Social Optimization Strategy in Unobserved Queueing Systems in Cognitive Radio Networks

Cuong T. Do, *Member, IEEE*, Nguyen H. Tran, *Member, IEEE*, Mui Van Nguyen, Choong Seon Hong, *Senior Member, IEEE*, and Sungwon Lee, *Member, IEEE*

***Abstract*—We study a noncooperative game problem for queueing control in the Cognitive Radio (CR) system where selfish Secondary User's (SU) data packets (a.k.a. "customers" in this work) are served by a CR base station (a.k.a. "server"). The scenario is modeled as an M/M/1 queueing game with server breakdowns where each customer wants to optimize their benefit in a selfish distributed manner. We first show that the game has an inefficient unique Nash Equilibrium (NE). In order to improve the outcome efficiency, we propose an appropriate admission fee that can be easily implemented at the server. We then show that the social welfare at the equilibrium point can be coincided the social welfare of the socially optimal strategy.**

***Index Terms*—Cognitive radio, game theory, M/M/1 queue.**

I. Introduction

RECENT works apply the queueing theory with server interruption model [1] and server-breakdown queueing model [2] to study about controlling the queues in CR networks. However, in [1], in order to oblige the customers to adopt the socially optimal queue length threshold n_{th}^{S}, Li et.al. proposed a pricing scheme. The current queue length can be received by a broadcast message from the CR base station. However, the authors did not consider the overhead broadcast message of in the paper. In addition, the multiple PU bands was not mentioned in [1]. On the other hand, in [2], Jagannathan et.al. did not mention the social optimization strategy. This paper only considers a monopolist system which want to maximize its own profit. Therefore, it is hard to know the efficient of the individual equilibrium strategy (p, q) to the whole system or the society. Then, we realized that it is hard to extend the model of both papers [1] and [2] in other scenarios such as the heterogeneous service values model, multiple PU bands, multiple priority classes due to the complex expression of the queue length distribution in [1] or the conditional expected delay time of the SU packets in [2].

The motivation of this paper is to overcome the drawback in the observable queue model used in [1]. SU's decisions in [1] are based on the queue length information, which increases the overhead due to the necessity of a signaling scheme. Furthermore, SU cannot get exact queue length information immediately. In contrast to [1], we use the unobservable queue model, where SUs make a decision just based on the average delay measurement that takes into account the presence of PU. In this paper, the queue of the CR base station can be modeled by using the server-breakdown queueing model. To understand the breakdown, we can consider the CR base station, which aggregates several connections, calls or packets of SUs to jobs that can be served over the PU band, as a server. When PUs emerge and occupy the PU band, the server has a breakdown, i.e. the service for SU customers is stopped. We consider an arrival process of SU customers, e.g., calls, packets, sessions or connections, arriving at the CR base station which is considered as the server. Each customer makes a decision whether to join or to balk the queue, i.e. discarding the packet. When a SU customer make a decision to join the queue, the waiting time in the queue will incur a cost. If the customer finishes its job then it will get a reward.

To conclude, the main contributions of this paper are: We consider the interaction among SU customers as a noncooperative game. Based on the queueing analysis for fixed SU customers' policies, we then analyze both individually optimal strategy and optimal social welfare strategy of SU customers. A pricing agent, e.g, the CR base station, announces an admission fee for the queuing system such that the individually optimal decision of customers coincides with the socially optimal strategy that optimizes the total welfare of the society. Furthermore, we can extend to other scenarios such as heterogenous service value, multiple PU bands etc.

