Low-PAPR Hybrid Filter for SC-FDMA

Shaharyar Kamal, Cesar A. Azurdia Meza, *Member, IEEE,* Nguyen H. Tran, *Member, IEEE,* and Kyesan Lee

***Abstract*—We propose a hybrid filter to reduce the peak-to-average power ratio (PAPR) on the transmitter side in a long-term evolution (LTE) uplink scheme. The design of the proposed filter is based on two key components: a finite impulse response (FIR) filter and a Nyquist-I pulse. We consider an envelope-constrained (EC) filter design to optimize the impulse response of the proposed filter in terms of PAPR reduction. Moreover, we propose a new family of Nyquist-I pulses, the exponential linear pulse (ELP), which has a new design parameter that helps reduce PAPR for a given roll-off factor and transmission scheme. Theoretical and numerical results show that the proposed filter outperforms existing filters in terms of PAPR and symbol-error-rate (SER), and it has a less computationally complex impulse response expression than existing filters for the interleaved subcarrier mode of single-carrier frequency-division multiple access (SC-FDMA).**

***Index Terms*—Intersymbol interference (ISI), Nyquist-I criterion, peak-to-average power ratio (PAPR), power amplifier, pulse shaping filters, SC-FDMA.**

I. Introduction

SINGLE-carrier frequency-division multiple access (SC-FDMA) in long-term evolution (LTE) is a popular uplink technology because of its low peak-to-average power ratio (PAPR) [1]. High-order modulations are suggested to achieve high spectral efficiencies in 5G because of small cells where the signal-to-noise ratio (SNR) can be very high [2]. However, the high-order modulations cause high PAPR in the SC-FDMA signals, especially for interleaved SC-FDMA (SC-IFDMA) with a small roll-off factor [3]. The high PAPR results in low efficiency in the high-power amplifiers and fails to satisfy the 5G requirements regarding energy efficiency [4]. Additionally, a low PAPR is desired in battery-oriented terminals because of power limitations [5].

Pulse shaping is required in single-carrier systems in order to limit the transmitted signal bandwidth [1]. A pulse with a highly reduced tail size is extremely desirable, because it has a large effect on PAPR reduction [6]. Such a reduction is achievable by optimizing the impulse response on the transmitter side for the SC-FDMA scheme [1], [6]. On the other hand, recent studies have concluded that, although PAPR can be reduced sufficiently on the transmitter side, it increases the complexity of SC-FDMA [7]. Unfortunately, no reliable method can directly optimize impulse response expression in the time domain. Therefore, we propose a hybrid filter for the transmitter side of SC-FDMA in which first we optimize a finite impulse response (FIR) filter and then use a desirable Nyquist-I pulse to form an impulse response with greatly reduced tail size. We use an envelope-constrained (EC) optimal filter [8] to formulate our hybrid filter design problem.

In this paper, we propose a hybrid filter with two principle components: an FIR filter and a Nyquist-I pulse. A rectangular pulse is used as an FIR filter for its simplicity. Moreover, we propose a new family of Nyquist-I pulses, the exponential linear pulse (ELP), which has an additional design parameter that provides an extra degree of freedom to minimize PAPR for a certain roll-off factor, α. The ELP has simple impulse response expression, which reduces the computational complexity of the proposed filter compared to existing pulses [1], [6]. We compare the PAPR and symbol-error-rate (SER) performance of various Nyquist-I pulses (the proposed ELP, parametric linear combination pulse (PLCP) [1], piece-wise linear Nyquist filter (PWL) [6], and raised-cosine (RC) pulse) in combination with our FIR filter to form a hybrid filter. In this work, we focused on SC-IFDMA; however, the proposed filter can be applied in any orthogonal frequency division multiplexing scheme.

II. SC-FDMA transmitter

A binary input sequence is transformed into a multilevel sequence of complex numbers by a baseband modulator using different modulation techniques. Then, the modulated symbols are grouped into blocks of N symbols. A frequency domain version of the input signal is generated by calculating the N-point discrete Fourier transform (DFT). During subcarrier mapping, each DFT output is mapped to each M orthogonal subcarrier ($M > N$). We implemented interleaving in the distributed subcarrier mode (DSM), SC-IFDMA, because it showed better PAPR performance than the localized mode [3]. In the DSM scheme, the DFT output is spread over the entire bandwidth, and unused subcarriers are padded with zeros. Additionally, the occupied subcarriers are equally spaced over the entire bandwidth. Here, the FIR filter is multiplied with the output of the subcarrier mapping block, as shown in Fig. 1. The frequency domain signal then becomes a complex signal in the time domain via the M-point inverse DFT (IDFT). A cyclic prefix is added to the IDFT output to prevent intersymbol interference (ISI). Finally, a linear filtering operation reduces the out-of-band (OOB) emissions and limits the transmitted signal bandwidth [1], [6].

