\lambda}=\frac{(\eta_0+\eta_1+P_1\mu)}{(\eta_0+\eta_1)(\mu P_0-\lambda)}=\frac{\gamma}{\bar{\mu}-\lambda}, \tag{18}$$

where $\gamma=\frac{(\eta_0+\eta_1+P_1\mu)}{(\eta_0+\eta_1)}$.

We have the expected queueing delay time of an SU's packet on channel i is $E[T^i]=\frac{\gamma^i}{\bar{\mu}^i-\lambda^i}$. Then, the expected queueing delay time on N channels $E[\bar{T}]$ in (1) has the following expression

$$E[\bar{T}]=\sum_{i=1}^{N}p^iE[T^i]=\sum_{i=1}^{N}p^i\frac{\gamma^i}{\bar{\mu}^i-\lambda^i}. \tag{19}$$

IV. A Lightweight Algorithm for Probability-based Spectrum Decision Scheme

The expected queueing delay time $E[\bar{T}]$ is a convex function, therefore, we will solve the Problem I in section II by using convex optimization theory. Based on the analysis of the solution, we then propose a lightweight algorithm to obtain the optimal distribution probability vector $\mathbf{p}^*$.

At first, we start solving Prolem I as follows. Multiplying (1) and (2) with λ_S, Problem I can be rewritten as Problem II:

$$\begin{aligned}&\min \textstyle\sum_{i=1}^{N}\frac{\lambda^i\gamma^i}{\bar{\mu}^i-\lambda^i}\\&s.t. \textstyle\sum_{i=1}^{N}\lambda^i=\lambda_S,\\&0\le\lambda^i<\bar{\mu}^i,\ i=1,2,...,N.\end{aligned} \tag{20}$$

We introduce the Lagrangian function of Prob. II

$$L(\lambda,\alpha,\beta)=\sum_{i=1}^{N}\frac{\lambda^i\gamma^i}{\bar{\mu}^i-\lambda^i}-\alpha\left(\sum_{i=1}^{N}\lambda^i-\lambda_S\right)-\beta^T\lambda, \tag{21}$$

where α, vector β are Lagrange multipliers and arrival rate vector $\lambda=(\lambda^1,\lambda^2,...,\lambda^N)$. Then we obtain the KKT conditions [7],[8] for the optimal solution $(\lambda^*,\alpha^*,\beta^*)$ as follows

$$\begin{aligned}&\lambda^*\ge 0,\\&\textstyle\sum_{i=1}^{N}\lambda^{*(i)}=\lambda_S,\\&\beta^*\ge 0,\lambda^{*(i)}\beta^{*(i)}=0,\ i=1,2,...,N,\end{aligned} \tag{22}$$

$$\frac{\partial L(\lambda^*,\alpha^*,\beta^*)}{\partial\lambda^i}=\frac{\gamma^i\bar{\mu}^i}{\left(\bar{\mu}^i-\lambda^{*(i)}\right)^2}-\alpha^*-\beta^{*(i)}=0. \tag{23}$$

By solving KKT condition, we obtain the optimal solution λ^* such as

$$\frac{\gamma^i\bar{\mu}^i}{\left(\bar{\mu}^i-\lambda^{*(i)}\right)^2}=\alpha^*,\quad \lambda^{*(i)}>0, \tag{24}$$

$$\frac{\gamma^i}{\bar{\mu}^i}\ge\alpha^*,\quad \lambda^{*(i)}=0. \tag{25}$$

As can be seen that $\lambda^{*(i)}>0$ if and only if $\alpha^*>\frac{\gamma^i}{\bar{\mu}^i}$. It implies that in the optimal solution, a threshold index i^* exists such that channel j is active ($p^j>0$) if and only if $j\le i^*$ and i^* satisfies

$$\frac{\gamma^1}{\bar{\mu}^1}\le\frac{\gamma^2}{\bar{\mu}^2}....\le\frac{\gamma^{i^*}}{\bar{\mu}^{i^*}}<\alpha^*\le\frac{\gamma^{i^*+1}}{\bar{\mu}^{i^*+1}}\le...\frac{\gamma^N}{\bar{\mu}^N}. \tag{26}$$

Based on (24), we obtain

$$\lambda^{*(i)}=\bar{\mu}^i-\frac{\sqrt{\gamma^i\bar{\mu}^i}}{\sqrt{\alpha^*}}, i=1,2,...,i^*. \tag{27}$$

Using (27) and $\sum_{i=1}^{i^*}\lambda^i=\lambda_S$, we get

$$\sqrt{\alpha^*}=\frac{\sum_{i=1}^{i^*}\sqrt{\gamma^i\bar{\mu}^i}}{\sum_{i=1}^{i^*}\bar{\mu}^i-\lambda_S}. \tag{28}$$

Finally, based on (26), the optimal value of i^* is obtained

$$i^*=\min\left\{i:\frac{\gamma^{i+1}}{\bar{\mu}^{i+1}}\ge\frac{\left(\sum_{j=1}^{i}\sqrt{\gamma^j\bar{\mu}^j}\right)^2}{\left(\sum_{j=1}^{i}\bar{\mu}^i-\lambda_S\right)^2}=\alpha^*\right\}. \tag{29}$$