II. System Model

We start by defining the model for a system with a single PU band. The PU band can be considered as a server who oscillates between two feasible states ON/OFF which can be modeled by using Markov ON/OFF channel model [4]. The base station in the CR system, using PU band, is considered as a server who serves the SUs. Due to the higher priority of PU, when PU emerge, the CR base station stop serving SU customer, i.e. it has a breakdown. The CR base station also operates as a server who oscillates between two modes ON/OFF denoted by 0 and 1 respectively. If the CR base station functions at state 0 (i.e. when the PU is absent) then it tends to jump randomly to the alternative state 1 (i.e, when the PU is present) with Poisson intensity η_1. And the reverse is also a Poisson process with intensity η_0. We assume that the SU customers are served with the service times which are exponentially distributed with rate μ with the absence of PUs. Poisson process is used for customer arrivals so that the

Manuscript received April 16, 2012. The associate editor coordinating the review of this letter and approving it for publication was D. Popescu.

This research was supported by the MKE, Korea, under the ITRC support program supervised by the NIPA (NIPA-2012-(H0301-12-2001)).

C. T. Do, N. H. Tran, M. Van Nguyen, C. S. Hong (corresponding author), and S. Lee are with the Dept. of Computer Engineering, Kyung Hee University, Korea (e-mail: {dtcuong, nguyenth, nvmui, cshong, drsungwon}@khu.ac.kr).

Digital Object Identifier 10.1109/LCOMM.2012.111412.120830

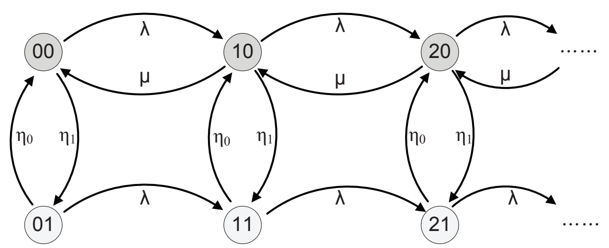

Fig. 1. Transition-rate diagram.

inter-arrival times are exponentially distributed. In general, the customer arrival rate is denoted by λ.

When a SU customer wants to be served at the CR base station, the SU (i.e., physical device) decides about whether to let the SU customer to join the CR base station's queue or leave it. A first-in-first-out (FIFO) rule can be implemented in the CR base station by letting each SU customer register its ID to the CR base station when it arrives [1]. Similar to [1], the net benefit of a customer that stays in the system for T time unit and completes the service successfully is: $R - CT$.Here, R is the reward or a service value for a completed service. C represents the penalty for the delay in the system.

III. Expected Sojourn Time Analysis

In this study, we use a M/M/1 queueing model subject to a breakdown server to analyze the expected sojourn time of SU customers. Based on that, the strategies of customers are analyzed in the next sections.

We start to derive the average number of customers in the system. We represent the state of the system at time t by a pair $(K(t), I(t))$, where $K(t)$ denotes the number of customers in the system and $I(t)$ denotes the working state of the CR server (0 : ON, 1 : OFF). The process $(K(t), I(t))$ is a continuous time Markov chain with non-zero transition rates. Fig. 1 shows the Markov process corresponding to the system evolution. The steady-state probability of the server working at the state ON is $P_0 = \eta_0/(\eta_0 + \eta_1)$, and at the state OFF is $P_1 = \eta_1/(\eta_0+\eta_1)$. Then the average service rate $\overline{\mu}$ is $\overline{\mu} = \mu P_0$. The Markov process is a positive recurrent if the average arrival rate is less than the average service rate. Then, the steady-state condition of the system is $\lambda - \overline{\mu} < 0$.

Under the above condition, the set of global balance equations are

$$(\lambda + \eta_0)P_{01} = \eta_1 P_{00}, \tag{1}$$

$$(\lambda + \eta_0)P_{k\,1} = \lambda P_{k-1\,1} + \eta_1 P_{k\,0}, \, k \geq 1, \tag{2}$$

$$(\lambda + \eta_1)P_{00} = \mu P_{10} + \eta_0 P_{01}, \tag{3}$$

$$(\lambda + \eta_1 + \mu)P_{k\,0} = \lambda P_{k-1\,0} + \eta_0 P_{k\,1} + \mu P_{k+1\,0}, \, k \geq 1. \tag{4}$$