Our objective is to minimize PAPR by decreasing the magnitude of the output side lobes of the impulse response

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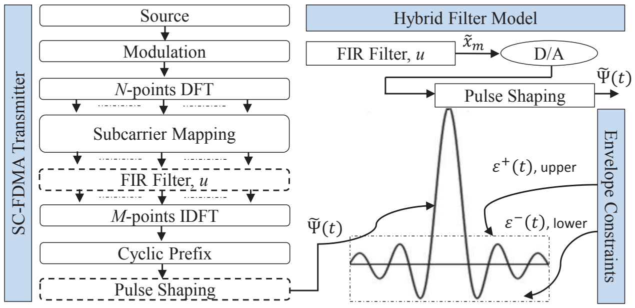

Fig. 1. Hybrid filter for SC-IFDMA Transmitter

$\Psi(t)$. Therefore, we optimize $\Psi(t)$ by applying the hybrid filter model on the transmitter side in SC-IFDMA, as shown in Fig. 1. The complex passband transmit signal for SC-FDMA is represented as [1]

$$\Psi(t) = e^{jw_c t} \sum_{m=0}^{M-1} x_m p(t - mT_s), \tag{1}$$

where w_c is the carrier frequency of the system, $p(t)$ is a Nyquist-I pulse used to limit the transmitted symbol bandwidth and reduce OOB radiation, T_s is the sampling period, and x_m represents discrete-time symbols after IDFT. The total PAPR of the transmit signal is given as

$$PAPR = \frac{\max_{0\le t\le MT_s} |\Psi(t)|^2}{\frac{1}{MT_s}\int_0^{MT_s} |\Psi(t)|^2}, \tag{2}$$

where a decrease in PAPR value is equivalent to a reduction in the maximum peak of an output signal $\Psi(t)$, and the average power of the output signal is a constant. Thus, the objective function is formulated as

$$\min \max_{0\le t\le MT_s} |\Psi(t)|^2 . \tag{3}$$

III. Problem Formulation

In this section, we analyze the EC optimal filter to create a hybrid filter design problem for data transmission in SC-IFDMA. The key idea is to develop a robust impulse response using digital techniques. We use an FIR filter to conceptualize the hybrid filter as a finite dimensional optimization problem, and we use the Nyquist-I pulse as a linear interpolator to smooth the impulse response.

A. FIR Filter

The discrete data are multiplied by a discrete-time impulse response after subcarrier mapping, as shown in Fig. 1. The discrete-time impulse response $u = [u_0, \ldots, u_{n-1}]^T \in R^n$ is the vector of the coefficients of the FIR filter. The discrete-time signal is expressed as

$$\tilde{x}_m = \left\{ \sum_{j=0}^{n-1} x[(m-j)T_s]u_j \right\}_{m=-\infty}^{\infty}, \tag{4}$$

where n is the filter order, x is the system input (i.e. $\{x(kT_s)\}_{k=-\infty}^{\infty}$ is square-summable), and the output $\tilde{x}_m$ is bounded by discrete-time filtering results.

B. Nyquist-I Pulse

A convolution occurs between the discrete-time modulated signal and the impulse response of a Nyquist-I pulse. The response $\tilde{\Psi}$ is given as

$$\tilde{\Psi}(t) = \sum_{m=-\infty}^{\infty} \tilde{x}_m p(t - mT_s),\ t \in [0, \infty). \tag{5}$$