By using (26), (27), (28) and (29), the solution of Problem I can be obtained and an algorithm to define the probability vector $\mathbf{p}^*$ is proposed. The SU firstly estimates the initial parameters λ_S, $\gamma^1, \gamma^2, ..., \gamma^N; \bar{\mu}^1, \bar{\mu}^2, ..., \bar{\mu}^N$ by existing methods [4]. Then, the algorithm executed in the SU to compute the probabilities $p^i = \lambda^i/\lambda_S$ is provided in detail as follows

1) Initializing $i^* = 1$
2) Sorting $\frac{\gamma^1}{\bar{\mu}^1} \leq \frac{\gamma^2}{\bar{\mu}^2} .... \leq \frac{\gamma^N}{\bar{\mu}^N}$;
3) Computing α^* by (28);
4) if α^* satisfies (26) then return i^* and α^*;
   else $i^* = i^* + 1$ and go back to step 3;
5) Calculating λ^i by (27) and $p^i = \lambda^i/\lambda_S$;

The sorting complexity in step 2 is $O(logN)$. The loop for step 3 and 4 has maximum $i^* < N$ iterations. Therefore,the computation complexity of the above algorithm is $O(N)$. Additionally, the proposed algorithm greatly reduces computational overheads and memory space.

V. Numerical Analysis

We consider a six-channel system with following traffic parameters: for PU's traffic $\eta_1^1 = \eta_1^2 = \eta_1^3 = \eta_1^4 = \eta_1^5 = \eta_1^6 = 0.05$; and $\eta_0^1 = 0.1466; \eta_0^2 = 0.1416; \eta_0^3 = 0.1366; \eta_0^4 = 0.1316; \eta_0^5 = 0.1266; \eta_0^6 = 0.1216$. These parameters show that the percentage of using channel by PUs are around 30%. For SU traffic, we assume SU's service rate of six channels are the same and equal $\mu = 0.2$. This means that the average service time of SU's packet is 1/4 the average service time of PU's packet.

From Fig. 3, one can see that when the average service time of SU's packets $1/\mu$ or SU arrival rate λ_S increases, the expected queueing delay time also rise. Fig. 3 also shows that utilizing free six PU channels can keep the queueing delay time of SU's packet equal or smaller than using the high fee dedicated channel with the same service rate. For example, if we have one high fee dedicated channel which is exclusive for SU's packets with $1/\mu = 5$. The average system time of SU's packet in a normal M/M/1 queue in the dedicated channel in case of $\lambda_S = 0.1$ is $1/(\mu - \lambda) = 1/(0.2 - 0.1) = 10$ that is equal to using six PU channels.

Fig. 4 shows the optimal distribution probability vector under various arrival rate of SU's packet. The channel 1 has the highest probability because channel 1 has the lightest traffic loads. Furthermore, as λ_S increases, SU's packets tend to select other channels to transmit data in order to balance the traffic loads in each channel and to reduce the queueing delay time. For example when $\lambda_S = 0.01$, SUs only use three channels 1,2,3 for operating channels. However, when $\lambda_S = 0.03$, four channels are selected and the optimal probability is (0.31,0.27,0.23,0.19,0,0). Inevitably, channel 1 is still chosen to be the operating channel with the largest probability.

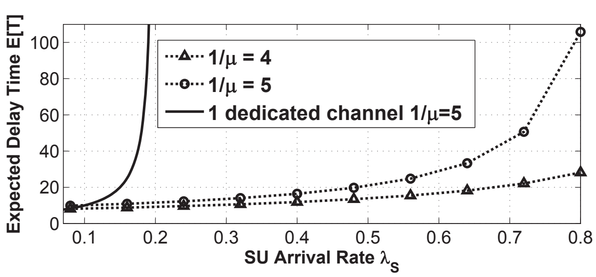

Fig. 3. Expected queueing delay time.

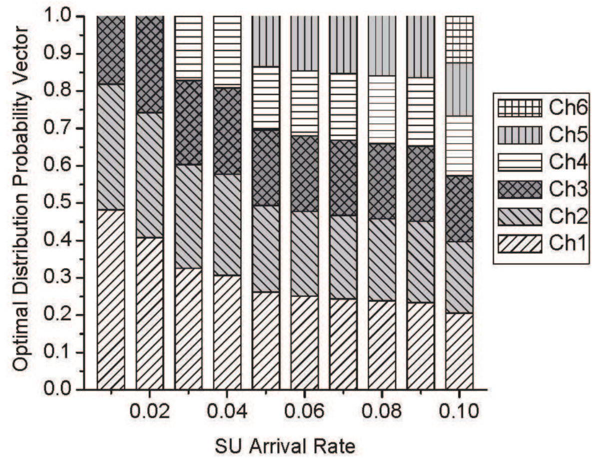

Fig. 4. Probability of Each Channel with $\mu = 0.2$.

VI. Conclusion

In this paper, an analytical framework is proposed to design the system parameters for the probability-based spectrum decision scheme. The proposed model integrated with the server-breakdown queueing systems can evaluate the effects of multiple interruptions, channel capacity on the queueing delay time of the SU's packets in the simple expression. Based on this analytical model, the optimal distribution probability vector can be obtained by a lightweight algorithm.

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