Employing vertical cuts in Fig. 1, we obtain

$$\lambda P_{k\,0} + \lambda P_{k\,1} = \mu P_{k+1\,0}, \, k \geq 0. \tag{5}$$

By summing (5) over k, we get

$$\lambda P_0 + \lambda P_1 = \mu(P_0 - P_{00}), \tag{6}$$

or

$$P_{00} = P_0 - \frac{\lambda}{\mu}, \tag{7}$$

where $P_i = \sum_{k=0}^{\infty} P_{k\,i}, i = 0, 1$. We define the partial generating function of the system as

$$G_i(z) = \sum_{k=0}^{\infty} z^k P_{k\,i}\,, i = 0, 1. \tag{8}$$

Multiplying each equation (1), (2), (3), (4) by z^k and sum over k, we obtain

$$(\lambda + \eta_0)G_1(z) = \lambda z G_1(z) + \eta_1 G_0(z), \tag{9}$$

$$\begin{aligned}(\lambda + \eta_1 + \mu)G_0(z) &= \lambda z G_0(z) + \eta_0 G_1(z)\\ &+\tfrac{\mu}{z}\left[G_0(z) - P_{00}\right] + \mu_0 P_{00}.\end{aligned} \tag{10}$$

In order to derive the explicit expression of $G_0(z)$ and $G_1(z)$, a polynomial of the second degree $g(z)$ is defined as:

$$g(z) = \lambda^2 z^2 - (\lambda^2 + \mu\lambda + \eta_1\lambda + \eta_0\lambda)z + (\mu\lambda + \eta_0\mu). \tag{11}$$

Combining (9), (10) and (11), we have

$$G_0(z) = \frac{P_{00}\mu\left[\eta_0 + \lambda(1 - z)\right]}{g(z)}, \tag{12}$$

$$G_1(z) = \frac{P_{00}\eta_1\mu}{g(z)}. \tag{13}$$

If we define $E[L_i] = \sum_{k=0}^{\infty} kP_{k\,i}$ as the contribution of state i to the mean number of customers in the system. Using (13), then it becomes

$$E[L_i] = (d/dz)G_i(z)|_{z=1}. \tag{14}$$

From (11), (12), (13) and (14), the (unconditional) expected number of customers in the system $E[L]$ is derived as

$$\begin{aligned}E[L] &= E[L_0] + E[L_1]\\ &= (d/dz)G_0(z)|_{z=1} + (d/dz)G_1(z)|_{z=1}\\ &= \frac{\lambda + P_1\frac{\lambda\mu}{\eta_0+\eta_1}}{\mu P_0 - \lambda}.\end{aligned} \tag{15}$$

This result was obtained as the special case of a model discussed by White et al [5]. By applying Little's formula to $E[L]$, we get the expected sojourn time of an arbitrary customer in the system as

$$\overline{T} = \frac{E[L]}{\lambda} = \frac{1 + P_1\frac{\mu}{\eta_0+\eta_1}}{\mu P_0 - \lambda} = \frac{\gamma}{\overline{\mu} - \lambda}, \tag{16}$$

where $\gamma = 1 + P_1\frac{\mu}{\eta_0+\eta_1}$.

IV. Unobservable Queuing Control in Single PU Band

In this section, we first analyze the customers' individual behavior in equilibrium. Then we turn out attention to social optimization. As Edelson and Hildebrand [3] shown that in unobserved queues the individual's decision deviates from the socially preferred one. Therefore, a correct admission fee must be computed and administratively imposed. Assuming that customers are risk neutral, that is, they maximize the expected value of their net benefit. Utility functions of individual customers are identical and additive, from the public (social) point of view. A decision to join is irrevocable, and reneging is not allowed. At the time a customer's need for service arises, the customer irrevocably either joins the queue or does not join. It is impossible for him to observe the queue length before making this decision.