Because the input signal is $[0,\infty)$, the index m can be a number from zero to infinity. In addition, the Nyquist-I pulse requires $t \in [0, T_s)$, e.g., bounded input-bounded output stability. Thus, $\tilde{\Psi}$ is bounded and continuous on $[0,\infty)$. When we sample the input with a finite sequence, as $\{s(kT_s)\}_{k=0}^{q-1}$, the output $\tilde{\Psi}(t)$ from (5) with finite sequences becomes

$$\tilde{\Psi}(t) = \sum_{m=0}^{\tilde{M}-1} \sum_{j=0}^{n-1} s[(m-j)T_s]u_j p(t - mT_s), \tag{6}$$

where $\tilde{M} = n + q - 1$, $\tilde{M}$ is the orthogonal subcarrier, and q is the number of samples. The above sum has a matrix form S with a stable Nyquist-I pulse. The matrix S is given as

$$S = \begin{bmatrix} s(0) & 0 & \ldots & 0 \\ \vdots & s(0) & & \vdots \\ s((q-1)T_s) & \vdots & \ddots & 0 \\ 0 & s((q-1)T_s) & & s(0) \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & 0 & s((q-1)T_s) \end{bmatrix}_{N\times n} \tag{7}$$

Here, we ignore the complex terms, i.e., $e^{jw_c t}$ from (1), to simplify the notation, similar to [9], but we applied $e^{jw_c t}$, in our analysis and numerical results. Thus, (1) and (6) are similar. The Nyquist ISI-free criterion for $p(t)$ is defined as

$$p(rT_s) = \begin{cases} 1, & \text{if } r = 0 \\ 0, & \text{if } r = \pm1, \pm2, \ldots \end{cases} \tag{8}$$

An increase in the exponential power of a Nyquist-I pulse reduces the magnitude of the impulse response's output side lobes [10]. Therefore, we increase the exponential power of a linear pulse (LP) for $n = 1$ [11], to minimize PAPR. In general, the impulse response of the proposed hybrid filter becomes:

$$p_{u^*_{ELP}}(t) = e^{-\pi(\beta/2)(t/T)^2} sinc(t/T) sinc(\alpha t/T), \tag{9}$$

where u^*_{ELP} is a product of the exponential expression and the LP, while a sinc function in LP is considered as a FIR filter. Similarly, a sinc function of each Nyquist-I pulse (PLCP [1], PWL [6] and RC) is used as the FIR filter to form a hybrid filter design. Moreover, α is the roll-off factor with a range $0 \le \alpha \le 1$. The new design parameter β, with the range $0 \le \beta \le 1$, controls the phase and side lobe magnitude of the required pulse. We use that particular range of β because an increase in β narrows the main lobe of the impulse response, and sufficiently reduces the tail size, but at the expense of a poor frequency response and performance degradation in terms of SER for $0.6 \le \beta \le 1$. However, the performance of the proposed filter with respect to the frequency domain and SER can be improved by observing an optimal value

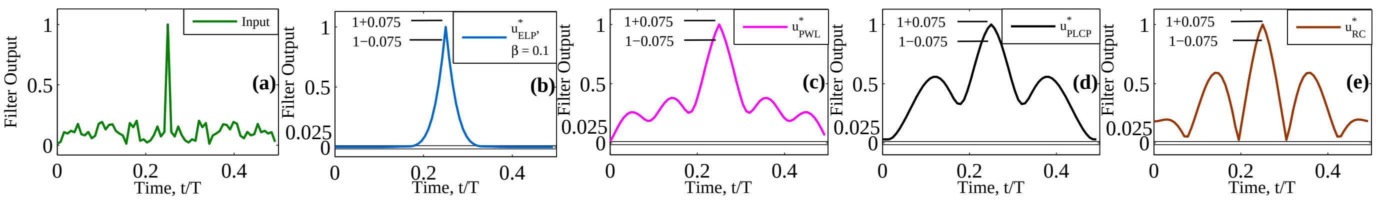

Fig. 2. Output mask with constrained side lobes up to ±0.025 and a main lobe peak of 1±0.075, and output of various filters at $\alpha = 0.35$: (a) input signal (b) optimal u^*_{ELP} (c) u^*_{PWL} (d) u^*_{PLCP} and (e) u^*_{RC}

of β within our defined range. In a trade-off between PAPR reduction and providing a low SER, we find an optimal value of β via theoretical and numerical simulations, as explained in the next section. For the sake of clarity and space, we do not discuss the frequency domain expression or behavior of the proposed hybrid filter. The pulse in (9) fulfills the Nyquist-I criterion from (8). We set $\alpha = 0.35$ to validate the comparison with the results given in [1], [6].