We consider customers' strategies described by a probability q which is the probability an SU customer decides to join the queue (thus, with probability $1-q$ it decides to leave the queue). Because customers are allowed to take their own decisions, then the system can be modeled as a noncooperative and symmetric game among the customers.

A. Individual equilibrium strategy

Thereupon, we discuss the individually optimal strategy of each SU. We start by analyzing the customers' behavior in equilibrium when the potential arrival rate is λ_0 (i.e, the arrival rate of customers who intends to arrive at the server).

There are two pure strategies available for a customer: to join the queue or not to join. A pure or mixed strategy can be described by a fraction q, $1 \geq q \geq 0$, which is the probability of joining. We denote the equilibrium (effective) arrival rate by λ_e (i.e, the arrival rate of customers who have already decided to join the queue) such that $\lambda_e = q_e\lambda_0 < \mu P_0$. We denote the expected waiting time by $\bar{T}(\lambda) = \frac{\gamma}{\overline{\mu}-\lambda}$ when the (effective) arrival rate is $\lambda < \mu P_0$. This function is continuous and monotonely increasing. The expected net benefit for a customer who joins the queue and completes its service is derived as: $R - C\bar{T}(\lambda)$.

To avoid a trivial solution, we make the following assumption: $C\bar{T}(0) = \gamma C/\overline{\mu} < R$. Then, we consider two cases:

1) $C\bar{T}(\lambda_0) \leq R$. All joining customers can get a non-negative benefit even if all potential customers join. Therefore, the strategy of joining with probability $q_e = 1$ is an equilibrium strategy and no other equilibrium is possible. Moreover, in this case, joining is a dominant strategy.
2) $C\bar{T}(0) < R < C\bar{T}(\lambda_0)$. If $q_e = 1$ then a customer who joins, suffers from a negative benefit. Hence, this cannot be an equilibrium strategy. At the same way, if $q_e = 0$, a customer who joins get a positive benefit, more than by leaving. Therefore, this too cannot be an equilibrium. Therefore, there exists a unique equilibrium strategy q_e satisfies $q_e = \frac{\lambda_e}{\lambda_0}$ and λ_e solves $C\bar{T}(\lambda_e) = R$.

The individuals optimizing strategy in this model is straightforward as follows. Suppose that the probability of joining is q. Because the customers's behavior are modeled as a symmetric game between customers then we have statement as follows: if $q < q_e$ then the unique best response is 1, if $q = q_e$ then any strategy between 0 and 1 is a best response, and if $q > q_e$ then the unique best response is 0. Since, a strategy is a symmetric equilibrium if it is a best response against itself, we conclude that q_e is a equilibrium point. We summarize the individual equilibrium joining strategy for two cases as follows

1) $C\bar{T}(\lambda_0) \leq R$: $\lambda_e = \lambda_0$ and $q_e = 1$.
2) $C\bar{T}(0) < R < C\bar{T}(\lambda_0)$: $\lambda_e = \overline{\mu} - \frac{\gamma C}{R}$ and $q_e = \frac{\overline{\mu} - \frac{\gamma C}{R}}{\lambda_0}$.

B. Social optimization

Now, we turn our attention to social optimization. A queueing system may also be considered from a social point of view. Then the social objective function is defined as

$$U_S = \lambda\left[R - C\bar{T}(\lambda)\right], \tag{17}$$

where λ is the (effective) arrival rate, defined as $\lambda = q\lambda_0$ when the joining probability is q.