C. Optimality of u

Here, we describe the transmit pulse design problem in terms of the vector u. According to the EC optimal filter [12], the magnitude of the impulse response's output side lobes must adhere to the defined envelope constraints, as shown in Fig. 1. The expression in (6) can be written as

$$\tilde{\Psi}(t) = y(t)^T u, \tag{10}$$

where $y(t)^T = [p(t), ..., p(t-(N-1)T_s)]\ S$. We use the EC optimal filter design to formulate the hybrid filter design problem as follows:

$$\begin{aligned} \min_{u \in R^n} \quad & u^T S u, \\ \text{subject to} \quad & \varepsilon^-(t) \le y^T(t) u \le \varepsilon^+(t), \forall t \in \Omega, \end{aligned} \tag{11}$$

where Ω is an interval for the filter output that must fit in the envelope. The problem in (11) is a quadratic programming (QP) problem with a continuum of constraints. It has a strict convex quadratic cost and convex constraints [12]. The solution for (11) is unique, if it exists. One way to solve it is by approximating the continuum of constraints using a finite number of constraints:

$$\begin{aligned} \min_{u \in R^n} \quad & u^T S u, \\ \text{subject to} \quad & \varepsilon^-(t_i) \le y^T(t_i) u \le \varepsilon^+(t_i), \forall i = 0, ..., M-1. \end{aligned} \tag{12}$$

Satisfaction of the constraints at t_i, $i = 0, ..., M-1$ can be guaranteed (if a feasible solution exists). This approximate problem is a standard QP problem with a unique solution and can be solved using the active set strategy [12].

IV. Simulation Results and Discussion

For this section, we used various Nyquist-I pulses, the proposed ELP, PLCP [1], PWL [6], and RC, in combination with the FIR filter to form a hybrid filter. These hybrid filters are represented as u^*_{ELP}, u^*_{PLCP}, u^*_{PWL} and u^*_{RC}, respectively. We compared them with the conventional SC-FDMA (RC) in terms of side lobe reduction, PAPR, and SER using 16QAM. We performed a transmission of 10^6 system blocks in the SC-FDMA scheme using interleaved allocation for PAPR and SER. All the simulations were based on the parameters given in Table I, which comply with the ones given in [1], [6]. Additionally, we determined an optimal value for β.

TABLE I
Simulation Parameters

Parameter	Value
Modulation	16QAM
Number of subcarriers	512
Input data block size	128
Spreading factor, Q	4
Transmission bandwidth	20 MHz
Block oversampling	4
Subcarrier allocation	Interleaved
Uniform random data points	10^6
Roll-off factor, α	0.35

A pulse with small-magnitude side lobes requires less transmit power [5] and sufficiently reduces PAPR [1], [6]. Therefore, we relate a reduction in PAPR value and the transmitted power of a pulse with a decrease in the magnitude of the impulse response's output side lobes. The coefficients of various hybrid filters in terms of side lobe reduction are discussed via theoretical and numerical simulations, respectively. Theoretically, we consider the compression of a discrete-time input signal and the output response of the hybrid filter to the input signal, as in [8]. However, the overall operation matrix in this work and [8] are different because of the modulation scheme. Furthermore, the QP problem in (12) is analyzed for various hybrid filters where the transmission matrix is based on the modulation scheme and impulse response of the hybrid filter, as shown in Fig. 2. The input and output signals are restricted within lower and upper bounds with allowable side lobes of ±0.025 and a main lobe peak of 1±0.075, similar to [8]. We consider a 16-tap FIR filter (because of the high-order modulation scheme) in combination with the various Nyquist-I pulses using the approximate solution given in (12) and applied in the MATLAB optimization toolbox for QP. In Fig. 2, the proposed hybrid filter response to the input signal stays within the boundary of the mask, and a sufficient reduction occurs in the magnitude of the side lobes. On the other hand, the responses of the existing hybrid filters contravene the envelope constraints by introducing large-magnitude side lobes. Numerically, we used the combination of the modulation scheme, FIR filter, and a Nyquist-I pulse to evaluate the various hybrid filters in terms of reduction in side lobe magnitude, as shown in Figs. 3(a, b). We considered different values of β as examples, and $u^*_{ELP} = \{0.5, 1\}$ had the smallest magnitude side lobes.