Let the socially optimal joining probability be q^*, and let the socially optimal joining rate be λ^*, where $\lambda^* = q^*\lambda_0$. Then λ^* can be obtained by solving

$$\lambda^* = \arg\max_{0<\lambda<\lambda_0}\left\{\lambda\left[R - C\bar{T}(\lambda)\right]\right\}. \tag{18}$$

Since, $\bar{T}(\lambda)$ is strictly convex, the function to be maximized is strictly concave and has a unique maximum. By substituting $\bar{T}(\lambda) = \frac{\gamma}{\overline{\mu}-\lambda}$, the optimal solution can be obtained as follows

$$\lambda^* = \max\left\{\min\left\{\lambda_0, \overline{\mu} - \sqrt{\frac{\gamma\overline{\mu}C}{R}}\right\}, 0\right\}. \tag{19}$$

Then, we summarizes the socially optimal strategy as

1) $\lambda_0 \leq \overline{\mu} - \sqrt{\frac{\gamma\overline{\mu}C}{R}}$: $\lambda^* = \lambda_0$ and $q^* = 1$.
2) $\lambda_0 \geq \overline{\mu} - \sqrt{\frac{\gamma\overline{\mu}C}{R}}$: $\lambda^* = \overline{\mu} - \sqrt{\frac{\gamma\overline{\mu}C}{R}}$ and $q^* = \frac{\overline{\mu} - \sqrt{\frac{\gamma\overline{\mu}C}{R}}}{\lambda_0}$.

The assumption $\overline{\mu}R > \gamma C$ infers that $q^* < q_e$. Similar to the observable queue model [1], individual optimization leads to a queue that is longer than that desired socially. This gap can be corrected by imposing an appropriate admission fee, as discussed in the next section.

C. Pricing

To oblige the customers to adopt the socially optimal strategy, one approach is to apply a pricing mechanism to reduce the individual joining probability q_e. Then, the server would act as a pricing agent and imposes an admission fee p, which is a constant given the potential arrival rate, the breakdown pattern, the reward and the cost.

When an admission fee p is imposed, the net benefit for a customer who joins the queue and completes its service is: $R - p - C\overline{T}(\lambda)$. Given p, we denote the equilibrium probability of joining by $q_e(p)$, and the corresponding equilibrium (effective) arrival rate by $\lambda_e(p)$ such that $\lambda_e(p) = q_e(p)\lambda_0 < \mu P_0$. If we assume that $p + C\bar{T}(0) < R < p + C\bar{T}(\lambda_0)$, then we obtain the equilibrium probability of joining by $q_e(p) = \frac{\lambda_e(p)}{\lambda_0}$, where $\lambda_e(p)$ solves the following equation

$$p + C\bar{T}(\lambda_e(p)) = R. \tag{20}$$

Solving (20), we get

$$q_e(p) = \frac{\overline{\mu} - \frac{\gamma C}{R-p}}{\lambda_0}. \tag{21}$$

Note that the admission fee p is not included in the objective function, since it implies a transfer of income from one social group (customers) to another (the server who imposes an admission fee). Therefore, the social objective function is

$$\begin{aligned} U_S &= \lambda\left[R - C\bar{T}(\lambda) - p\right] + \lambda p \\ &= \lambda\left[R - C\bar{T}(\lambda)\right], \end{aligned} \tag{22}$$

By comparing (22) with (17), we conclude that the social objective function when the server impose an admission fee p is the same as in subsection B.

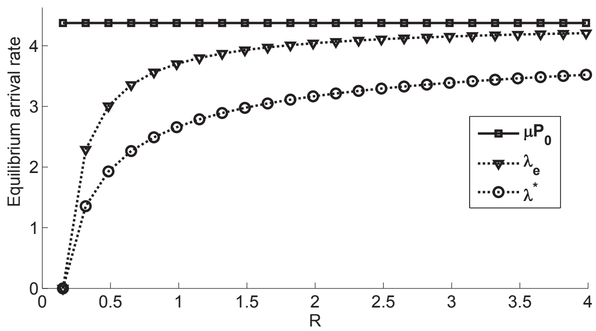

Fig. 2. Individual and social equilibrium arrival rate vs. service value R.