The PAPR performance of various hybrid filters is shown in

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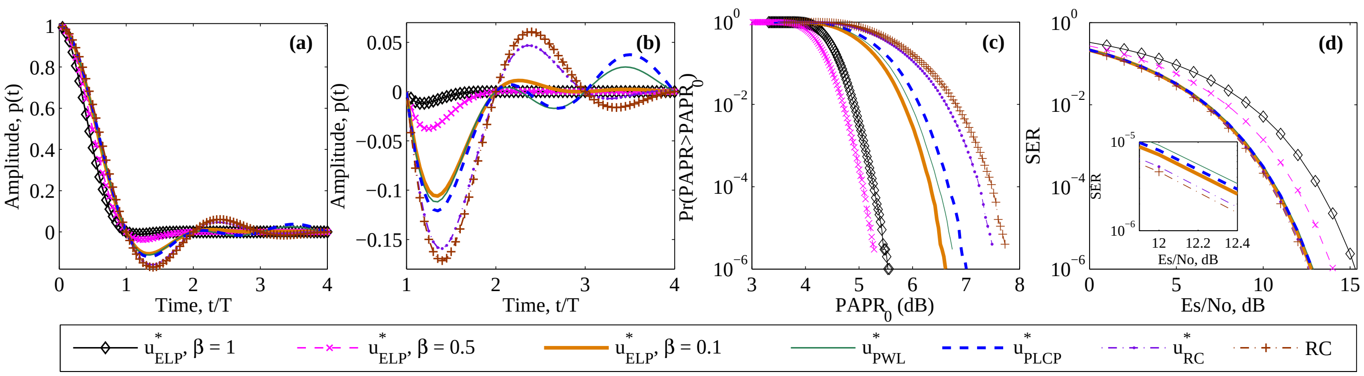

Fig. 3. Comparison of various filters applied in SC-IFDMA system using 16QAM, at α = 0.35: (a) Impulse response (b) Magnitude of the impulse response's output side lobes (c) CCDF of PAPR for optimal value selection of β value (d) SER performance.

Fig. 3(c). The total PAPR on the transmitter side is determined by the combination of the modulation scheme, FIR filter, and a Nyquist-I pulse. The resultant data are presented as an empirical complementary cumulative distribution function after calculating the PAPR of each block. Here, $\beta = 0.5$ shows the lowest PAPR value, and (9) is good enough to reduce transmit power and PAPR due to small magnitude side lobes.

We considered the tic and toc commands in MATLAB to evaluate the time complexity by estimating the average elapsed time of the convolution operation, as in [13]. In this work, we considered the same computer system specifications used in [13], and the convolution matrix is based on the modulated symbols and the impulse response of the hybrid filter. The average elapsed time of the convolution operation for u^*_{PWL}, u^*_{ELP}, u^*_{RC}, u^*_{PLCP} are 2.07 x 10^{-4} sec, 2.15 x 10^{-4} sec, 2.17 x 10^{-4} sec, and 2.20 x 10^{-4} sec, respectively. The u^*_{PWL} achieved the lowest average elapsed time because it uses the Nyquist-I given in [6], which is based on simple arithmetic operations. However, the u^*_{ELP} in (9), achieved a lower average elapsed time than u^*_{RC} and u^*_{PLCP}, and the proposed filter can be effective in resource-limited systems.

Next, we evaluated various hybrid filters in terms of SER for an SC-IFDMA scheme over the ITU pedestrian outdoor channel with AWGN and MMSE equalization, as in [6]. In Fig. 3(d), the conventional SC-FDMA (RC) shows lowest SER. However, $\beta = 0.1$ is the optimal value because it provides the optimal trade-off between PAPR and SER performance, and it shows better performance than the existing filters in terms of side lobe reduction, PAPR, and SER. In general, there is an optimal β for every roll-off factor and transmission scheme, although it might not be unique. Since the conventional SC-FDMA (RC) shows less SER than proposed filter, a technique called subcarrier combining can be used to improve SER [14].

V. Conclusion

We proposed a hybrid filter to optimize the impulse response in terms of PAPR reduction on the transmitter side for the SC-IFDMA scheme. The design of the proposed filter is based on an FIR filter and a Nyquist-I pulse. We used an EC optimal filter to create a hybrid filter design problem and proposed a novel family of Nyquist-I pulses, the exponential linear pulse (ELP). The ELP has a new design parameter, β. The optimal β of the proposed filter (to reduce PAPR in SC-IFDMA) was 0.1 for $\alpha = 0.35$. Theoretical and numerical simulations showed that the proposed hybrid filter outperformed existing filters in terms of side lobe magnitude reduction, PAPR, and SER, and it has less computationally complex impulse response expression than existing filters. However, SER of the proposed filter can be further improved using subcarrier combining technique.

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