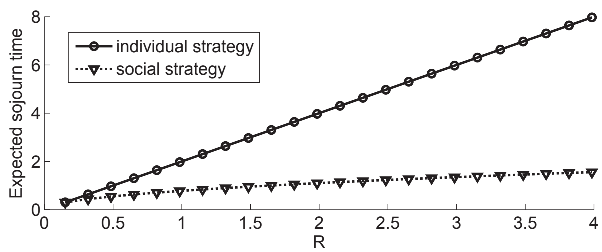

Fig. 3. The expected sojourn time vs. service value R.

The admission fee p^*, which is to correct the gap between the individual optimal strategy and the social optimal strategy, makes $q_e(p*)$ identically equal to q^*.

In case of $\lambda^* < \lambda_0$, from subsection B and (21), solving $q^* = q_e(p^*)$, we get

$$p^* = R - \sqrt{\gamma RC/\overline{\mu}} = R - \frac{\gamma C}{\overline{\mu} - \lambda^*}. \tag{23}$$

It can be seen that $p^* + C\bar{T}(0) < R$ with the assumption $\gamma C < \overline{\mu} R$.

In case of $\lambda^* = \lambda_0$, the server chooses the admission fee, that is, $p^* = R - \frac{\gamma C}{\overline{\mu} - \lambda_0}$. A social planner would choose this fee, or any smaller fee, since any such choice induces the same optimal arrival rate, λ_0.

In conclusion, the CR base station imposes the admission fee p^* to correct the gap between individually optimal strategy and socially optimal strategy. The base station can notice each SUs this admission fee in the first time it connects to the CR base station.

D. Numerical Analysis

The numerical parameters are $\mu = 7$, $\eta_0 = 5$, $\eta_1 = 3$, $C = 0.5$, $\lambda_0 = 5$. It can be seen that the gap between the individual equilibrium arrival rate and social arrival rate in Fig. 2. By imposing appropriate admission fee, we can regulate the arrival rate of customers and avoid individual behavior which make the expected sojourn time of customer very large as in Fig. 3. In contrast, the social behavior can keep the expected sojourn time stable and acceptable for customers.

When we reduce the potential arrival rate to $\lambda_0 = 3.5$, Fig.4 shows that when the service value R increases, the joining probability also increases but the individual joining probability rises up more quickly than the social joining probability.

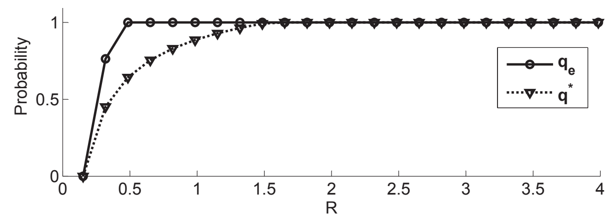

Fig. 4. The equilibrium joining probability vs. service value R.

V. Extension Scenarios

In case of the service value R is not a constant but a concave function of arrival rate $V(\lambda)$ [6]. Since $\bar{T}(\lambda)$ is strictly convex, using similar approach in [6], we obtain the socially optimal arrival rate $\lambda^* = \arg\max_{0<\lambda} \{V(\lambda) - \lambda C\bar{T}(\lambda)\}$ and the optimal admission fee $p^* = \lambda^* C\bar{T}'(\lambda^*)$.

An other scenario is N PU bands system, one possible approach is to use N queues in the CR base station for N different types of SU customers. The system now is formally modeled as having N servers, each server can be analyzed independently by the similar approach in the previous sections. Then, the admission fee p_i^* is calculated to force the SU customers to adopt the socially optimal strategy for each type *i*th of SU customer. In case the CR base station is a monopolistic server who apply the same admission fee to all N type *i*th of SU customer to maximize his profit, using numerical analysis to solve for the optimal admission fee p^m, we can maximize the profit of the server.

VI. Conclusion

Generally, this paper provides a theoretical understanding of decision processes of SU customers in unobserved queues model in CR system. There are still many problems and extensions that can be examined.

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